Distributed feedforward control of wind farms: prospects and open problems

Maxim Kristalny

Department of Automatic Control, LTH

Joint work with Daria Madjidian and Anders Rantzer

Partially based on PhD advised by Leonid Mirkin

Haifa, April 2011

Preliminaries

Wind power constitutes

- 2% of power production over the world
- more than 20% of power production in Denmark

Demand in wind power continues to grow

Preliminaries

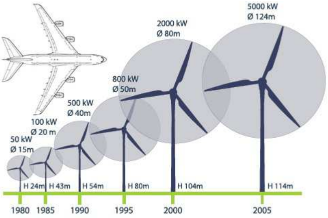
Wind power constitutes

- 2% of power production over the world
- more than 20% of power production in Denmark

Demand in wind power continues to grow

Growth of turbines dimensions

- complex systems
  producing up to 7.5 MW
- structural loads
  become a central issue



by L. Y. Pao and K. E. Johnson, ACC 2009

Preliminaries

Wind power constitutes

- 2% of power production over the world
- more than 20% of power production in Denmark

Demand in wind power continues to grow

Construction of large scale wind farms (economically beneficial)

- today each turbine in a farm is controlled separately

Are there better options?

Distribution of power between turbines

Wake effects - upwind turbine partially block wind flow

Nowadays, each turbine tries to extract the maximum

Is this an optimal way?

- Decrease in upwind turbine power may increase overall power

(D. Madjidian and A. Rantzer, 2011)

Load reduction in turbines

Load control is essential in modern wind turbines

Load reduction contributes to cost efficiency of turbines

Load reduction in turbines located in farms

The main idea:

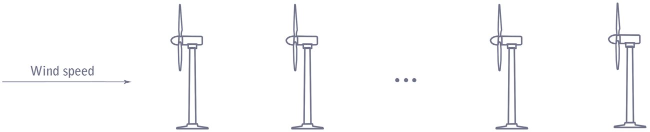
Accounting for and communicating with neighbours might be beneficial

- sharing wind speed measurements
- cooperation in terms of power production

Load reduction in turbines located in farms

The main idea:

Accounting for and communicating with neighbours might be beneficial



- sharing wind speed measurements
- cooperation in terms of power production

Wind farm models contain delays due to wind propagation

Distributed control is required due to large scale and modularity demand

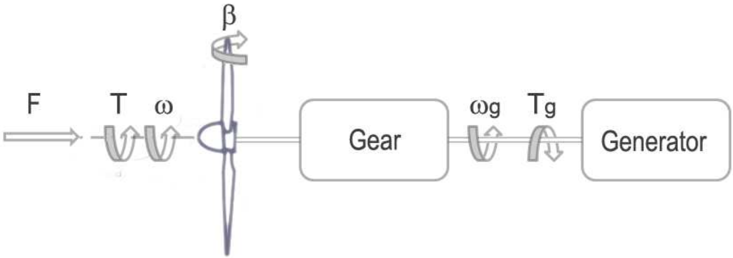
Outline

1. Preview control of individual turbine
   - individual turbine model
   - problem solution
   - simulation results
2. Cooperative control of entire farm
   - wind farm model
   - quadratic invariance
   - solution
   - preliminary simulation results

Outline

1. Preview control of individual turbine
2. Cooperative control of entire farm

Turbine model

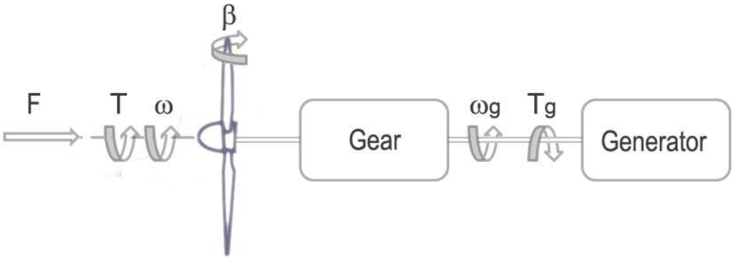


T - rotor torque
F - thrust force
ω - rotor speed

β - pitch angle
ω_g - generator shaft speed
T_g - generator torque

Produced power: $P = T_g \cdot \omega_g$

Turbine model



T - rotor torque
F - thrust force
ω - rotor speed

β - pitch angle
ω_g - generator shaft speed
T_g - generator torque

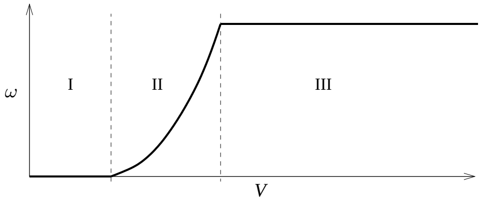
Produced power: $P = T_g \cdot \omega_g$

Internal controller adjusts β and T_g to maintain operating point

Standard internal controller

Several logical switches and PIDs

Maintains rotor speed according to:



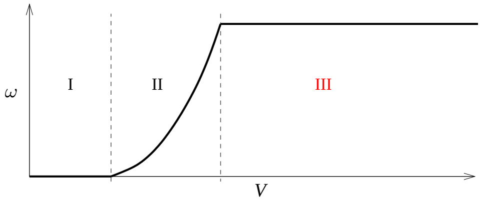
Three operating regions:

- I Wind speed too low to produce power (the rotor is frozen)
- II Energy available from wind is less then required ($\beta = 0$, ω depends on wind)
- III Energy available from wind is more then required (rated ω, β depends on wind)

Standard internal controller

Several logical switches and PIDs

Maintains rotor speed according to:



Three operating regions:

- I Wind speed too low to produce power (the rotor is frozen)
- II Energy available from wind is less then required ($\beta = 0$, ω depends on wind)
- III Energy available from wind is more then required (rated ω, β depends on wind)

Assume operation in region III

Standard internal controller

Keeping standard internal controller is restrictive

- prevents direct access to pitch and generator torque
- leaves power reference as the only control signal

At the same time, this

- simplifies the problem
- facilitates experiments in existing wind farms

Standard internal controller

Keeping standard internal controller is restrictive

- prevents direct access to pitch and generator torque
- leaves power reference as the only control signal

At the same time, this

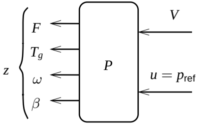
- simplifies the problem
- facilitates experiments in existing wind farms

Problems formulations discussed below

can be extended for the case without internal controller

Turbine with internal controller

NREL 5 MW turbine with standard internal controller (in operating region III)



Inputs:

V - wind speed; p_{ref} - power reference;

Outputs:

F - thrust force; T_g - generator torque
ω - rotor speed; β - pitch angle;

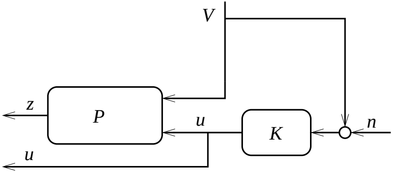
Model neglects electrical circuit dynamics (power production equals power reference)

Model is linearized around operating point

(all signals represent deviations from nominal values)

Problem formulation

Measured disturbance attenuation

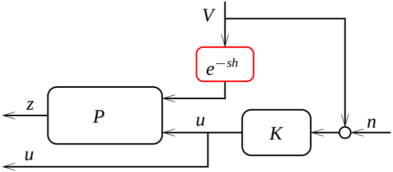


Deviations of wind speed $V \quad = \quad$ disturbances measured with noise n

The aim is to keep deviations of turbine outputs small

Problem formulation

Measured disturbance attenuation



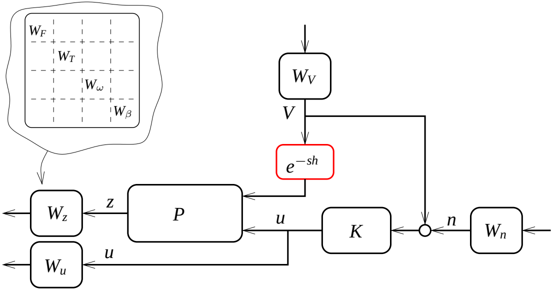
Deviations of wind speed V = disturbances measured with noise n

The aim is to keep deviations of turbine outputs small

Availability of preview is captured by delay operator e^{-sh}

Problem formulation

Measured disturbance attenuation



Deviations of wind speed V $=$ disturbances measured with noise n

The aim is to keep deviations of turbine outputs small

Availability of preview is captured by delay operator e^{-sh}

Weights

Thrust force - accounts for tower oscillations

$$W_F = k_F \frac{s + \omega_{\text{twr}}}{s^2 + \omega_{\text{twr}}^2}.$$

Generator torque - accounts for drive train oscillations

$$W_T = k_T \frac{s + \omega_{\text{shf}}}{s^2 + \omega_{\text{sft}}^2}.$$

Control signal - (power reference) no change of set point for permanent wind changes
prevents tower damping by means of pitch oscillations

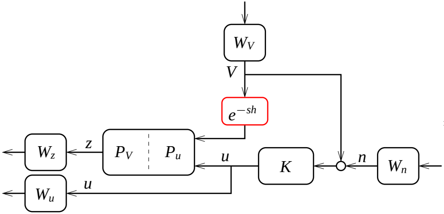
$$W_u = k_p \frac{(0.1s + 1)}{s} \cdot \frac{(s + \omega_{\text{twr}})^2}{s^2 + 0.02\, \omega_{\text{twr}} + \omega_{\text{twr}}^2}.$$

The rest of the weights are static.

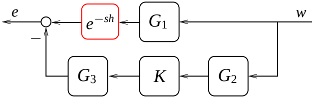
Model matching

The problem can be cast as model matching -

unified framework for estimation and feedforward control

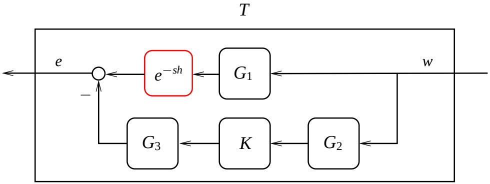


$\Rightarrow$



$$G_1 = \begin{bmatrix} W_z P_V W_V & 0 \\ 0 & 0 \end{bmatrix} \qquad G_2 = \begin{bmatrix} W_V & W_n \end{bmatrix} \qquad G_3 = \begin{bmatrix} -W_z P_u \\ -W_z \end{bmatrix}$$

Model matching

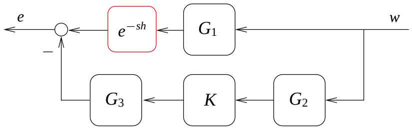


Problem statement

Given LTI G_1, G_2, G_3 and $h > 0$, find stable and causal K such that

- T is input/output stable (asymptotic performance)
- $||T||_2$ is minimal (transient performance)

Model matching



One-side problem with either $G_2 = I$ or $G_3 = I$

- corresponds to problem without measurement noise
- solved in both H^2 and H^∞ settings

(Tomizuka, 1975; Moelia and Meinsma, 2006; Mirkin and Tadmor, 2007)

Extension to general two-side problem is not trivial
(due to combination of asymptotic and transient performance requirements)

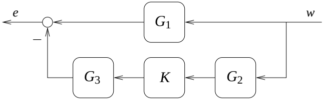
- without stability constraints the extension could be straightforward
- with stability constraints solved,

yet is not readily extendable for the case with preview.

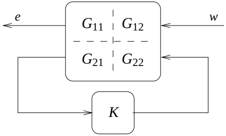
(Liu, Zhang and Mita, 1997)

Model-matching vs standard four-block stabilization

Model matching stabilization



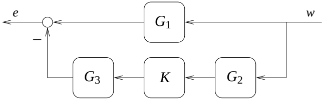
Standard four-block stabilization



- Unstable modes ⇔ physical instabilities
- Internal stability requirement
- Unstable cancelations prohibited

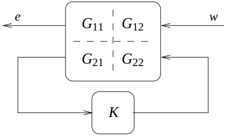
Model-matching vs standard four-block stabilization

Model matching stabilization



- Unstable modes ⇔ external signal dynamics
- No internal stability requirement
- Unstable cancelations eligible

Standard four-block stabilization



- Unstable modes ⇔ physical instabilities
- Internal stability requirement
- Unstable cancelations prohibited

Not a special case of the standard problem

Stabilization

Two-side stabilization is more complicated than one-side

- related to bilateral Diophantine equations
- state-space solution involves Sylvester equations
- cumbersome parameterization of stabilizing solutions

Stabilization

Two-side stabilization is more complicated than one-side

- related to bilateral Diophantine equations
- state-space solution involves Sylvester equations
- cumbersome parameterization of stabilizing solutions

Assumptions

$\mathcal{A}_1$ Unstable poles of G_2 do not coincide with zeros of G_3

$\mathcal{A}_2$ Unstable poles of G_3 do not coincide with zeros of G_2

satisfied in practical problems and lead to convenient parameterization

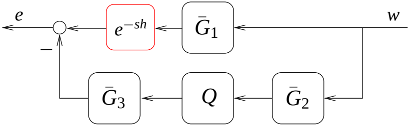
$$K = K_p + \tilde{M}_3 Q M_2, \quad Q \in H^\infty$$

Stability constraints released without changing problem structure

$$T = \underbrace{(G_1 - G_3 K_p G_2)}_{\bar{G}_1} + \underbrace{\tilde{N}_3}_{\bar{G}_3} Q \underbrace{N_2}_{\bar{G}_2}$$

Optimization

$$\min_{Q \in H^2} ||e^{-sh}\bar{G}_1 - \bar{G}_3 Q \bar{G}_2||_2$$



Following the steps of the one-side solution

- reduction to one-block problem (square completion)
- one-block problem solution (projection theorem)
- handling preview element (completion operator)

Nontrivial steps required to derive state-space formulae for two-side case

Optimization

Modified Riccati equations

$$(A+B_2F_t)'X + X(A+B_2F_t) - (XB_2 + C'D_3')(B_2'X + D_3C) + C'C = 0,$$
$$(A+L_tC_2)Y + Y(A+L_tC_2)' - (YC_2' + BD_2')(C_2Y + D_2B') + BB' = 0$$

- standard H^2 AREs with shifted A-matrices
- F_t and L_t defined by Sylvester equation (from stabilization solution)
- solvable for any stabilizable problem
- solution satisfies standard Riccati (but is not stabilizing for standard Riccati)

Optimization

Modified Riccati equations

$$(A+B_2F_t)'X + X(A+B_2F_t) - (XB_2 + C'D_3')(B_2'X + D_3C) + C'C = 0,$$
$$(A+L_tC_2)Y + Y(A+L_tC_2)' - (YC_2' + BD_2')(C_2Y + D_2B') + BB' = 0$$

- standard H^2 AREs with shifted A-matrices
- F_t and L_t defined by Sylvester equation (from stabilization solution)
- solvable for any stabilizable problem
- solution satisfies standard Riccati (but is not stabilizing for standard Riccati)

Alternative to the notion of semi-stabilizing ARE solutions

proposed in Liu, Zhang and Mita, 1997

Solution

Based on:

Two Sylvester equations (stabilization)

Two Riccati equations (optimization)

- *not affected by preview length*
- *shifted by terms from Sylvester*

Solution

Based on:

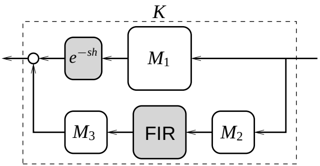
Two Sylvester equations (stabilization)

Two Riccati equations (optimization)

- *not affected by preview length*
- *shifted by terms from Sylvester*

Optimal solution structure:

$M_{1/2/3}$	finite dimensional h - independent
M_1	optimal for $h = 0$



Solution

Based on:

Two Sylvester equations (stabilization)

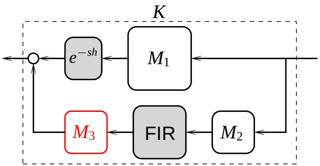
Two Riccati equations (optimization)

- *not affected by preview length*
- *shifted by terms from Sylvester*

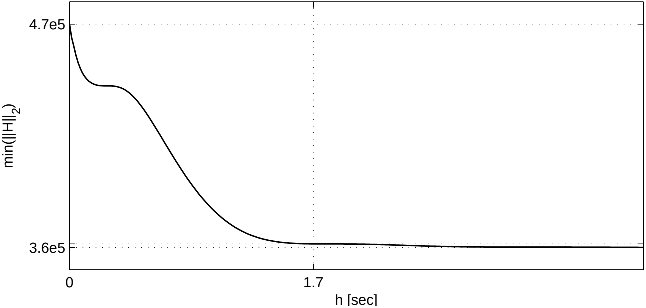
Optimal solution structure:

$M_{1/2/3}$ finite dimensional
h - independent

M_1 optimal for $h = 0$

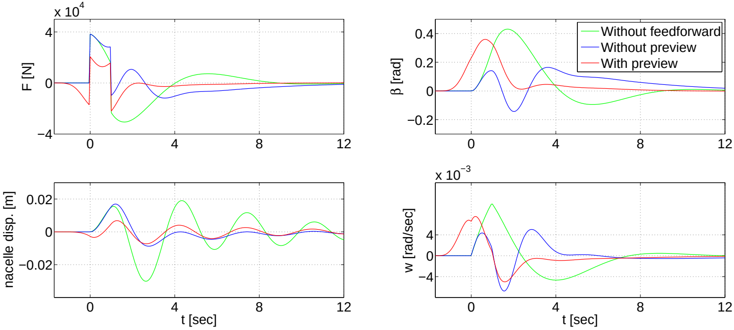


Performance vs. preview length



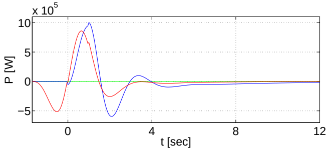
- The relevant scale of preview is a number of seconds
- 98 % of improvement achieved with 1.7 sec preview

Simulation results



Control signal

(change of power production)



Outline

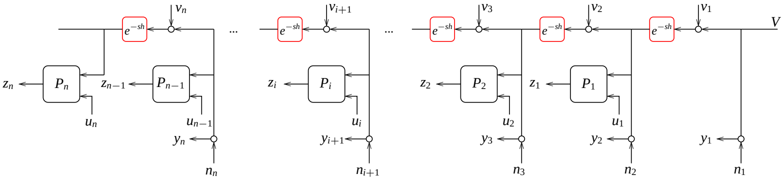
1. Preview control of individual turbine
2. Cooperative control of entire farm

Wind farm model

Wake effects: mean speed deficit, increase of turbulence

- important in quasi-static analysis of farm
  (distribution of nominal powers among turbines in farm)
- less important for dynamics around specified operating point

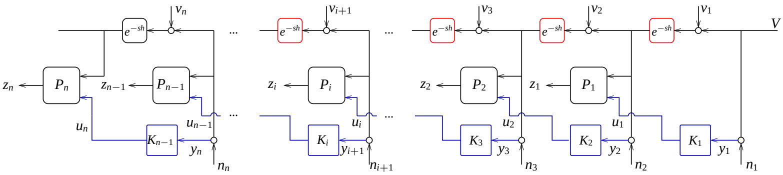
Neglect influence of pitch on wind flow



Wind propagation modeled as delay and additive noise

Distributed feedforward control

Preview control from previous part can be applied to each turbine in farm



Drawback:

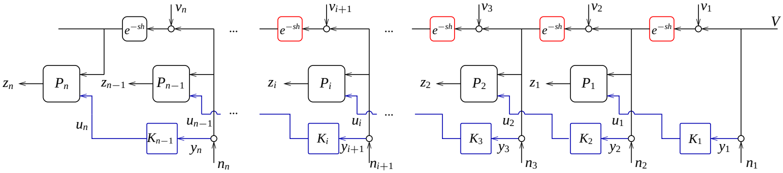
adjustment of turbines power ⇒ fluctuations in overall power production

Cooperation between turbines may be beneficial

- requires formulation that takes the entire farm into account

Distributed feedforward control

We keeping this control scheme, but optimize for the entire farm

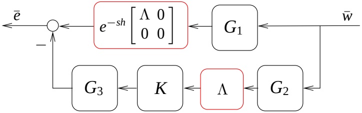


Inputs: $\bar{w} := \begin{bmatrix} V & v_1 & \cdots & v_N & n_1 & \cdots & n_N \end{bmatrix}'$

Outputs: $\bar{e} := \begin{bmatrix} z_1 & \cdots & z_N & u_1 & \cdots & u_N & \sum u_i \end{bmatrix}'$

The problem can be cast as model matching

Decentralized model matching



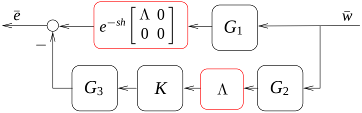
$$\Lambda := \text{diag}\{1, e^{-sh}, e^{-2sh}, \dots\}$$

$$K = \text{diag}\{K_1 \dots K_N\} \in H^\infty$$

$$G_1 := \left[\begin{array}{c:cccc:cccc}
P_{1,V} & P_{1,V} & 0 & \cdots & 0 & 0 & 0 & \cdots & 0\\
P_{2,V} & P_{2,V} & P_{2,V} & \cdots & 0 & 0 & 0 & \cdots & 0\\
\vdots & \vdots & \vdots & \ddots & 0 & \vdots & \vdots & \ddots & \vdots\\
P_{N,V} & P_{N,V} & P_{N,V} & \cdots & P_{N,V} & 0 & 0 & \cdots & 0\\
\hdashline
0 & 0 & 0 & \cdots & 0 & 0 & 0 & \cdots & 0\\
\vdots & \vdots & \vdots & & 0 & \vdots & \vdots & & \vdots\\
0 & 0 & 0 & \cdots & 0 & 0 & 0 & \cdots & 0\\
\hdashline
0 & 0 & 0 & \cdots & 0 & 0 & 0 & \cdots & 0
\end{array}\right]
\quad
G_3 := \left[\begin{array}{ccc}
P_{1,u} & \cdots & 0\\
\vdots & \ddots & 0\\
0 & \cdots & P_{N,u}\\
\hdashline
I & \cdots & 0\\
\vdots & \ddots & \vdots\\
0 & \cdots & I\\
\hdashline
1 & \cdots & 1
\end{array}\right]$$

$$G_2 := \left[\begin{array}{c:cccc:cccc}
1 & 0 & \cdots & 0 & 0 & 1 & 0 & \cdots & 0\\
 & & \ddots & \vdots & \vdots & & & & \\
1 & 1 & & & & 0 & 1 & \cdots & 0\\
\vdots & \vdots & \ddots & 0 & 0 & \vdots & \vdots & \ddots & \vdots\\
1 & 1 & \cdots & 1 & 0 & 0 & 0 & \cdots & 1
\end{array}\right]$$

Decentralized model matching



$$\Lambda := \text{diag}\{1, e^{-sh}, e^{-2sh}, \dots\}$$

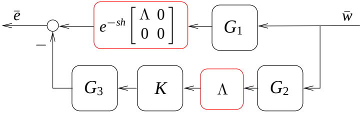
$$K = \text{diag}\{K_1 \dots K_N\} \in H^\infty$$

More complicated than in previous part:

- Parameter K constrained to be diagonal
  - Communication only with closest upwind neighbors
  - Various communication patterns can be considered

    (various structural constraints on K)
- Complicated structure of delays
  - Uniform delay e^{-sh} corresponds to availability of preview
  - Multichannel delay Λ is due to wind propagation

Decentralized model matching



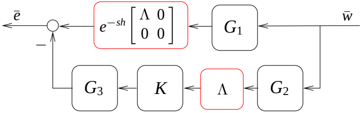
$$\Lambda := \text{diag}\{1, e^{-sh}, e^{-2sh}, \dots\}$$

$$K = \text{diag}\{K_1 \dots K_N\} \in H^\infty$$

Open problems:

- Decentralized model matching stabilization
  - May stability constraints be released without changing problem structure?
- Decentralized model matching optimization
  - Relevant in various distributed control problems
- How to handle delays?
  - Discretization leads to numerical difficulties
  - Does there exist a convenient solution like in centralized case?

Decentralized model matching



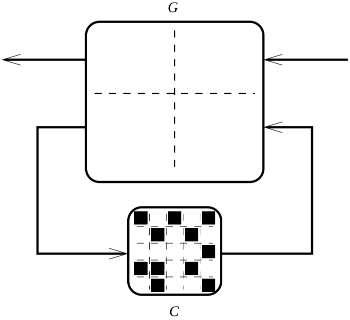
$$\Lambda := \text{diag}\{1, e^{-sh}, e^{-2sh}, \dots\}$$

$$K = \text{diag}\{K_1 \dots K_N\} \in H^\infty$$

Open problems:

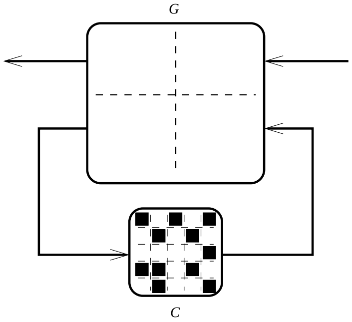
- Decentralized model matching stabilization
  - May stability constraints be released without changing problem structure?
- Decentralized model matching optimization
  - Relevant in various distributed control problems
- How to handle delays?
  - Discretization leads to numerical difficulties
  - Does there exist a convenient solution like in centralized case?

Distributed control



- Constraint on structure of controller (plant may also have structure)
- No analytical solutions available in the literature
- Generally, optimal controller may not be linear (H. S. Witsenhausen, 1968)

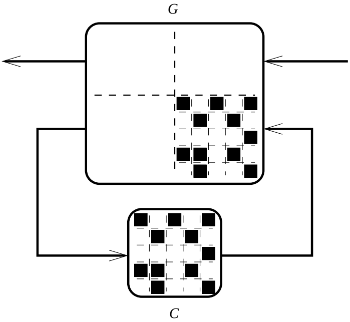
Distributed control



Hope to find tractable solutions for some special cases:

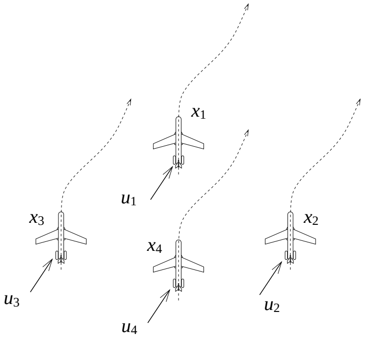
- positive systems (Tanaka and Langbord, 2010; Rantzer, 2011)
- quadratically invariant problems (Rotkowitz and Lall, 2006)

Quadratically invariant problems



- Relation between structures of controller and G_{22}
- This is a “small” class of distributed control problems
- Yet, it has practical motivation

Formation control



Goal: Follow trajectory,
while keeping formation

u_i - individual control signal

x_i - absolute position

- Each agent has individual control input
- Absolute position of each agent is measured separately

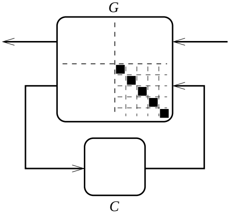
$\Downarrow$

- G_{22} has diagonal structure

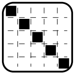
Formation control

Quadratically invariant
communication patterns

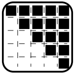
(for diagonal G_{22})



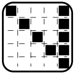
No communication
(each agent measures only its own position)



Hierarchical communication
(each agent measures himself and all predecessors)



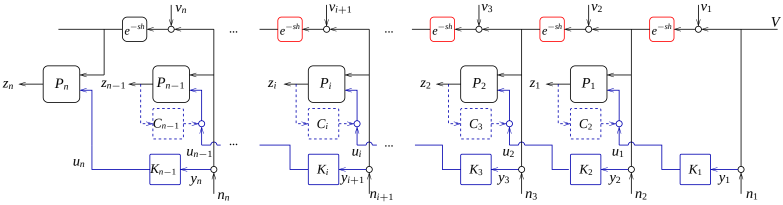
Leader based alignment
(each agent measures himself and the leader)



Other examples (quadratic invariance)

Wind farm without internal controllers is quadratically invariant

(feedbacks based on local measurements)



More generally, problems, in which agents

- have independent dynamics
- are coupled with common disturb. and control objectives

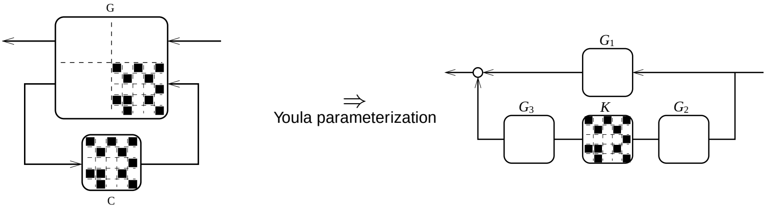
are quadratically invariant

Quadratic invariance

Youla parameterization can be applied

- reduces the problem to model matching
- constraint on Youla parameter structure

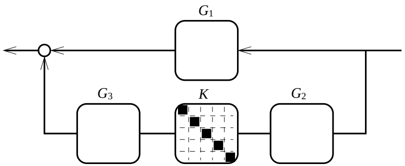
(the same constraint as on original controller)



Arbitrary structure can be transformed into diagonal (Rotkowitz, 2010)

Decentralized model matching

Any quad. inv. problem can be reduced to decentralized model matching



Analytical solution is missing:

- Is it possible to derive closed form formulae for optimal solution?
- What is the structure of optimal solution?
- What is the order of optimal solution?

So far, only three agents special case with triangular structure is solved

(Swigart and Lall, 2010)

Notation

Kronecker product

$$\begin{bmatrix} a_{11} & a_{12} \\ a_{21} & a_{22} \end{bmatrix} \otimes \begin{bmatrix} b_{11} & b_{12} \\ b_{21} & b_{22} \end{bmatrix} = \begin{bmatrix} a_{11}b_{11} & a_{11}b_{12} & a_{12}b_{11} & a_{12}b_{12} \\ a_{11}b_{21} & a_{11}b_{22} & a_{12}b_{21} & a_{12}b_{22} \\ a_{21}b_{11} & a_{21}b_{12} & a_{22}b_{11} & a_{22}b_{12} \\ a_{21}b_{21} & a_{21}b_{22} & a_{22}b_{21} & a_{22}b_{22} \end{bmatrix}$$

Khatri-Rao product

$$\begin{bmatrix} a_{11} & a_{12} \\ a_{21} & a_{22} \end{bmatrix} \odot \begin{bmatrix} b_{11} & b_{12} \\ b_{21} & b_{22} \end{bmatrix} = \begin{bmatrix} a_{11}b_{11} & a_{12}b_{12} \\ a_{11}b_{21} & a_{12}b_{22} \\ a_{21}b_{11} & a_{22}b_{12} \\ a_{21}b_{21} & a_{22}b_{22} \end{bmatrix}$$

Hadamard product

(element wise)

$$\begin{bmatrix} a_{11} & a_{12} \\ a_{21} & a_{22} \end{bmatrix} \circ \begin{bmatrix} b_{11} & b_{12} \\ b_{21} & b_{22} \end{bmatrix} = \begin{bmatrix} a_{11}b_{11} & a_{12}b_{12} \\ a_{21}b_{21} & a_{22}b_{22} \end{bmatrix}$$

Operators vec and dvec

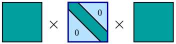
$$\text{vec}\left(\begin{bmatrix} a_{11} & a_{12} \\ a_{21} & a_{22} \end{bmatrix}\right) = \begin{bmatrix} a_{11} \\ a_{21} \\ a_{12} \\ a_{22} \end{bmatrix}, \qquad \text{dvec}\left(\begin{bmatrix} a_{11} & 0 \\ 0 & a_{22} \end{bmatrix}\right) = \begin{bmatrix} a_{11} \\ a_{22} \end{bmatrix}$$

Idea to apply Kronecker product to decentralized model matching by K. Park, 2008

Reduction to one-side problem

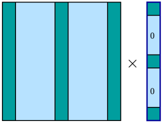
Applying vec operator

$$\text{vec}(T) = \text{vec}(G_1) - \text{vec}(G_3 K G_2)$$



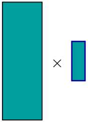
Using properties of Kronecker product

$$\text{vec}(T) = \text{vec}(G_1) - (G_2' \otimes G_3)\text{vec}(K)$$



Removing redundant columns

$$\text{vec}(T) = \text{vec}(G_1) - (G_2' \odot G_3)\text{dvec}(K)$$



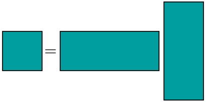
Problem reduced to one-side model matching without structural constraints
The number of columns does not grow

Decentralized model matching and Hadamard product

Solution in terms of spectral factorization

$$U^{\sim}U = (G_2' \odot G_3)^{\sim}(G_2' \odot G_3)$$

Original problem dimensions are preserved



Using properties of Khatri-Rao product

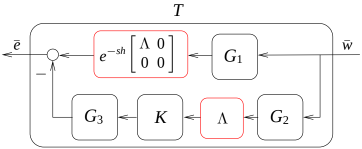
$$U^{\sim}U = (G_2 G_2^{\sim})' \circ (G_3^{\sim} G_3)$$

Spectral factorization of Hadamard product of matrices associated with centralized solution

---

Explicit state-space formulae for Hadamard product are needed

Structure of delays



$$\Lambda = \text{diag}\{1, e^{-sh}, e^{-2sh}, \dots\}$$

$$K = \text{diag}\{K_1 \dots K_N\} \in H^\infty$$

Neglect v_i, n_i and assume $N = 2$

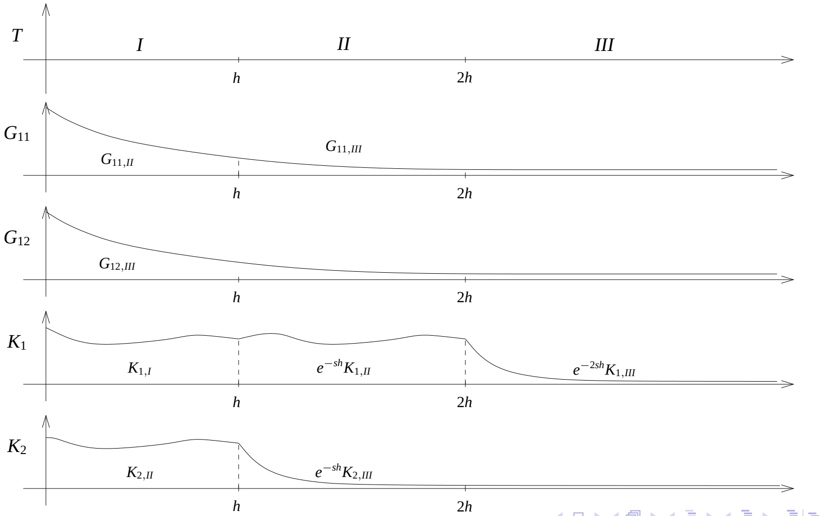
$$T = \begin{bmatrix} e^{-sh}P_{1v} \\ e^{-2sh}P_{2v} \\ 0 \\ 0 \\ 0 \end{bmatrix} - \begin{bmatrix} P_{1u} & 0 \\ 0 & P_{2u} \\ 1 & 0 \\ 0 & 1 \\ 1 & 1 \end{bmatrix} \begin{bmatrix} 1 & 0 \\ 0 & e^{-sh} \end{bmatrix} \begin{bmatrix} K_1 \\ K_2 \end{bmatrix}$$

Can it be solved via splitting the time axes?

(similarly to A. Moelja and G. Meinsma, 2006; G. Marro and E. Zattoni, 2005; G. Tadmor, 1997)

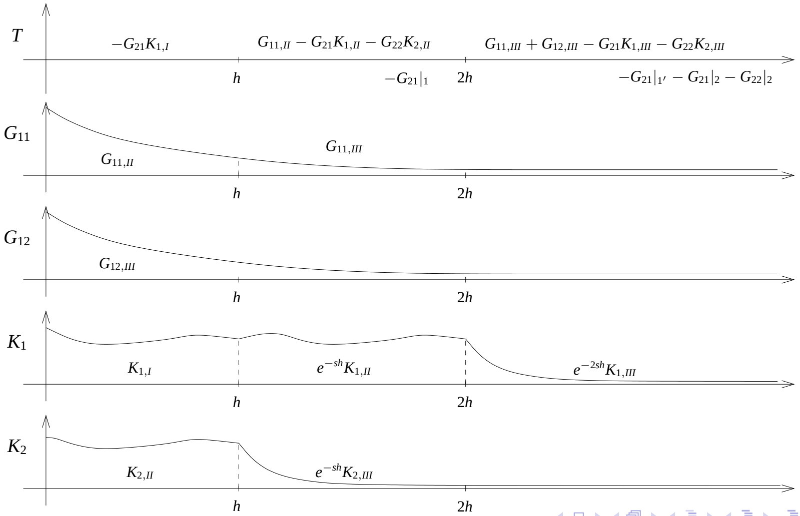
Splitting the time axis

$$T = e^{-sh}G_{11} + e^{-2sh}G_{12} - G_{21}K_1 - e^{-sh}G_{22}K_2$$

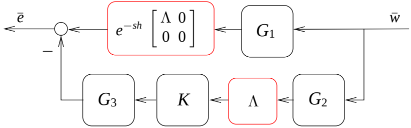


Splitting the time axis

$$T = e^{-sh}G_{11} + e^{-2sh}G_{12} - G_{21}K_1 - e^{-sh}G_{22}K_2$$



Approximate solution



$$\Lambda = \text{diag}\{1, e^{-sh}, e^{-2sh}, \ldots\}$$

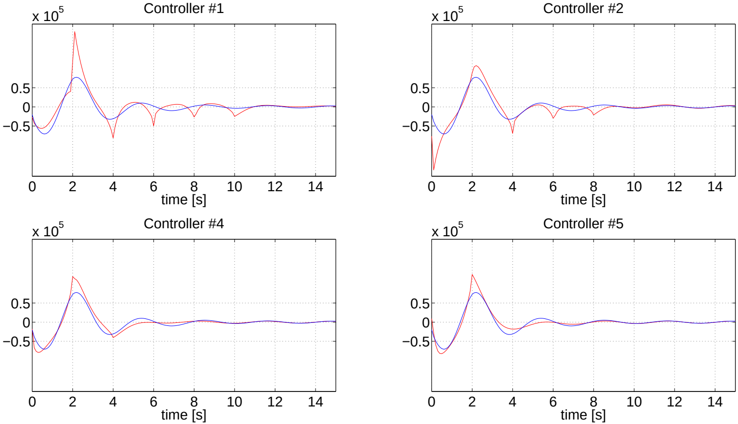
$$K = \text{diag}\{K_1 \ldots K_N\} \in H^{\infty}$$

Open problems:

- Decentralized model matching stabilization
  Shifting imaginary axis poles to OLHP
- Decentralized model matching optimization
  Frequency domain solution in terms of Hadamard product
- How to handle delays?
  Discretization of time axis (feasible for small number of turbines)

Optimal controllers

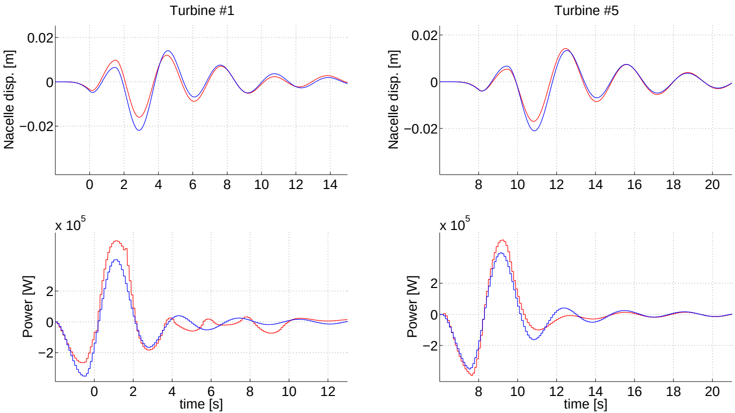
Impulse responses of optimal controllers



Each controller takes care of downwind turbines

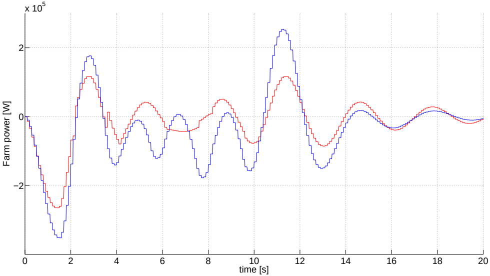
(peaks in time multiples of h)

Simulation results



- Slight decrease in tower oscillations
- Demands more deviations in individual turbine powers

Simulation results - improvement in overall power production



- decrease in deviations of overall farm power production
- without deterioration in terms of load reduction

Summary

- Feedforward control based on previewed wind speed measurements
  - individual turbine control
  - distributed control of entire farm
- Cooperation and use of preview are advantageous
- The results motivate more detailed study
  - without internal controllers
  - other communication patterns
  - realistic model of wind propagation

Summary

- Feedforward control based on previewed wind speed measurements
  - individual turbine control
  - distributed control of entire farm
- Cooperation and use of preview are advantageous
- The results motivate more detailed study
  - without internal controllers
  - other communication patterns
  - realistic model of wind propagation

Open theoretical problems:

Decentralized model matching with complicated structure of delays

- stabilization
- optimization (solution based on Hadamard product)

Thank you for attention!