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An Example of Modified Smith Predictor Design with QFT

Per-Olof Gutman

- The example – a double integrator with uncertain gain and delay
- A straightforward lead-lag design using QFT
- The Modified Smith Predictor
- Loop shaping for (the plant + the Modified Smith Predictor)
- A successful design
- Discussion

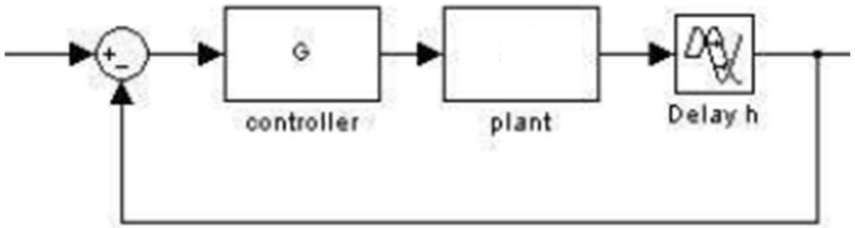
The example – a double integrator plant with uncertain gain and delay

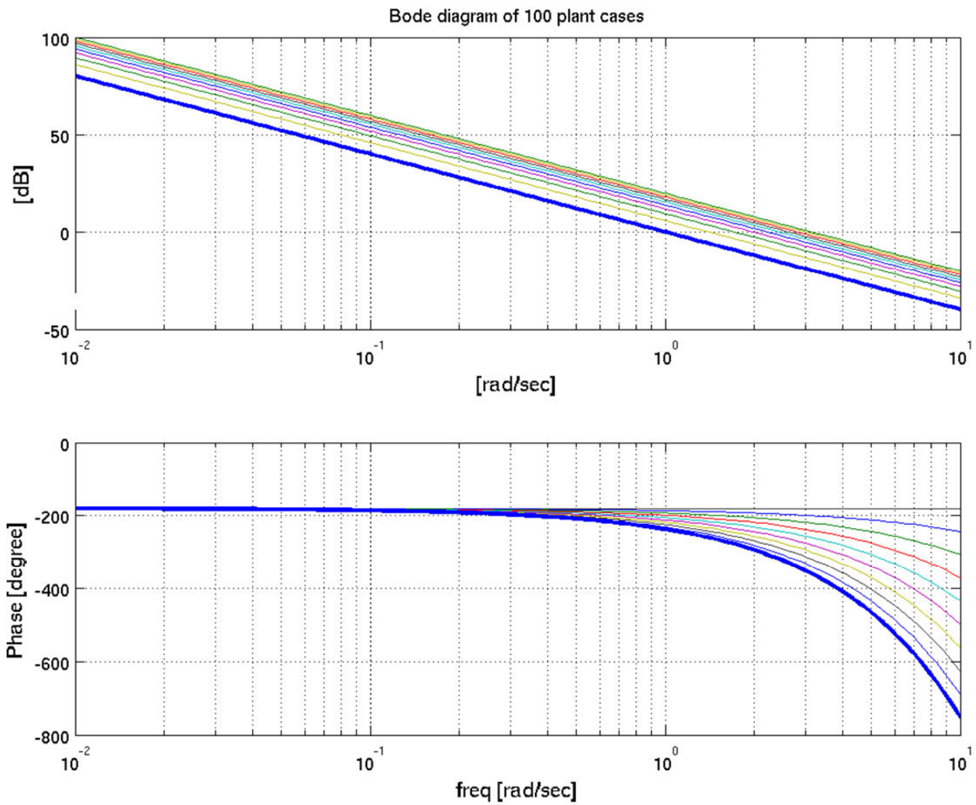
$$P(s) = \frac{k}{s^2} e^{-sh}$$

$$k \in [1, 10] \quad h \in [0, 1]$$

Specification:

$$|S| = \left| \frac{1}{1 + PG} \right| \le 2$$





The example – a double integrator plant with uncertain gain and delay

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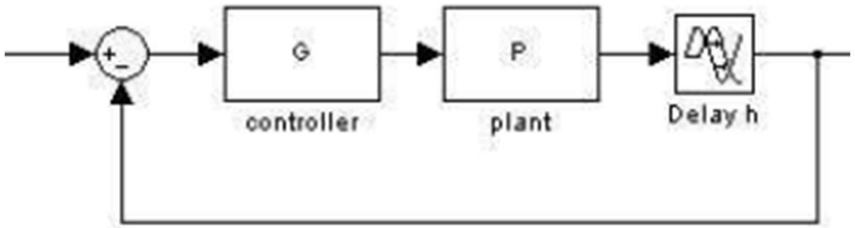
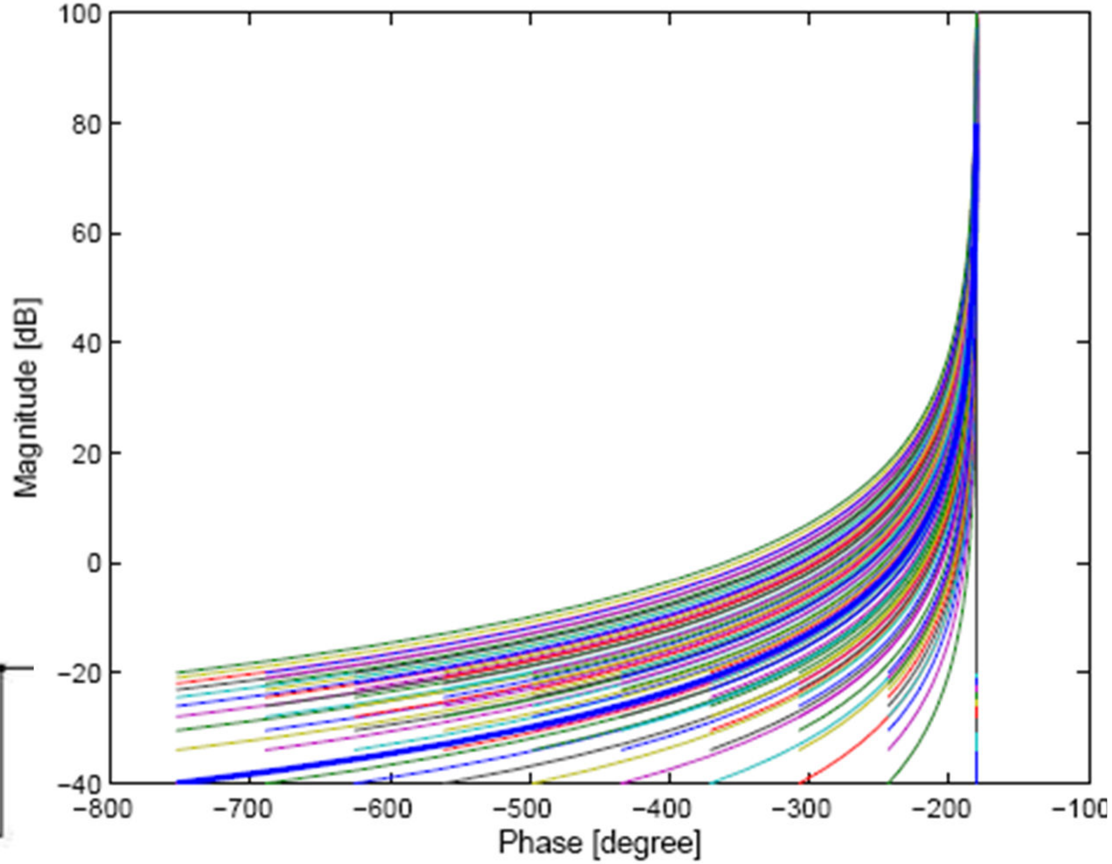
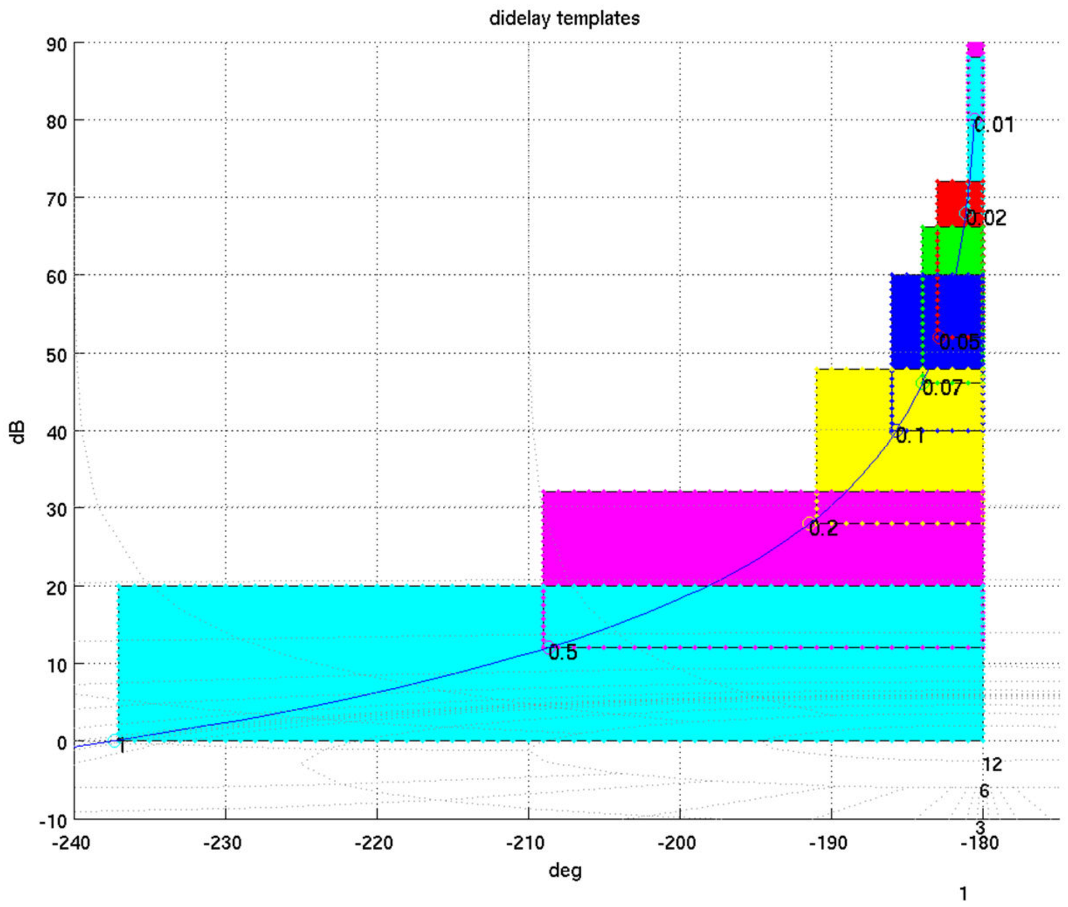




Templates of the double integrator plant



The arbitrary nominal plant is chosen as

$$P_{nom}(s) = \frac{e^{-s}}{s^2}$$

Note: For a plant w/o uncertainty, and delay = *h* [sec],

$$max(\omega_c) = \frac{1}{h} \text{ [rad/s]}$$

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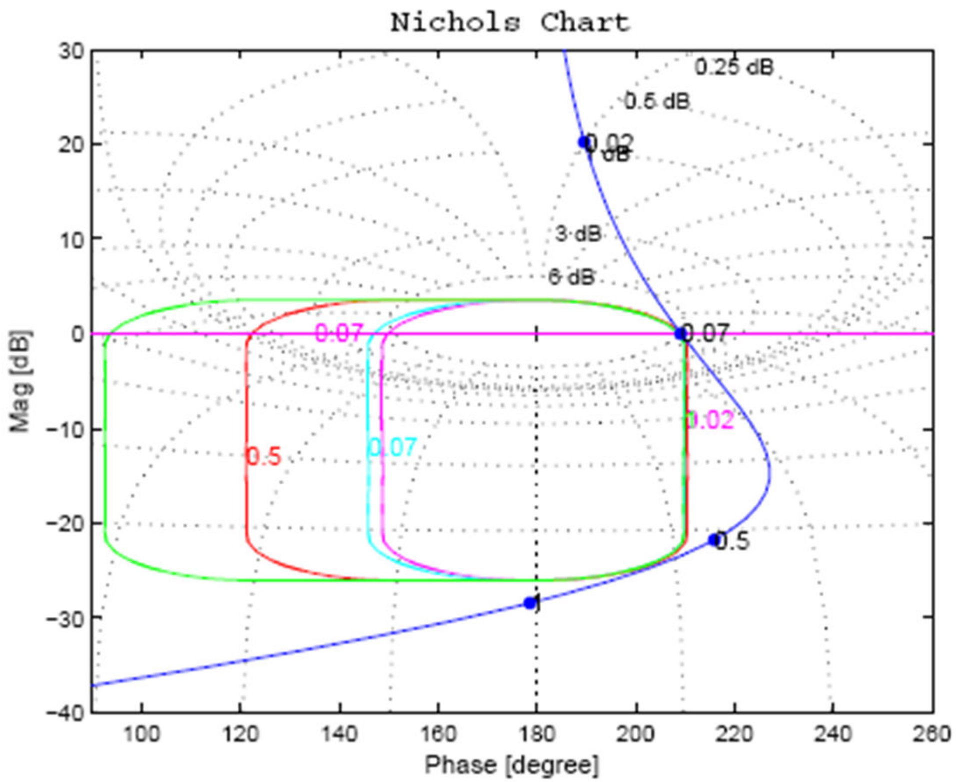
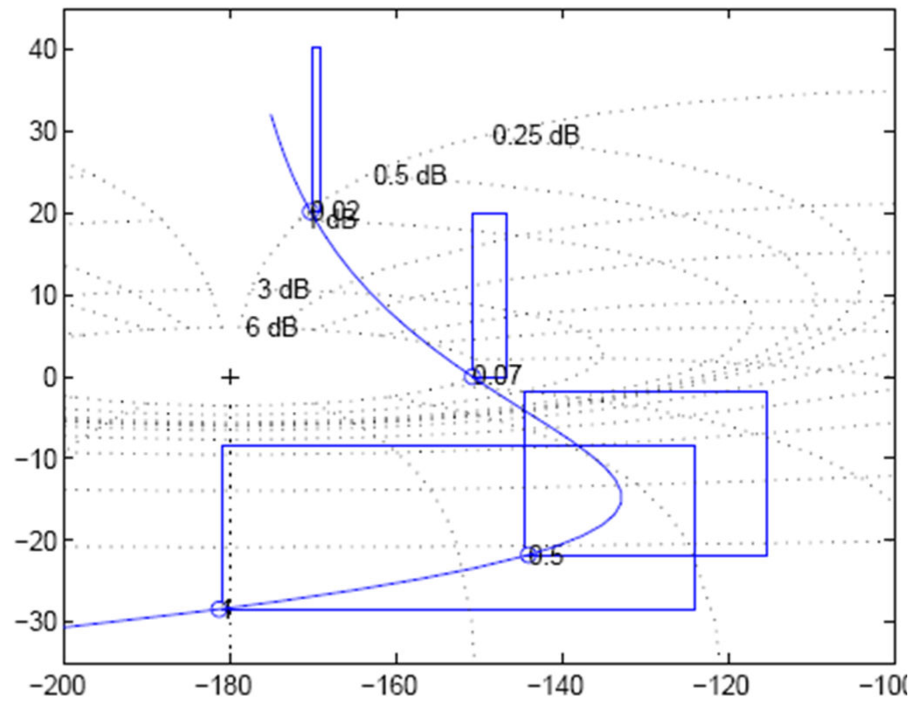
A straightforward lead-lag design

$$g304(s) = \frac{0.965(s+0.1)(s+0.3878)}{(s+0.4)(s+2.327)(s+10)}$$





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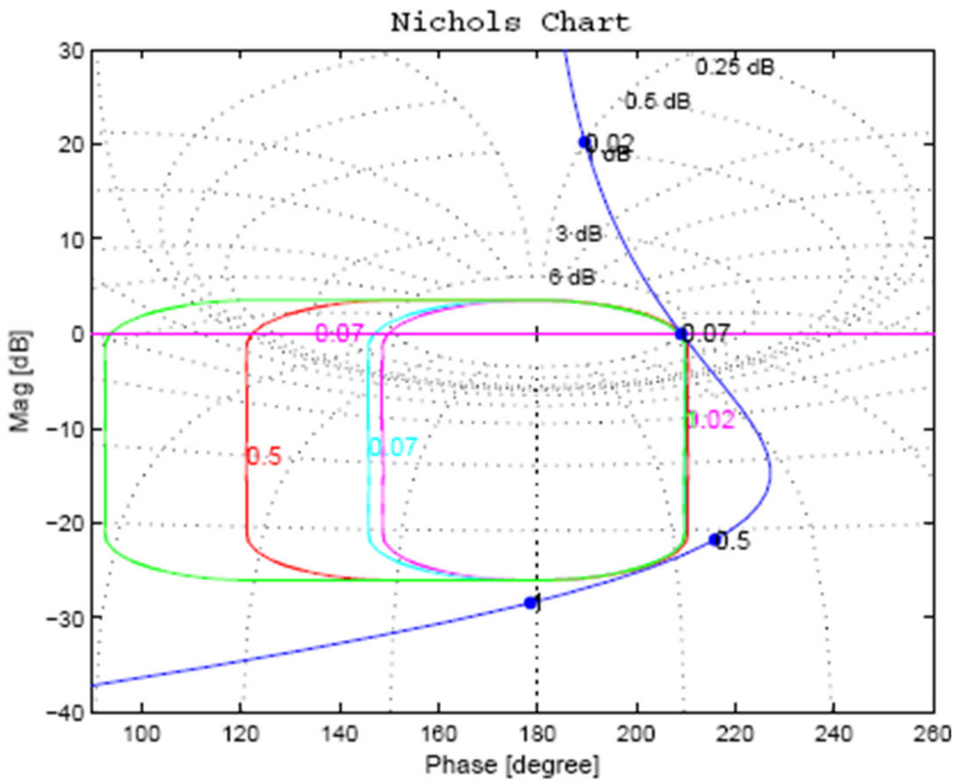
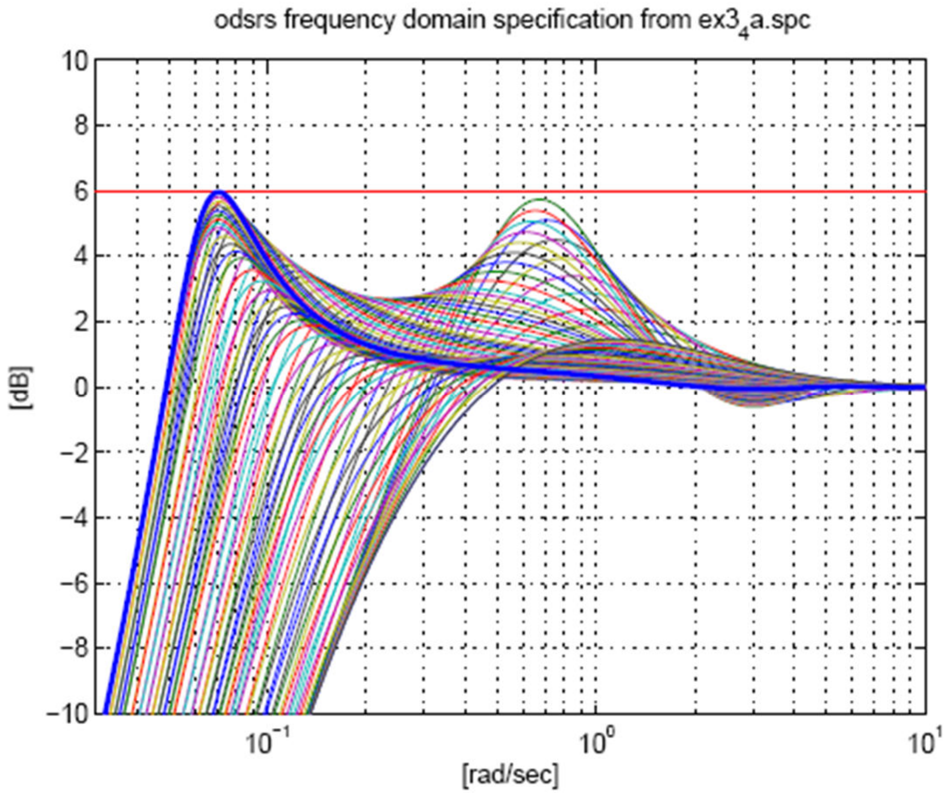
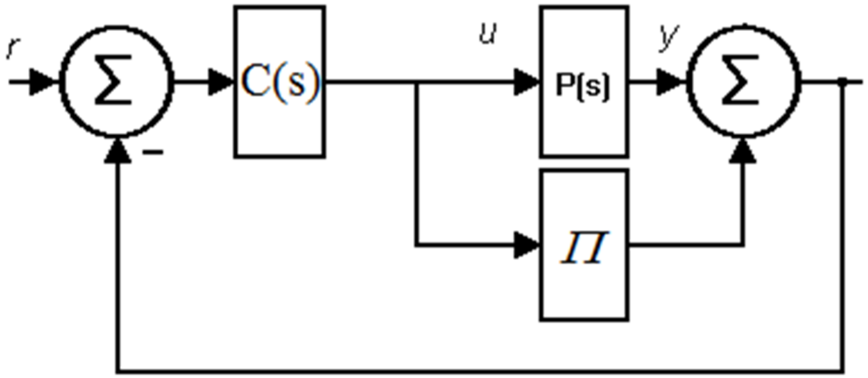
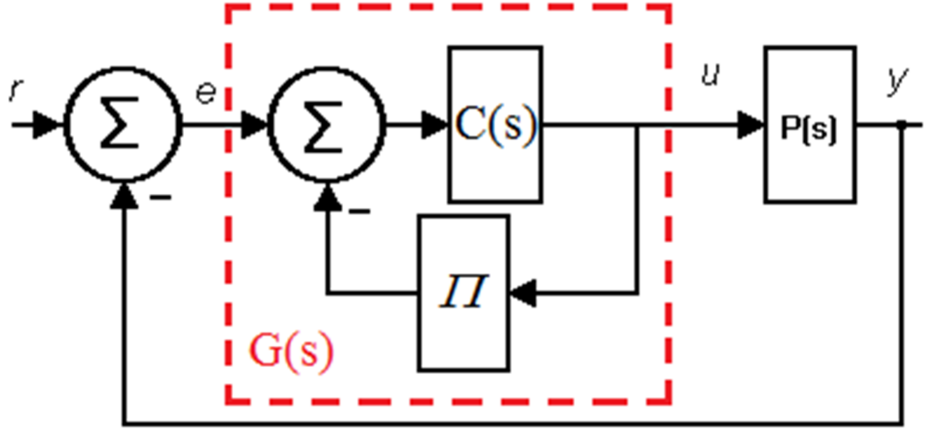




The Modified Smith Predictor





For the unstable plant

$$P(s) = \frac{k}{s^2} e^{-hs}$$

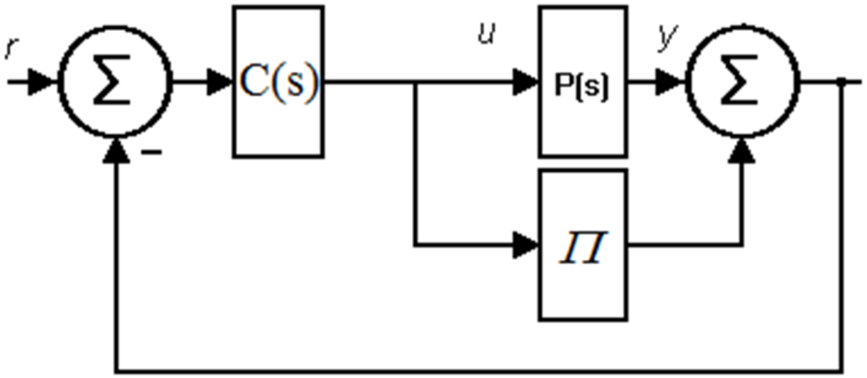
one may choose

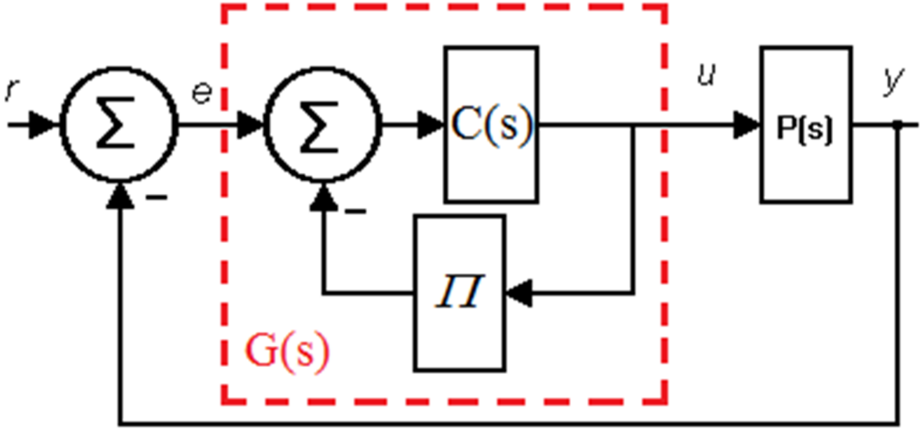
$$\pi(s) = \frac{k_s}{s^2}\left(1 - h_s s + \frac{{h_s}^2 s^2}{2} - e^{-h_s s}\right)$$

where k_s, h_s are chosen as one of the possible values of k, h.

$$G(s) = \frac{C(s)}{1 + C(s)\pi(s)}$$

Loop shaping with MSP





$$G(s) = \frac{C(s)}{1 + C(s)\pi(s)}$$

While PG=-1 ≡ $C(P+\pi)$=-1,

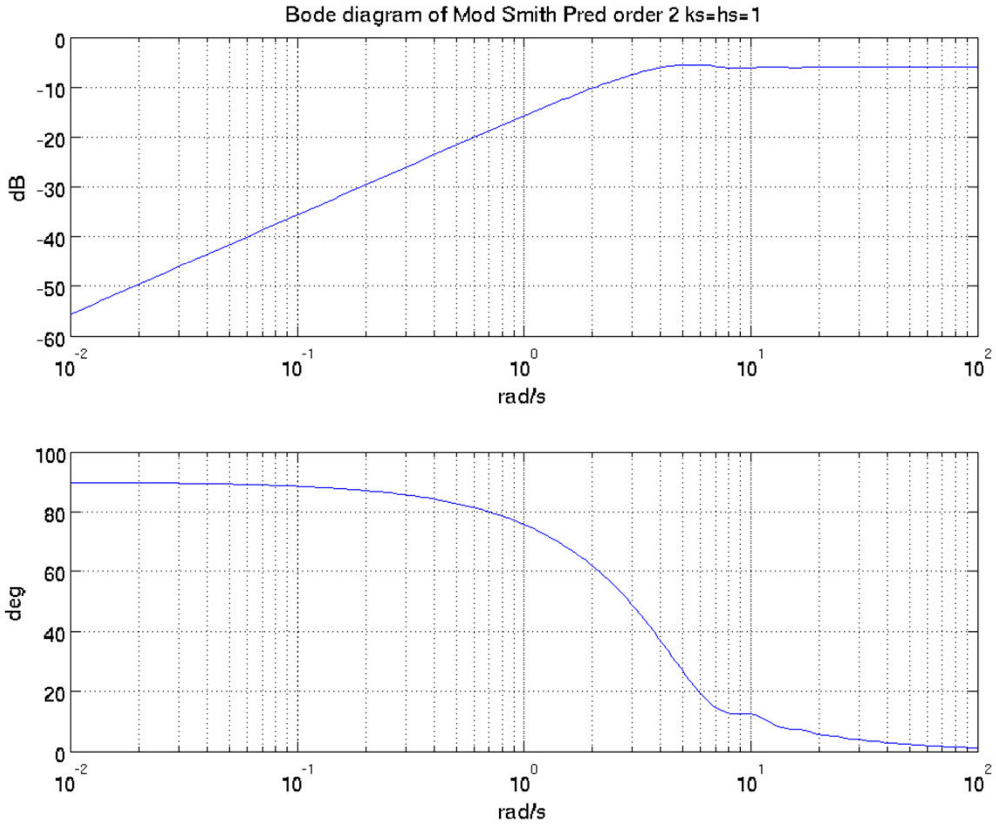
$$\left|\frac{1}{1 + \frac{CP}{1 + C\pi}}\right| \neq \left|\frac{1}{1 + C(P + \pi)}\right|$$

So, one should loop shape PG by changing factors of C within G, but that is not intuitive. So, let us try to loop shape $C(P+\pi)$ under the "specification"

$$\left|\frac{1}{1 + C(P + \pi)}\right| \leq 2$$

and see what happens …

Bode Diagram of MSP



Note that for low frequencies, the gain of MSP is low and the gain of P is high, so for low frequencies it holds that $C(P+\pi) \approx CP$, and $\left|\frac{1}{1+\frac{CP}{1+C\pi}}\right| \approx \left|\frac{1}{1+C(P+\pi)}\right|$ so there is some hope that loop shaping C(P+π) under the "specification"

$$\left|\frac{1}{1+C(P+\pi)}\right| \leq 2$$

will yield something useful…

$$\pi(s) = \frac{k_s}{s^2}\left(1 - h_s s + \frac{{h_s}^2 s^2}{2} - e^{-h_s s}\right) \text{ with } k_s = h_s = 1$$

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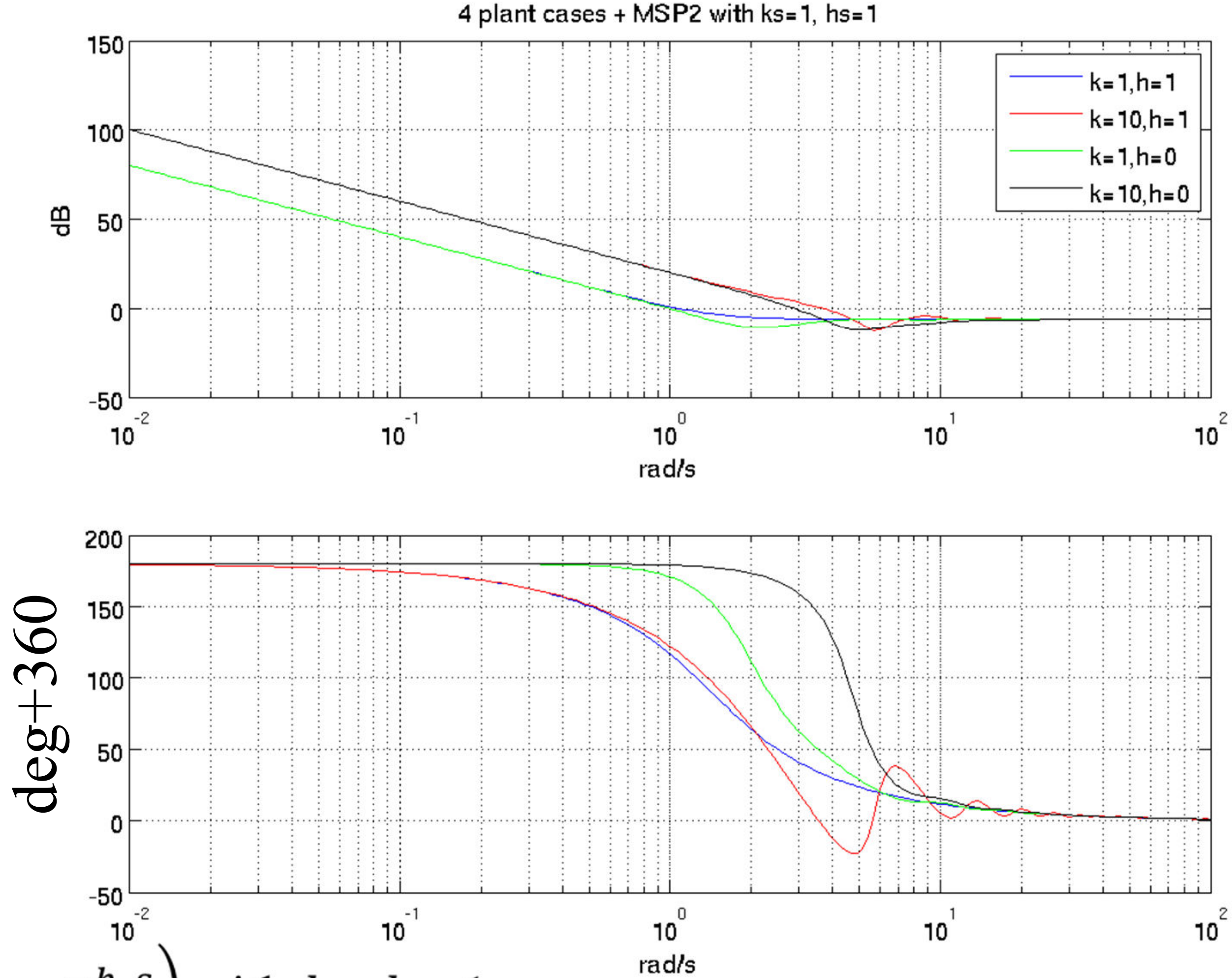
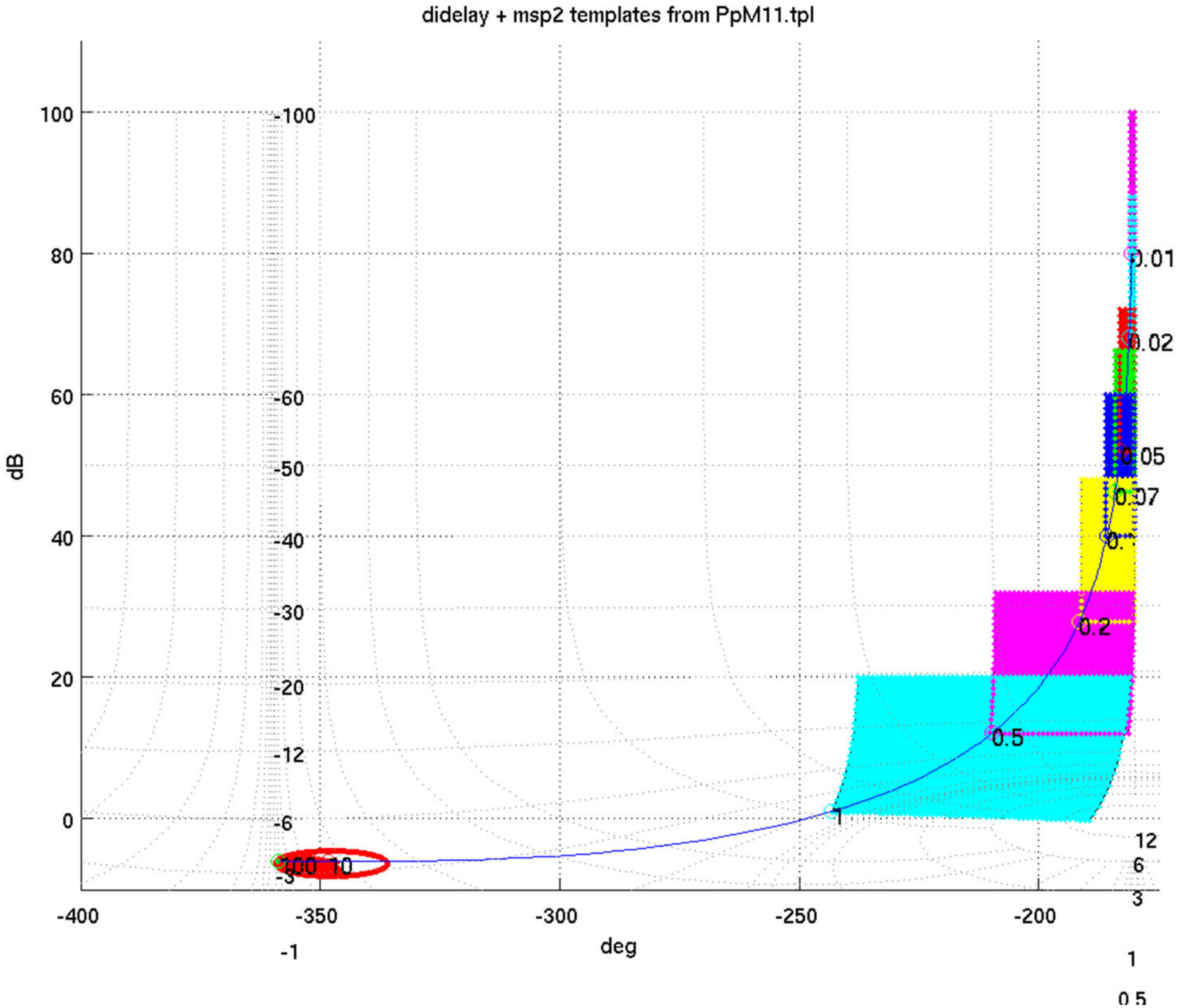
P(s) + π(s)

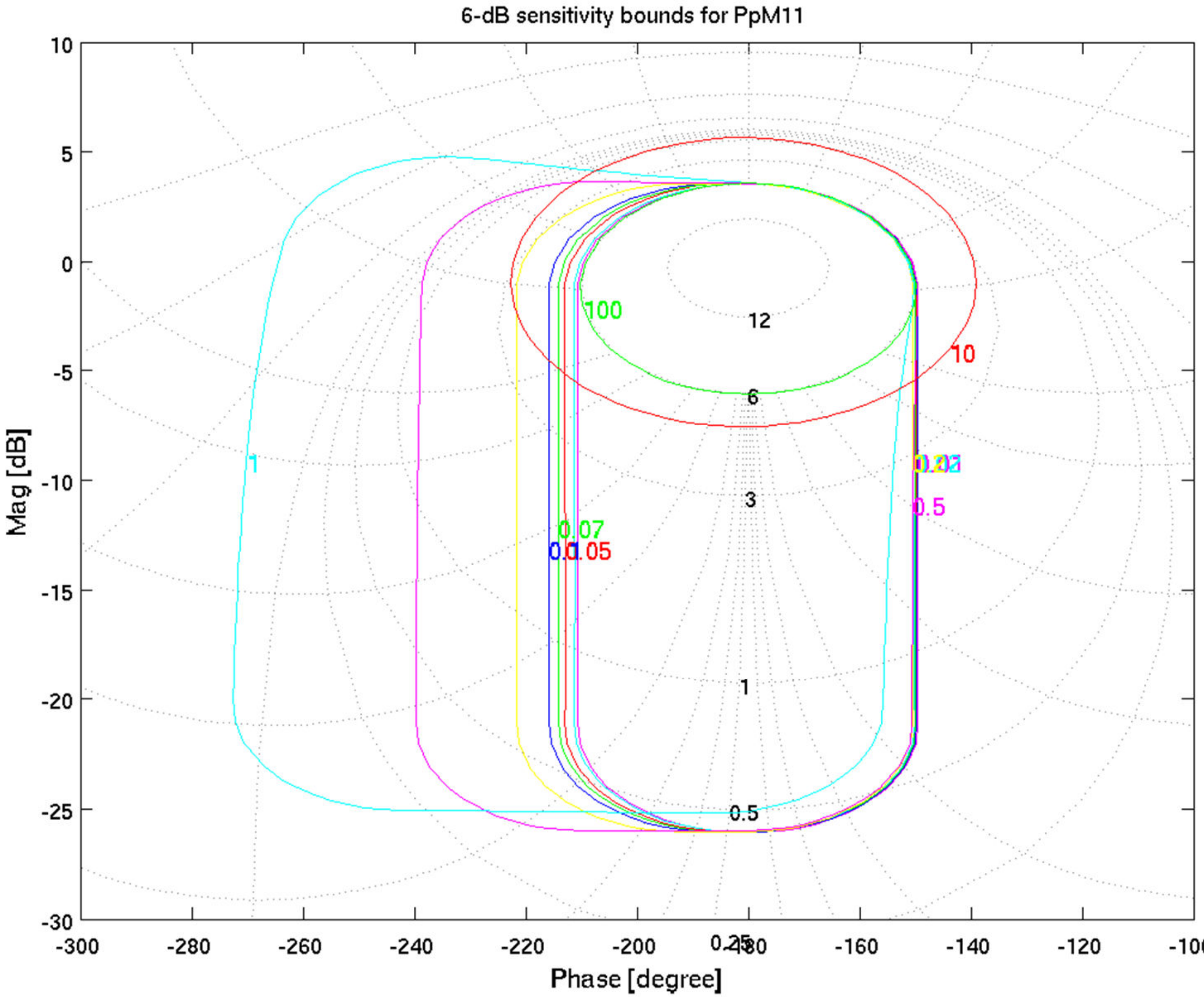


$$\pi(s) = \frac{k_s}{s^2}\left(1 - h_s s + \frac{{h_s}^2 s^2}{2} - e^{-h_s s}\right) \text{ with } k_s{=}h_s{=}1$$

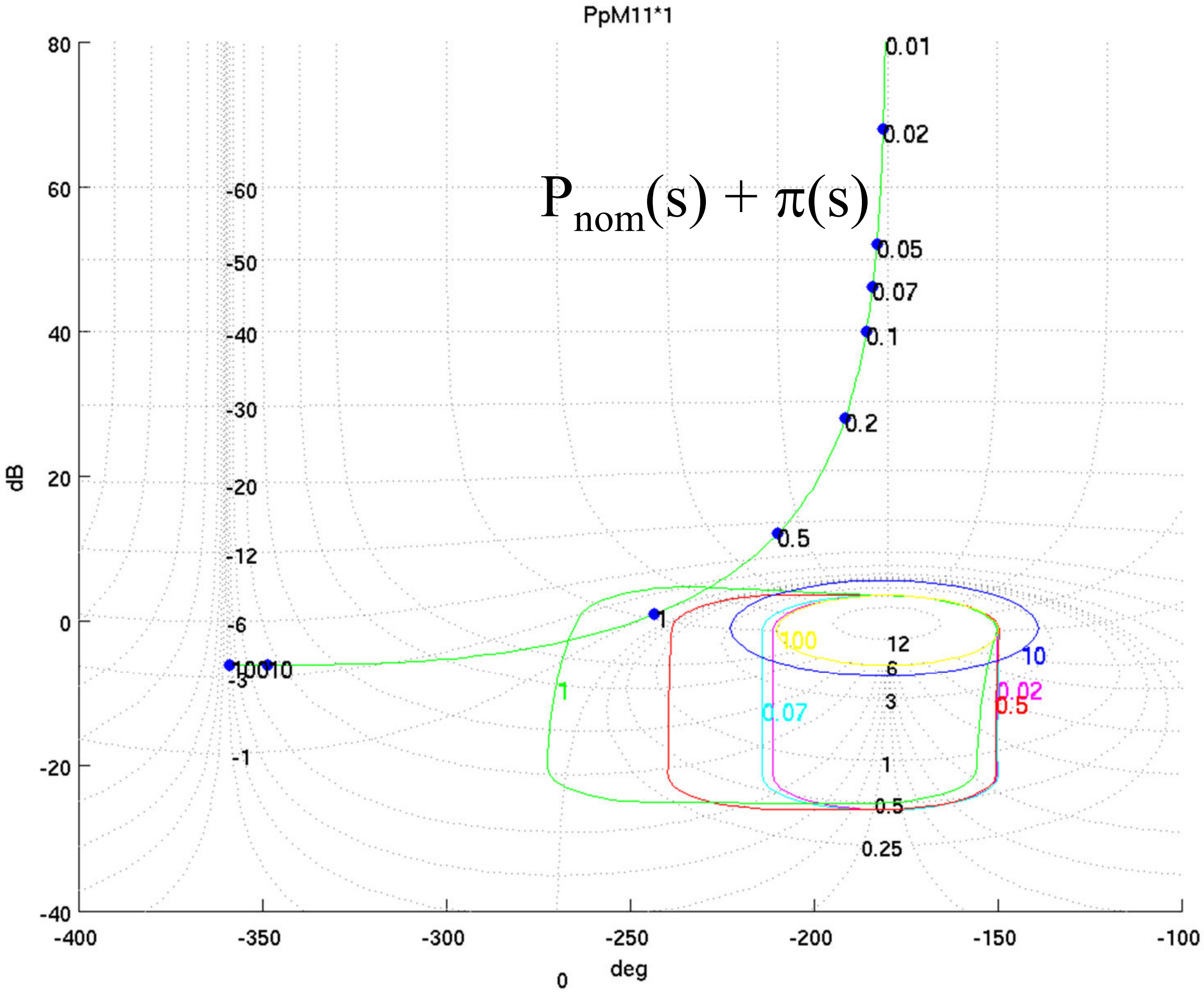
Templates of P(s) + π(s)



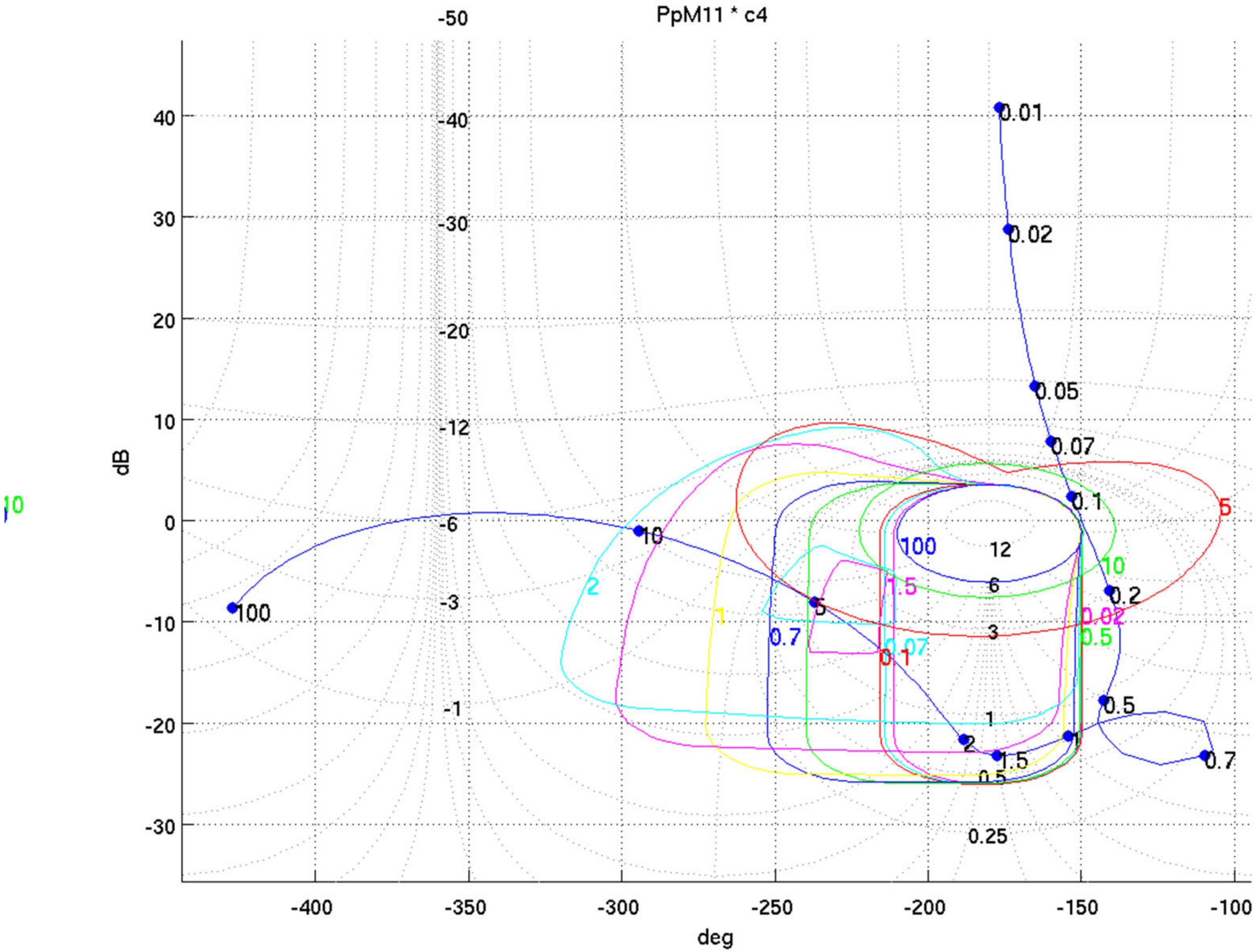
Bounds for $C(P_{nom}(s) + \pi(s))$



Bounds for $C(P_{nom}(s) + \pi(s))$



Design $C_4(P_{nom}(s) + \pi(s))$



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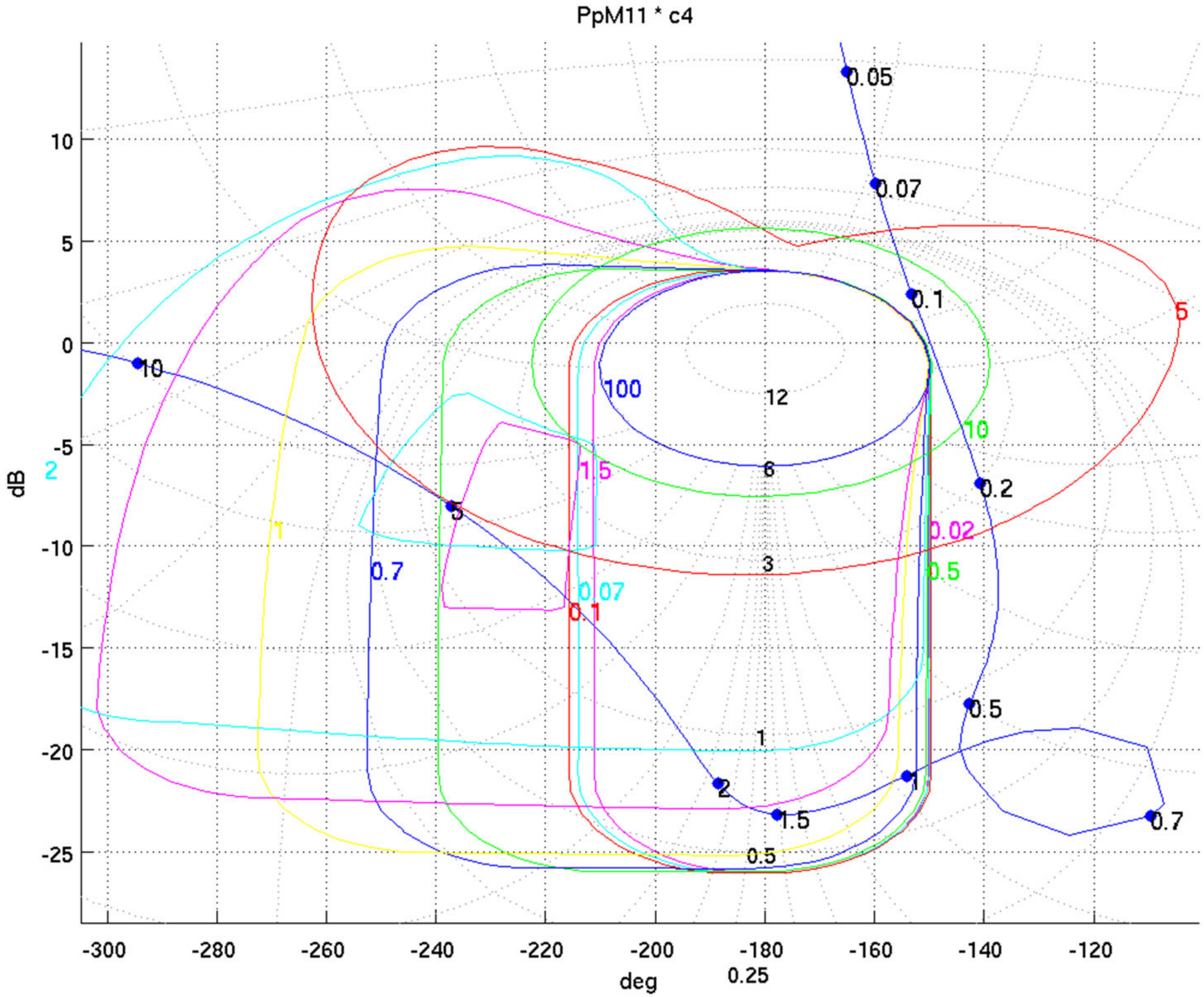
Design $C_4(P_{nom}(s) + \pi(s))$



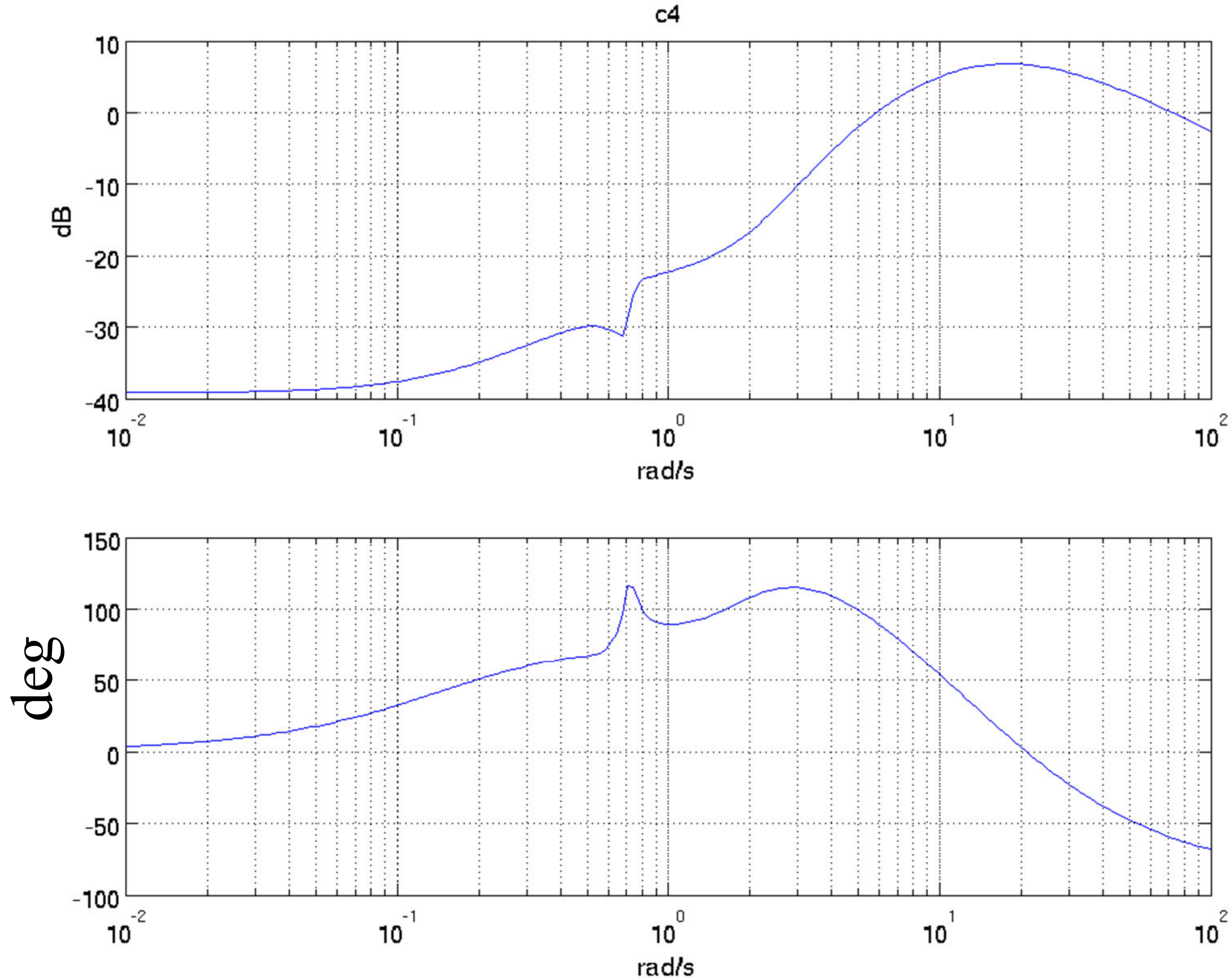
$C_4(s)$

$$C_4(s) = 0.011\,\frac{\left(1+\frac{s}{0.15}\right)\left(1+\frac{2\cdot 0.074s}{0.598}+\frac{s^2}{0.598^2}\right)}{\left(1+\frac{s}{2.327}\right)\left(1+\frac{s}{12}\right)\left(1+\frac{2\cdot 0.08s}{0.6}+\frac{s^2}{0.6^2}\right)}\cdot$$

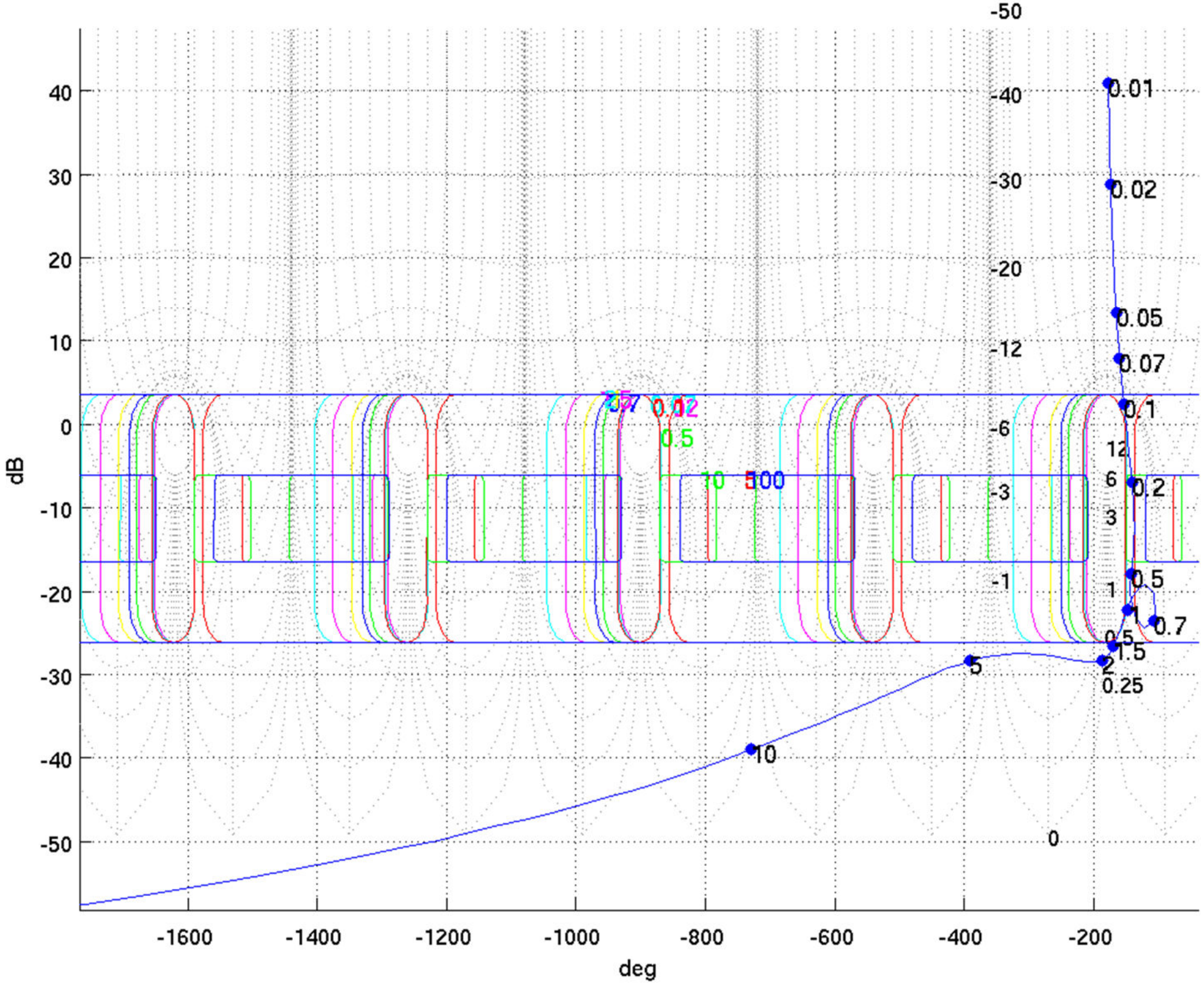
$$\cdot\,\frac{\left(1+\frac{2\cdot 0.06s}{0.69}+\frac{s^2}{0.69^2}\right)\left(1+\frac{2\cdot 0.6s}{1.9}+\frac{s^2}{1.9^2}\right)}{\left(1+\frac{2\cdot 0.09s}{0.725}+\frac{s^2}{0.725^2}\right)\left(1+\frac{2\cdot 1.2s}{11}+\frac{s^2}{11^2}\right)} \rightarrow$$

$$G_4(s) = \frac{C_4(s)}{1+C_4(s)\pi(s)}$$

Bode diagram of $G_4(s)$



Result $G_4(s)P(s)$



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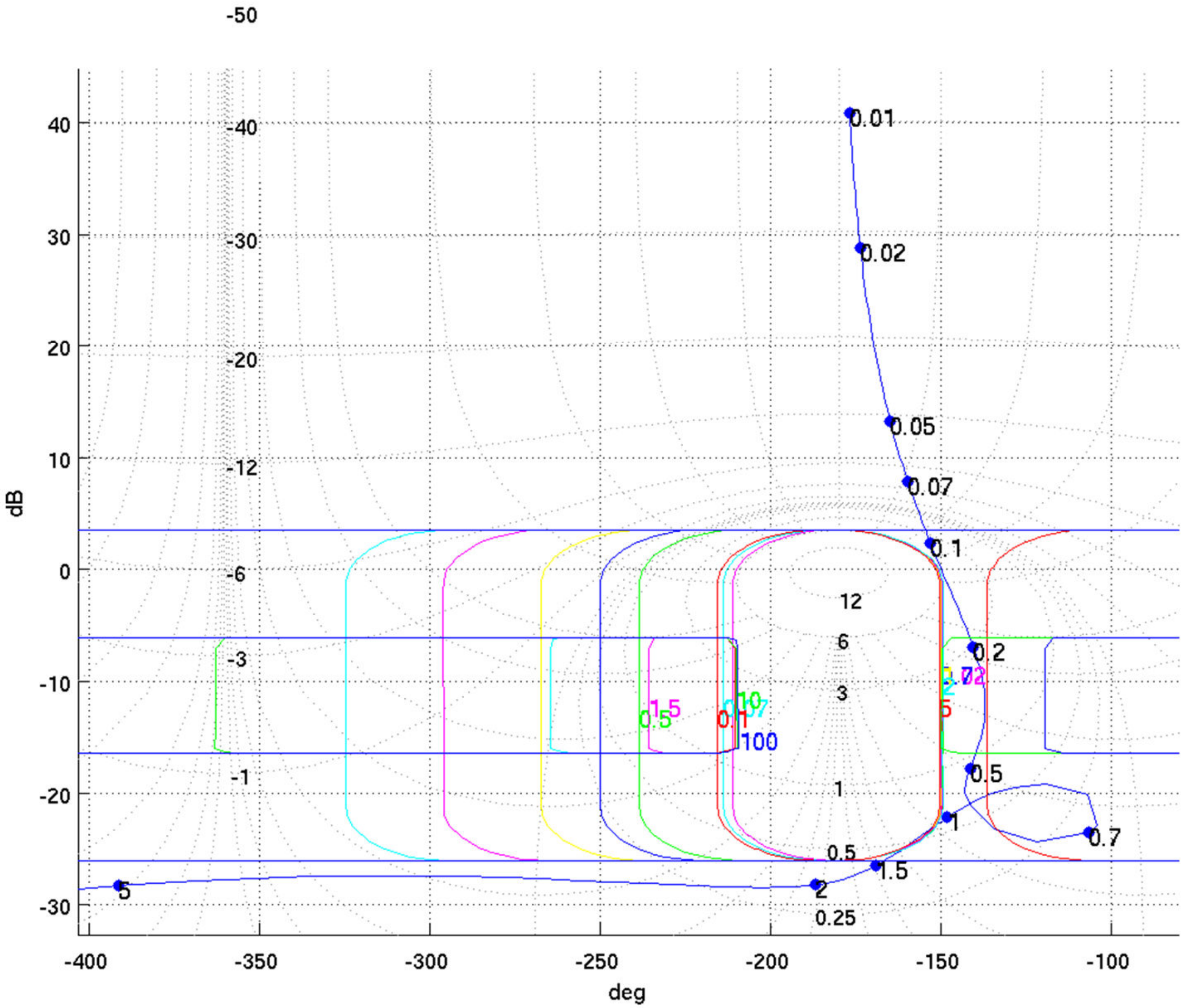

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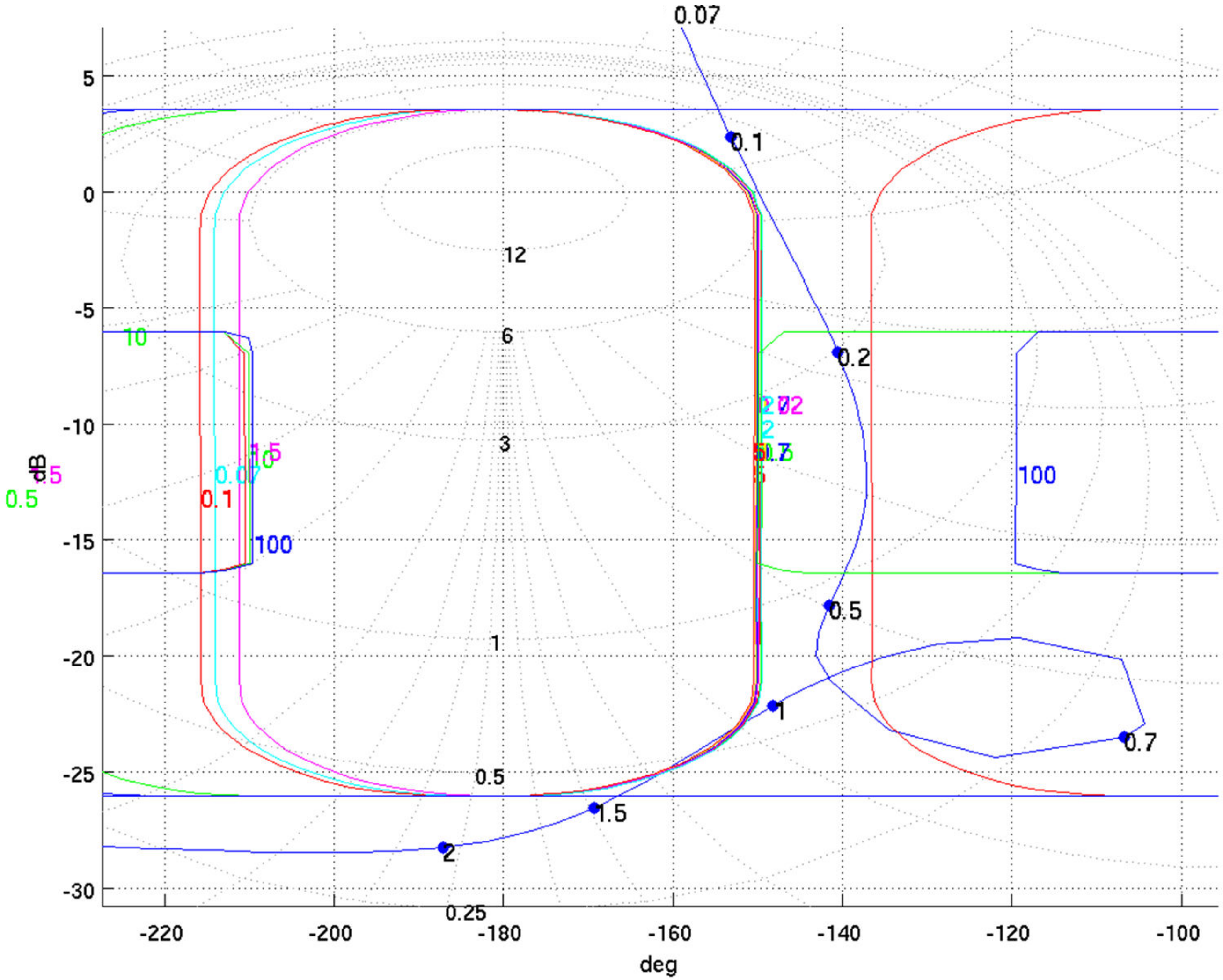
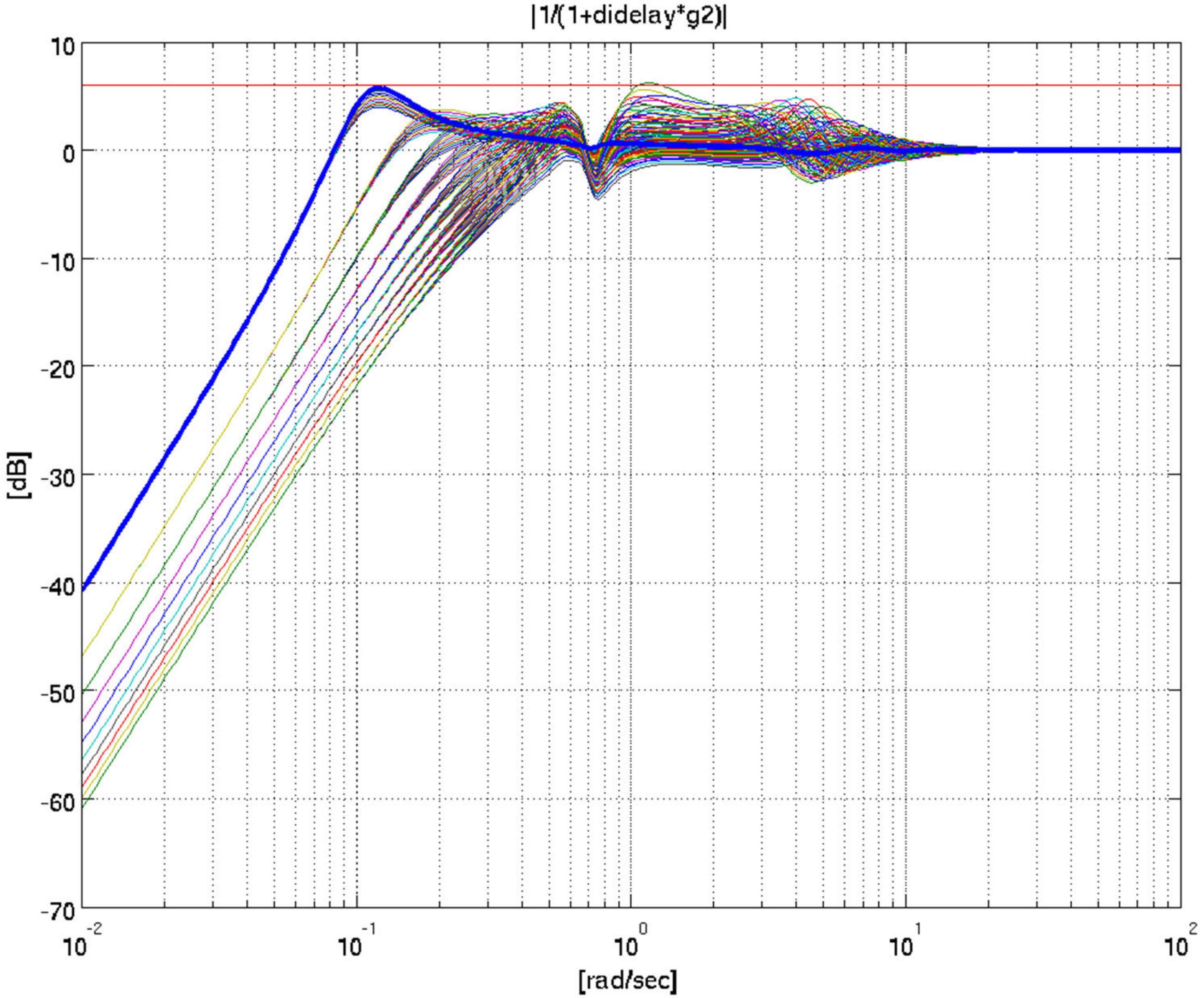
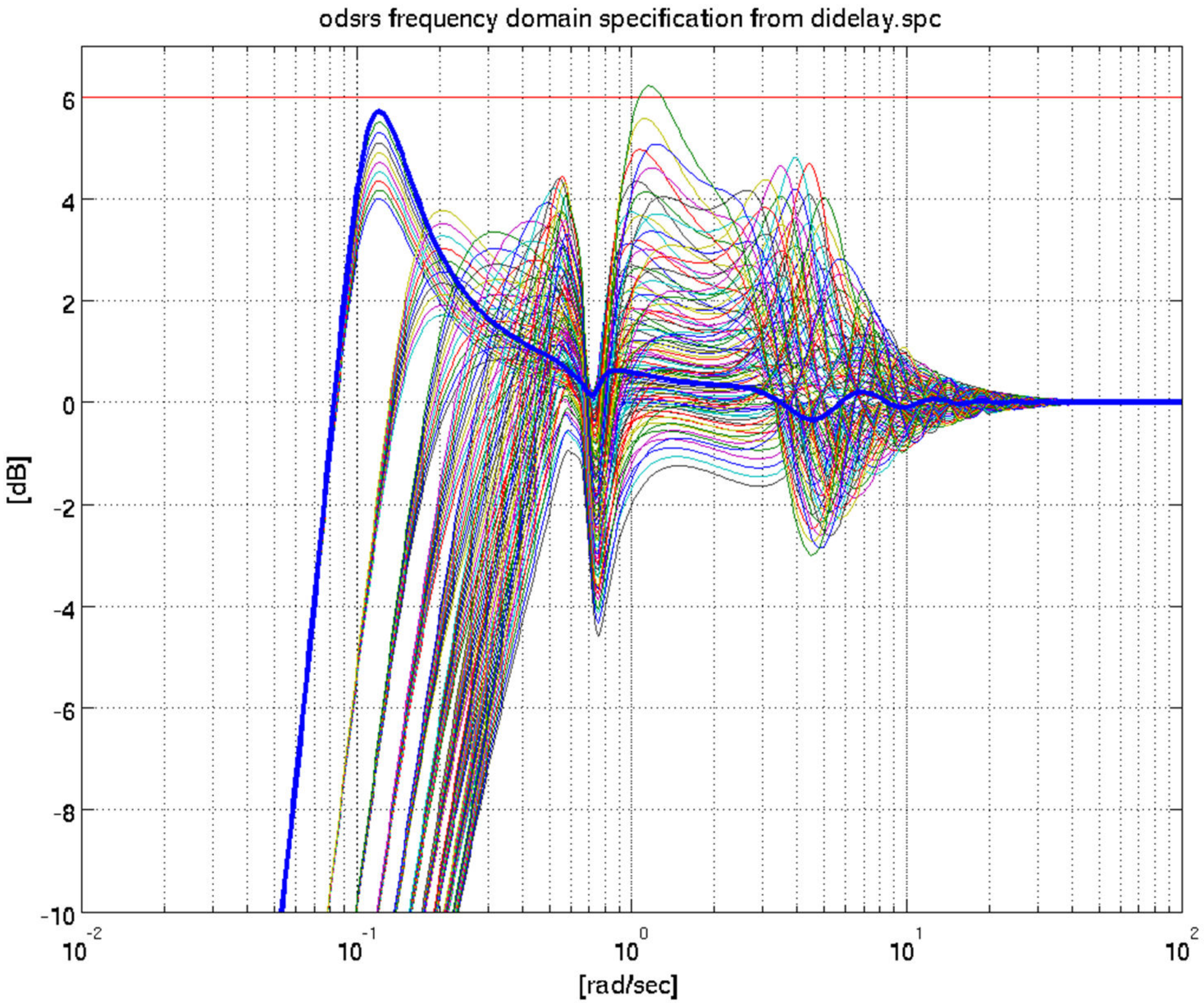
Result $G_4(s)P(s)$



$|1+G_4(s)P(s)|$


|1/(1+didelay*g2)|
10
0
-10
-20
-30
-40
-50
-60
-70
[dB]
10^-2
10^-1
10^0
10^1
10^2
[rad/sec]

$|1+G_4(s)P(s)|$


odsrs frequency domain specification from didelay.spc
6
4
2
0
-2
-4
-6
-8
-10
[dB]
10^-2
10^-1
10^0
10^1
10^2
[rad/sec]

Discussion

- It was easier to loop shape with the MSP in parallel with the plant than with no MSP at all, and higher bandwidth was achieved
- It is more difficult to loop shape when the MSP is postulated to be inside the feedback of the controller, although that is the correct way to do it
- It was not tested how the choice of the MSP parameters influences the design process.