Multiple Delay Dead-Time Compensation: From Stability- to Performance-Driven Configurations

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Based on a collaboration with *G. Meinsma, Z. J. Palmor,* and *D. Shneiderman*

The theme of this talk

Problem-oriented control scheme for control of dead-time systems,

- dead-time compensation (DTC)

Talk outline

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Prolog: a friendly intro to dead-time compensation

Smith predictor: the first dead-time compensator

Single-delay dead-time compensation: analytical justifications

Multiple-delay dead-time compensation

Feedforward action Smith predictor (FASP)

From stability- to performance-oriented

Conclusions

Prolog: a friendly intro to dead-time compensation

Time-delay stabilization problem for n-dimensional equation

$$x[k+1] = Ax[k] + Bu[k-h]$$

where x and u measurable and (A, B) controllable is equivalent to

Delay-free stabilization problem for $(n+h)$-dimensional state equation

$$\underbrace{\begin{bmatrix} u[k] \\ u[k-1] \\ \vdots \\ u[k-h+1] \\ x[k+1] \end{bmatrix}}_{x_a[k+1]} = \underbrace{\begin{bmatrix} 0 & \cdots & 0 & 0 & 0 \\ 1 & \cdots & 0 & 0 & 0 \\ \vdots & \ddots & \vdots & \vdots & \vdots \\ 0 & \cdots & 1 & 0 & 0 \\ 0 & \cdots & 0 & B & A \end{bmatrix}}_{A_a} \underbrace{\begin{bmatrix} u[k-1] \\ u[k-2] \\ \vdots \\ u[k-h] \\ x[k] \end{bmatrix}}_{x_a[k]} + \underbrace{\begin{bmatrix} 1 \\ 0 \\ \vdots \\ 0 \\ 0 \end{bmatrix}}_{B_a} u[k]$$

where x_a and u measurable and (A_a, B_a) controllable.

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where x_a and u measurable and (A_a, B_a) controllable.

Pole placement

Controllability $\implies$ $\operatorname{spec}(A_\mathrm{a} + B_\mathrm{a} F_\mathrm{a})$ can be assigned arbitrarily. But

- $A_\mathrm{a} = \left[\begin{array}{cccc|c} 0 & \cdots & 0 & 0 & 0 \\ 1 & \cdots & 0 & 0 & 0 \\ \vdots & \ddots & \vdots & \vdots & \vdots \\ 0 & \cdots & 1 & 0 & 0 \\ \hline 0 & \cdots & 0 & B & A \end{array}\right]$ has h eigenvalues at the origin,

which might make sense to keep untouched. With this in mind, applying the Ackermann's formula to

$$\chi_{\mathrm{cl,a}}(z) = \chi_{\mathrm{cl}}(z) z^h, \qquad \text{for arbitrary } n\text{-order } \chi_{\mathrm{cl}}(z)$$

and exploiting the structure of A_a and B_a, we end up with

$$F_\mathrm{a} = F \left[\begin{array}{ccccc} B & AB & \cdots & A^{h-1}B & A^h \end{array}\right],$$

where F solves $\det(A + BF) = \chi_{\mathrm{cl}}(z)$, which is n-dimensional (delay free).

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Control law

The control law we just derived,

$$u[k] = F\left(A^h x[k] + \sum_{i=1}^{h} A^{i-1} B u[k-i]\right) \stackrel{\text{calc}}{=} F x[k+h],$$

can be interpreted as predictive state feedback.

Controllers of the form

here, $\Pi(z) = \sum_{i=1}^{h} A^{i-1} B z^{-i}$

where

- internal feedback aims at rendering $\tilde{x}$ a prediction of x
- primary part designed for a delay-free plant

called *dead-time compensators* (DTCs).

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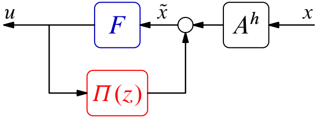

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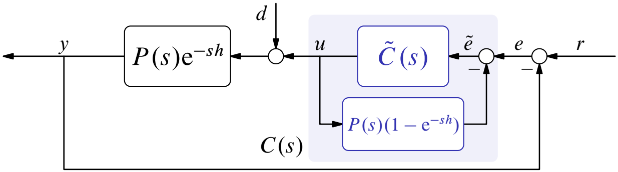
Smith controller



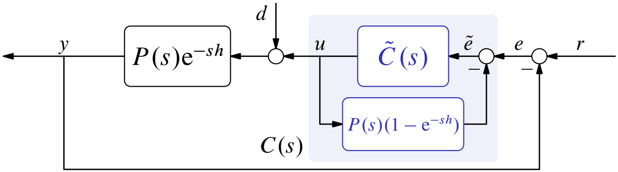
Introduced by Otto J. M. Smith in 1957, comprises

- $P(s)(1-e^{-sh})$ — Smith predictor

  w/o d, we have that $\tilde{e} = (r-y) - P(1-e^{-sh})u = r - Pu$ is prediction of delay-free e
- $\tilde{C}(s)$ — primary controller

  designed for delay-free $P(s)$

Closed-loop system



Closed-loop response

$$y = \mathrm{e}^{-sh} \frac{P\tilde{C}}{1 + P\tilde{C}}\, r + \frac{1 + P\tilde{C}(1 - \mathrm{e}^{-sh})}{1 + P\tilde{C}}\, Pd.$$

has no delay in denominators, meaning that

- delay eliminated from the characteristic equation
- reference response greatly simplified

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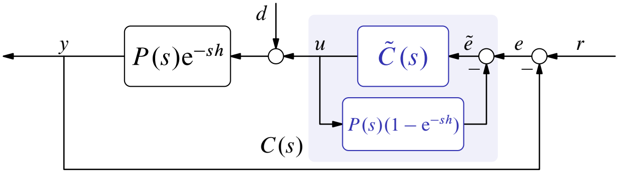

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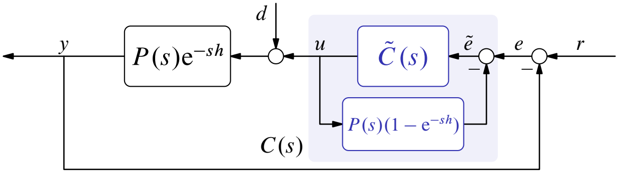
Success story



- intriguing properties

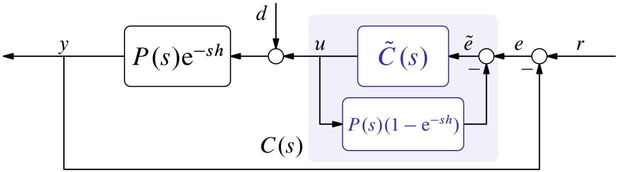
  numerous studies (zillions of papers, many book chapters)
- numerous modifications / generalizations

  modified Smith predictor, finite spectrum assignment, etcetera
- it does work

  many industrial applications, part of commercial controllers

Otto J. M. Smith listed in the ISA “Leaders of the Pack” list (2003) as one of the 50 most influential industry innovators since 1774.

Success story



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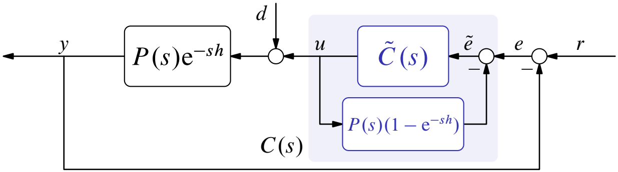
"A process engineer's crystal ball" (V. J. VanDoren)



So far as the structure of DTC is concerned:

- empirical approaches dominate
  driven mostly by engineering insight, with few attempts to be rigorous
- many extensions/modifications, but few analytically justified
  structure postulated and then its properties analyzed
- simulations as proofs

$\Downarrow$

- quite a few misconceptions

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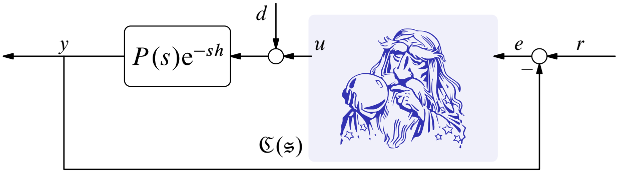

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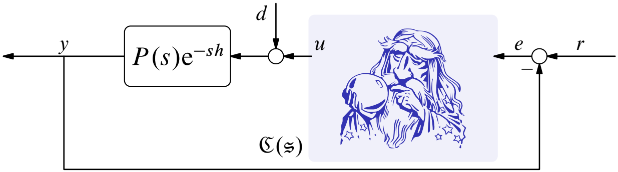

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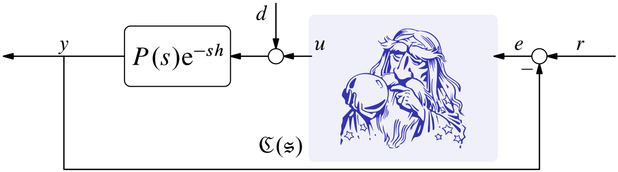

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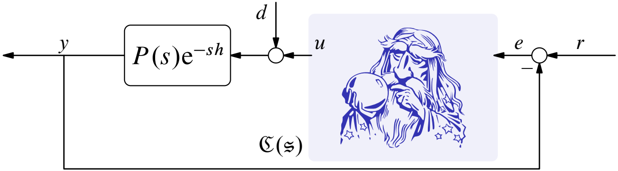
Urban legends (partial list)



- not efficient in disturbance attenuation
- intrinsically poor robustness
- stabilization induces “true” DTC controller structure
- artificial loop delays might be advantageous in multiple-delay systems

Talk outline

Stability

- **Stabilization** (Watanabe & Ito, 1981)
  stabilization via eliminating delays from the characteristic equation for unstable plants, introduced the modified Smith predictor (MSP)
- **Finite spectrum assignment** (Manitius & Olbrot, 1979; Lewis, 1979; Furukawa & Shimemura, 1983; et alii)
  state-predictor, observer-predictor; equivalent to MSP (Mirkin & Raskin, 2003)
- **Youla parametrization** (Mirkin & Raskin, 2003)
  parametrization of all stabilizing controllers, MSP
- **Coprime factorization** (Curtain, Weiss, & Weiss, 1996)
  MSP is an integral part of it
- **Robust stability** (Morari & Zafiriou, 1989; Zwart & Bontsema, 1997; Mirkin & Raskin, 2003)
  DTCs maximize robustness radius for several uncertainty descriptions
- **Nonlinear systems** (Krstic, 2009)
  Lyapunov analysis

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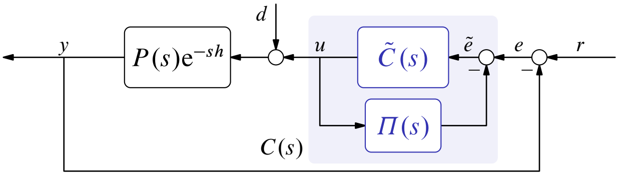
Stability-induced controller structure

Analytical approaches bring about[1] DTC controller structure



consisting of

- irrational $\Pi(s)$ in internal feedback

  only limited to be stable and such that $\tilde{P}(s) := P(s)e^{-sh} + \Pi(s)$ rational; always exists

  - Smith predictor corresponds to $\Pi(s) = P(s)(1 - e^{-sh})$, works only for stable P
  - modified Smith predictor yields $\Pi(s)$ with finite impulse response (FIR)

- rational $\tilde{C}(s)$ (primary controller)

  stabilizes rational $\tilde{P}(s)$

[1] Controller structure results from solution procedure, not postulated beforehand.

Performance

Early results:

- LQG optimal control (Kleinman, 1969)
  effectively invented observer-predictor controller (unfortunately, overlooked)
- SISO minimum variance control (Palmor, 1982)
  optimal controller can always be cast as Smith predictor if plant stable

Recent problem-oriented solutions

- H^∞ (Tadmor, 2000; Meinsma & Zwart, 2000; Mirkin, 2003)
- H^2 / LQG (Mirkin & Raskin, 2003)
- L^1 (Mirkin, 2006; Di Loreto *et al*, 2008)

demonstrated that

- DTC is an intrinsic part of general optimal solutions

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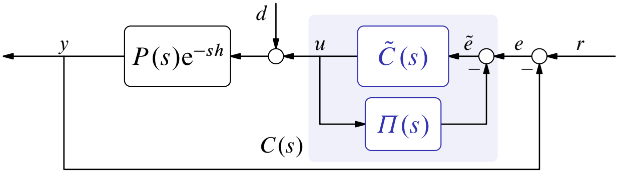


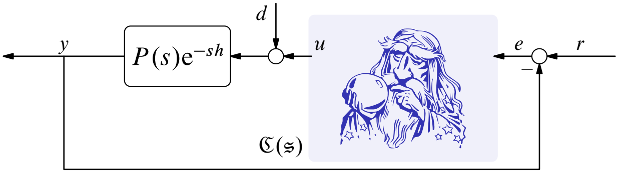
- irrational stable $\Pi(s)$ in internal feedback
  - MSP in H^2 and L^1 cases
  - MSP + feedforward SP of (Palmor & Powers, 1985) for worst-case d in H^∞ case
- rational $\tilde{C}(s)$

  solves corresponding problem for rational $\tilde{P}(s) := P(s)e^{-sh} + \Pi(s)$

Stability vs. performance: single-delay case

No conflict

MythBusters



- not efficient in disturbance attenuation ⟵ busted
- intrinsically poor robustness ⟵ busted
- stabilization induces "true" DTC controller structure ⟵ plausible
- artificial loop delays might be advantageous ⟵ busted

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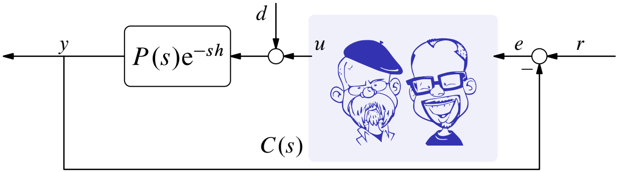

- not efficient in disturbance attenuation $\longleftarrow$ busted
  - $\hookrightarrow$ optimal disturbance attenuators (H^2, H^∞, L^1) are all DTCs
- intrinsically poor robustness $\longleftarrow$ busted
- stabilization induces "true" DTC controller structure $\longleftarrow$ plausible
- artificial loop delays might be advantageous $\longleftarrow$ busted

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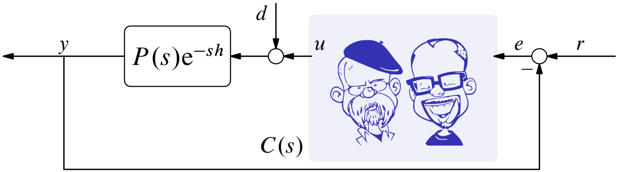

- not efficient in disturbance attenuation ⟵ busted
- intrinsically poor robustness ⟵ busted
  - ↪ controllers maximizing complex robustness radii (H^∞, L^1) are all DTCs
  - ↪ SP preserves robustness of delay-free design to uncertainty in $P(s)$
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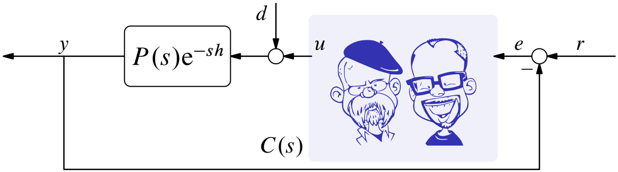

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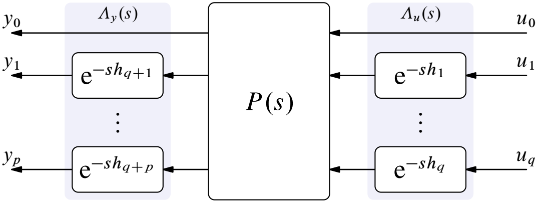
Multiple I/O delay systems

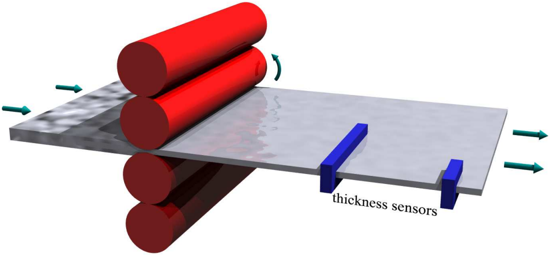
Different delays in different I/O channels:



where

$$\Lambda_u(s) = \begin{bmatrix} I & & & \\ & e^{-sh_1} I & & \\ & & \ddots & \\ & & & e^{-sh_q} I \end{bmatrix} \quad \text{and} \quad \Lambda_y(s) = \begin{bmatrix} I & & & \\ & e^{-sh_{q+1}} I & & \\ & & \ddots & \\ & & & e^{-sh_{q+p}} I \end{bmatrix}$$

Example: hot strip mill profile control



- sensors for measuring thickness at edges and at the centerline located at different distances from the stand exit

Stability

Direct extensions of single-delay results:

- Youla parametrizaion (Raskin, 2001; Moelja & Meinsma, 2003)
- finite spectrum assignment (Kwon & Pearson, 1980; Artstein, 1982; Fiagbedzi & Pearson, 1990)

Induced controller structure:

- irrational $\Pi(s)$ in internal feedback
  stable and such that $\tilde{P}(s) := \Delta_y(s)P(s)\Delta_u(s) + \Pi(s)$ rational; always exists
- rational $\tilde{C}(s)$ (primary controller)
  stabilizes rational $\tilde{P}(s)$

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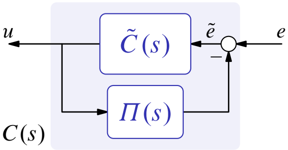

- irrational $\Pi(s)$ in internal feedback

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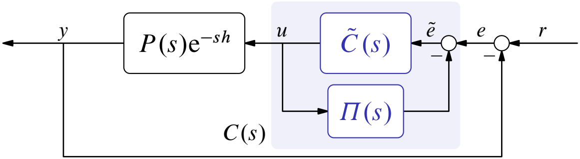
Performance

Choosing any stable DTC $\Pi = \tilde{P} - P\mathrm{e}^{-sh}$ with rational $\tilde{P}$,



Single input delay: because $P\mathrm{e}^{-sh} = \mathrm{e}^{-sh}P$,

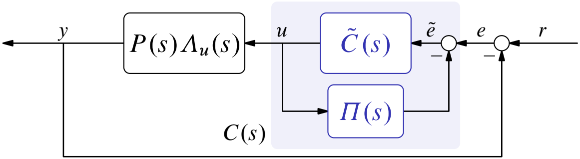
$$T_{ry} = P\mathrm{e}^{-sh}\tilde{C}(I + \tilde{P}\tilde{C})^{-1} = \underbrace{\mathrm{e}^{-sh}}_{\text{non-invertible}} \underbrace{P\tilde{C}(I + \tilde{P}\tilde{C})^{-1}}_{\text{rational}}$$

- ☺ irrational non-invertible inner (i.e., energy preserving) part is isolated
- ☺ $\tilde{C}$ designed ignoring e^{-sh}

  more precisely, not accounting for e^{-sh} explicitly

Performance

Choosing any stable DTC $\Pi = \tilde{P} - P\Lambda_u$ with rational $\tilde{P}$,



Multiple input delays: in general $P\Lambda_u \neq \Lambda_u P$ (input channels mix up in y), hence

$$T_{ry} = P\Lambda_u\tilde{C}(I + \tilde{P}\tilde{C})^{-1}$$

- ☹ hard to analyze

  non-invertible irrational Λ_u stuck in the middle
- ☹ poor performance if $\tilde{C}$ designed ignoring Λ_u

  noticed by Jerome & Ray (1986), who analyzed prior attempts

State of the art: GMDC (Jerome & Ray, 1986)

Generalized multidelay compensator (GMDC) attempts to pull Λ_u through the output (the rearrangement test):

$$P\Lambda_u = \Lambda_u \tilde{P}_{\mathrm{JR}} \quad \text{for some (irrational) causally invertible } \tilde{P}_{\mathrm{JR}}.$$

If possible: set $\Lambda_a = I$

If impossible: find artificial delays Λ_a such that $P\Lambda_u\Lambda_a$ passes the test

always exists, just think of $\Lambda_u = \begin{bmatrix} 1 & \\ & e^{-sh} \end{bmatrix}$, for which $\Lambda_a = \begin{bmatrix} e^{-sh} & \\ & 1 \end{bmatrix}$ results in $\Lambda_u\Lambda_a = e^{-sh}I$ (single delay)

Then,

- choose $\Pi = \tilde{P}_{\mathrm{JR}} - P\Lambda_u\Lambda_a$
- design $\tilde{C}$ for (possibly irrational) $\tilde{P}_{\mathrm{JR}}$

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State of the art: GMDC (Jerome & Ray, 1986)

Generalized multidelay compensator (GMDC) attempts to pull Λ_u through the output (the rearrangement test):

$$P\Lambda_u = \Lambda_u \tilde{P}_{\mathrm{JR}} \quad \text{for some (irrational) causally invertible } \tilde{P}_{\mathrm{JR}}.$$

If possible: set $\Lambda_a = I$

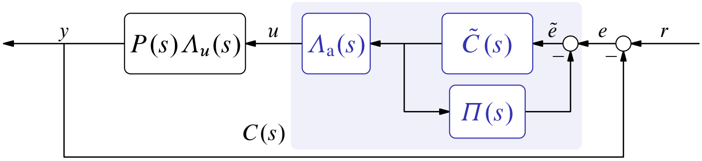
If impossible: find artificial delays Λ_a such that $P\Lambda_u\Lambda_a$ passes the test

always exists, just think of $\Lambda_u = \begin{bmatrix} 1 & \\ & e^{-sh} \end{bmatrix}$, for which $\Lambda_a = \begin{bmatrix} e^{-sh} & \\ & 1 \end{bmatrix}$ results in $\Lambda_u\Lambda_a = e^{-sh} I$ (single delay)

Then,

- choose $\Pi = \tilde{P}_{\mathrm{JR}} - P\Lambda_u\Lambda_a$
- design $\tilde{C}$ for (possibly irrational) $\tilde{P}_{\mathrm{JR}}$

GMDC (multiple input delays)

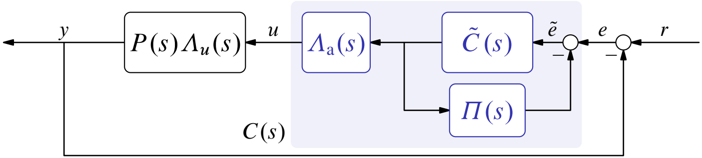


Two important aspects:

1. artificial loop delays might be added
2. delays might no longer be eliminated from the characteristic equation

   Jerome & Ray argued that this might be necessary to improve performance

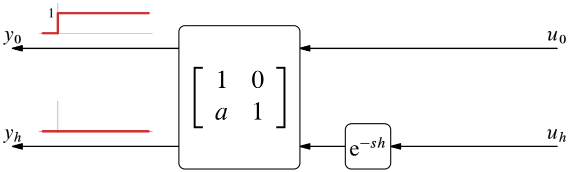
GMDC (multiple input delays)



Two important aspects:

1. artificial loop delays might be added ⟵ appears counterintuitive
2. delays might no longer be eliminated from the characteristic equation
   Jerome & Ray argued that this might be necessary to improve performance

Artificial delays: additional motivation



Goals:

$y_0(t) = u_0(t)$ tracks the unit step $\mathbb{1}(t)$

$y_h(t) = u_h(t-h) + a u_0(t)$ stays at zero

Arguments of Holt & Morari (1985), dynamic resilience theory:

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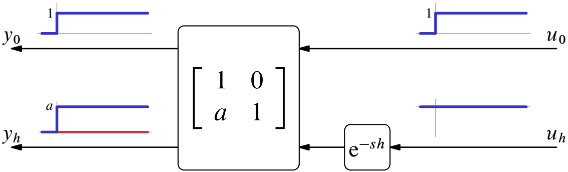

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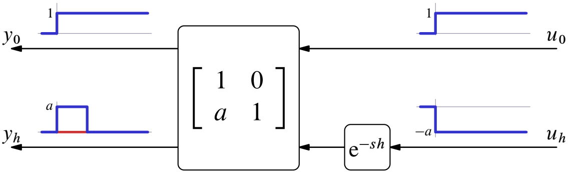
Arguments of Holt & Morari (1985), dynamic resilience theory:

- undesirable effect of $u_0(t) = \mathbb{1}(t)$ on y_h (coupling)
- only partially compensated by $u_h(t) = -a\mathbb{1}(t)$ because of the delay
- fully compensated by artificial delay in u_0, but at the expense of y_0
- hence, introducing artificial input delay may improve performance

Artificial delays: additional motivation



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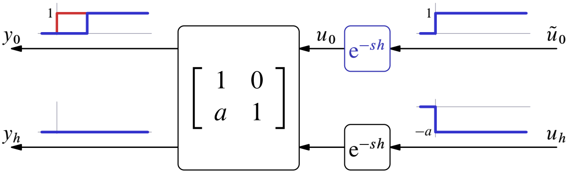

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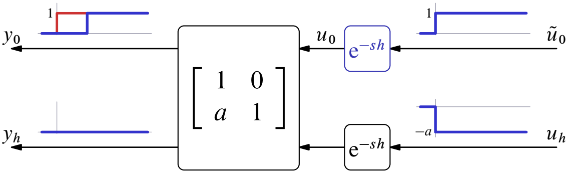

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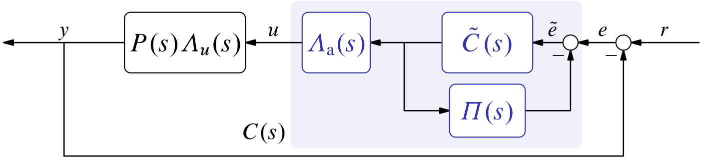
GMDC (multiple input delays)



Two important aspects:

1. artificial loop delays might be added
2. delays might no longer be eliminated from the characteristic equation
   Jerome & Ray argued that this might be necessary to improve performance

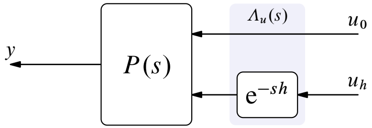
Stability vs. performance: multiple-delay case

Might conflict

Talk outline

Input adobe delay

To simplify exposition, consider



i.e., assume that

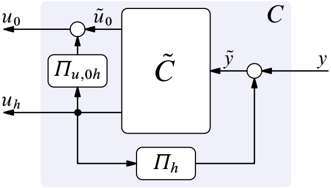
$$\Lambda_u(s) = \begin{bmatrix} I & 0 \\ 0 & e^{-sh} I \end{bmatrix} \quad \text{and} \quad \Lambda_y(s) = I.$$

- simplest nontrivial generalization of single-delay case
- captures the essence
- general case solved via nested recursion of adobe problems

Starting point

Technical outcome of

- multiple delay H^∞ solution (Meinsma & Mirkin, 2005)



with

1. rational $\tilde{C}$
2. irrational (FIR) Π_h
3. irrational (FIR) $\Pi_{u,0h}$

Resolved in (Mirkin, Palmor, & Shneiderman, 2011).

Starting point

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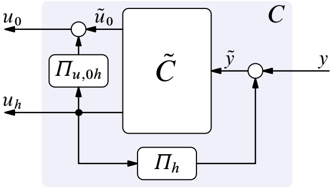


with

1. rational $\tilde{C}$ $\longleftarrow$ conventional
2. irrational (FIR) Π_h $\longleftarrow$ conventional
3. irrational (FIR) $\Pi_{u,0h}$ $\longleftarrow$ unorthodox

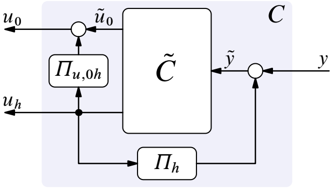
Called

- FASP — feedforward action Smith predictor

Starting point

Technical outcome of

- multiple delay H^∞ solution (Meinsma & Mirkin, 2005)



with

1. rational $\tilde{C}$
2. irrational (FIR) Π_h
3. irrational (FIR) $\Pi_{u,0h}$ $\longleftarrow$ rationale ???

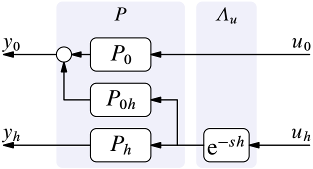
Resolved in (Mirkin, Palmor, & Shneiderman, 2011).

Upper triangular case

Assume[2] first that

$$P(s) = \begin{bmatrix} P_0(s) & P_{0h}(s) \\ 0 & P_h(s) \end{bmatrix} \text{ with square and invertible } P_0(s)$$

or

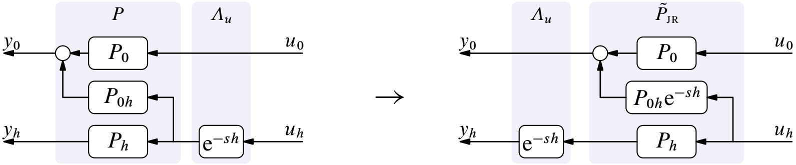


where

- y_0 affected by mixture of delay-free and delayed input channels
- y_h affected only by delayed input channel

[2] This is the only class passing the rearrangement test.

Upper triangular case: pulling Λ_u through y



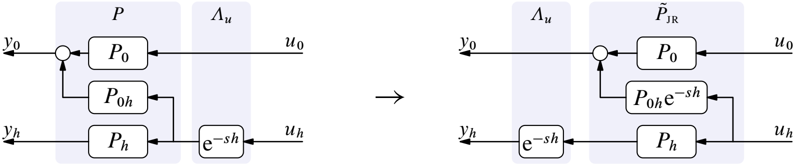
- this is where Jerome & Ray stopped

$$\tilde{P}_{\mathrm{JR}} = \begin{bmatrix} P_0 & P_{0h}e^{-sh} \\ 0 & P_h \end{bmatrix} \quad \text{is causally invertible}$$

where rational $\tilde{P}_{0h}$ and stable (irrational) $\Pi_{u,0h}$ are such that

$$P_0^{-1}P_{0h}e^{-sh} = P_0^{-1}\tilde{P}_{0h} - \Pi_{u,0h}$$

Upper triangular case: pulling Λ_u through y



$$
\begin{aligned}
y_0 &= P_0 u_0 + P_{0h} e^{-sh} u_h \\
&= P_0 (u_0 + P_0^{-1} P_{0h} e^{-sh} u_h) \\
&= P_0 (u_0 + (P_0^{-1} \tilde{P}_{0h} - \Pi_{u,0h}) u_h) \\
&= P_0 (u_0 - \Pi_{u,0h} u_h) + \tilde{P}_{0h} u_h,
\end{aligned}
$$

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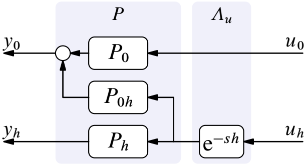
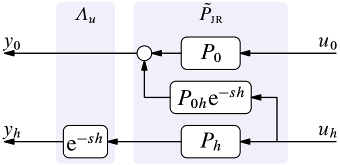


$\rightarrow$



$\downarrow$

$$
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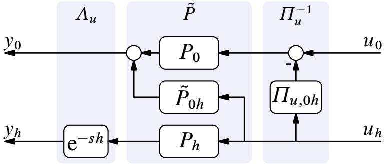

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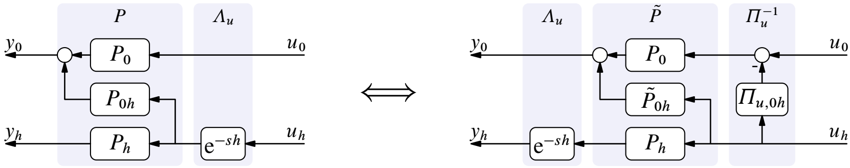
$$
P_0^{-1} P_{0h} \mathrm{e}^{-sh} = P_0^{-1} \tilde{P}_{0h} - \Pi_{u,0h}
$$

Upper triangular case: pulling Λ_u through y (contd)



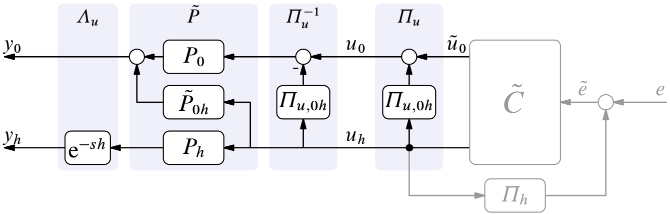
where

- Λ_u is (non-invertible) output delay
- $\tilde{P}$ is rational
- irrational

$$\Pi_u := \begin{bmatrix} I & \Pi_{u,0h} \\ 0 & I \end{bmatrix} \quad \left(\text{with } \Pi_u^{-1} = \begin{bmatrix} I & -\Pi_{u,0h} \\ 0 & I \end{bmatrix}\right)$$

is bi-stable $\implies$ can be canceled (by control law $u = \Pi_u \tilde{u}$).

Upper triangular case: canceling Π_u^{-1} by FASP



Thus, now

y_0 affected only by delay-free input channel

y_h affected only by delayed input channel

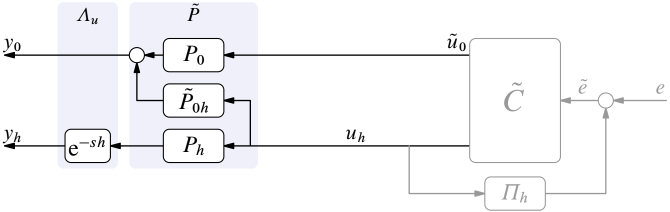
In other words, FASP interchannel block

- $\Pi_{u,0h}$ compensates for delay in "delay-free" output y_0

and then FASP feedback block

- Π_h eliminates delays from characteristic equation

Upper triangular case: canceling Π_u^{-1} by FASP



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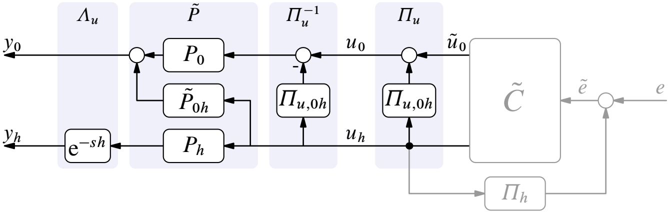

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and then FASP feedback block

- Π_h eliminates delays from characteristic equation

General case

Instead of $P\Lambda_u = \Lambda_u \tilde{P} \Pi_u^{-1}$, factor (quite technical)

$$P(s)\Lambda_u(s) = \Psi_y(s)\tilde{P}(s)\Pi_u^{-1}(s)$$

where

- $\tilde{P}(s)$ is rational

  having the same order and poles as $P(s)$

- $\Pi_u(s) := \begin{bmatrix} I & \Pi_{u,0h}(s) \\ 0 & I \end{bmatrix}$ is bi-stable

  with FIR $\Pi_{u,0h}$, exactly as in the triangle case

- $\Psi_y(s)$ is inner (i.e., stable and energy preserving)

  also FIR; the next best thing to Λ_u

The role of Π_u

Partition $P = \begin{bmatrix} P_0 & P_h \end{bmatrix}$ (compatibly with input channels). Then

$$y = P\Lambda_u u = P_0\, u_0 + \mathrm{e}^{-sh} P_h\, u_h$$

has delay-free and delayed parts mixed in it. Feedforward compensation of FASP, $u = \Pi_u \tilde{u}$, yields

$$y = P\Lambda_u \Pi_u \tilde{u} = \underbrace{P_0\, \tilde{u}_0}_{y_0} + \mathrm{e}^{-sh} \underbrace{(I - Q_0) P_h\, u_h}_{y_h}$$

where

- $Q_0(\mathrm{j}\omega)$ is orthogonal projection onto the image of $P_0(\mathrm{j}\omega)$

so that $P_0(\mathrm{j}\omega) \perp (I - Q_0(\mathrm{j}\omega)) P_h(\mathrm{j}\omega)$ and, hence, $y_0 \perp y_h$.

Thus, feedforward part of FASP

- renders delay-free (y_0) and delayed (y_h) parts of y orthogonal (in $L^2(\mathbb{R})$)

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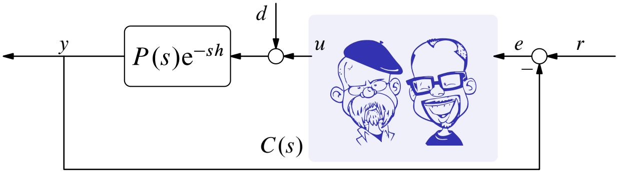
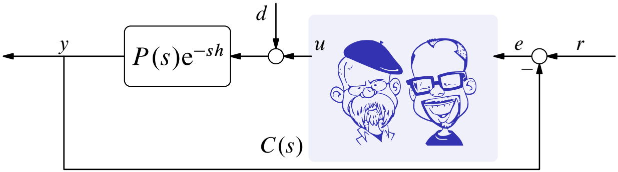
MythBusters



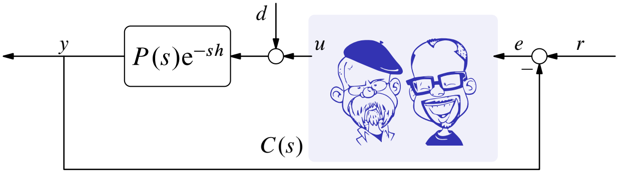
- not efficient in disturbance attenuation $\longleftarrow$ busted
- intrinsically poor robustness $\longleftarrow$ busted
- stabilization induces “true” DTC controller structure $\longleftarrow$ plausible
- artificial loop delays might be advantageous $\longleftarrow$ busted

MythBusters



- not efficient in disturbance attenuation $\longleftarrow$ busted
- intrinsically poor robustness $\longleftarrow$ busted
- stabilization induces “true” DTC controller structure $\longleftarrow$ busted
  - $\hookrightarrow$ stability-induced DTC might conflict with performance requirements
  - $\hookrightarrow$ multiple-delay DTC should be performance dependent
- artificial loop delays might be advantageous $\longleftarrow$ busted

MythBusters



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- intrinsically poor robustness ⟵ busted
- stabilization induces "true" DTC controller structure ⟵ busted
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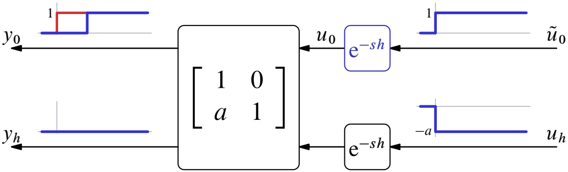
  ↪ at least, not for the reason they've been believed so hitherto

adding artificial delays

GMDC:



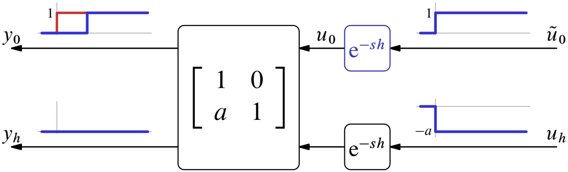
error energy: $\|e\|_2^2 = h$ (independent of a).

FASP (here $\eta := 1/(1+a^2)$):

error energy: $\|e\|_2^2 = \frac{a^2}{1+a^2} h < h$ (always better, especially if $|a|$ small).

No need in adding artificial delays

GMDC:



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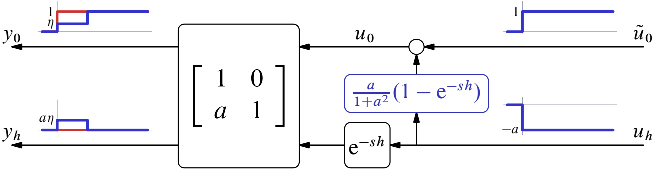

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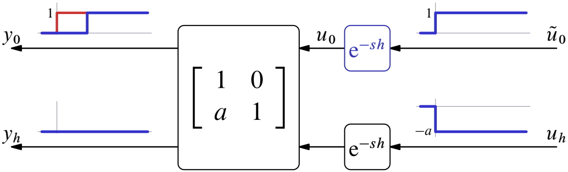
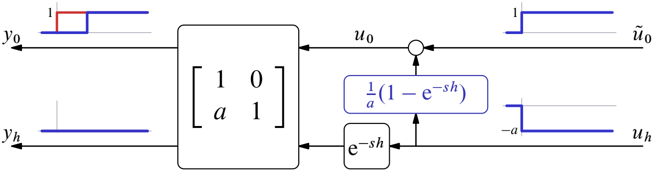
Doing it FASP way

GMDC:



☹ robustness harmed anyway

FASP:



☺ decoupling without sacrificing robustness

☹ not universal

Stability vs. performance: multiple-delay case

No conflict

Talk outline

Prolog: a friendly intro to dead-time compensation

Smith predictor: the first dead-time compensator

Single-delay dead-time compensation: analytical justifications

Multiple-delay dead-time compensation

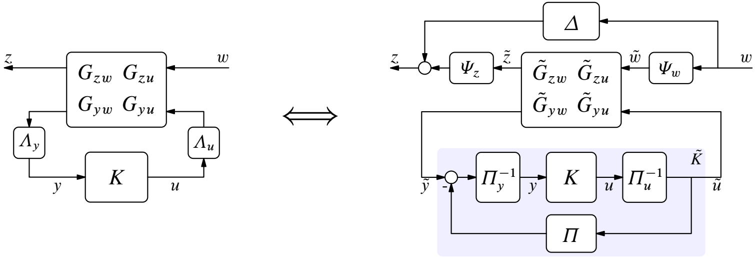
Feedforward action Smith predictor (FASP)

From stability- to performance-oriented

Conclusions

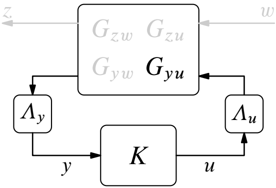
General case (Mirkin, Palmor, & Shneiderman, 2011)

By applying adobe loop shifting recursively as in (Meinsma & Mirkin, 2005)



- $\tilde{G}$ rational with the same "A" matrix as G
- Ψ_z and Ψ_w inner and FIR
- Δ stable, FIR, and $\Delta \perp \Psi_z T_{\tilde{z}\tilde{w}} \Psi_w$ in H^2 for every $T_{\tilde{z}\tilde{w}} \in H^2$
- Π_u and Π_y bistable and FIR
- Π stable and FIR

Stability

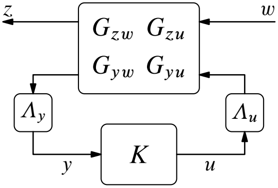


Effectively,

- only "$\Lambda_y - G_{yu} - \Lambda_u - K$" loop

needs to be analyzed

Performance



Closed-loop system is

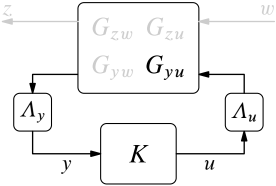
$$T_{zw} = G_{zw} + G_{zu}\Lambda_u K(I - \Lambda_y G_{yu}\Lambda_u K)^{-1}\Lambda_y G_{yw}$$

and it

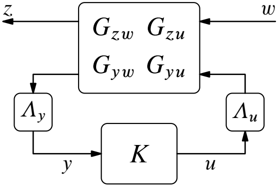
- depends on all components of G

Qualitative difference

Conventional DTC: structure depends only on G_{yu} (real plant)

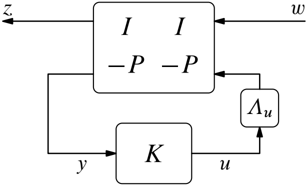


FASP: structure also depends on G_{zu} (effect of u on performance measure z) and G_{yw} (effect of exogeneous signal w on y)



Example

$S_{\text{in}} = (I + \Lambda_u K P)^{-1}$:

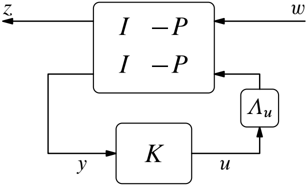


It

- induces conventional DTC

($G_{zu} = I$ commutes with Λ_u).

$S_{\text{out}} = (I + P \Lambda_u K)^{-1}$:



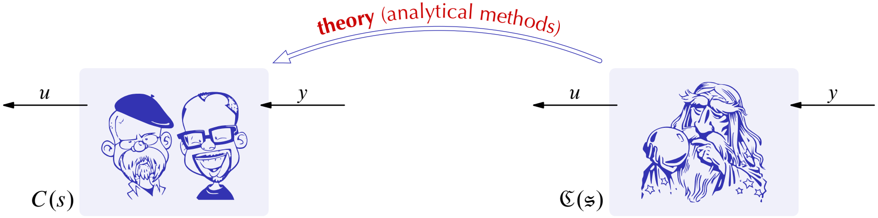
It

- induces FASP

($G_{zu} = -P$ might not commute with Λ_u).

Talk outline

First message



Second message

Paradigm shift is in order