



Design and Implementation of the Novel Feedforward Action Smith Predictor (FASP) on the Quadruple Tank Process with Multiple Delays.

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Goals & method

- ❑ To check the feasibility and to examine some implementation issues of the Feedforward Action Smith Predictor (FASP) controller.
- ❑ Performed by designing and applying the FASP to the Quadruple Tank Process with DTs (QTPwDT) lab setup.
- ❑ Compared performances with the state of the art DTC-GMDC

Collaborator - Alexey German

Outline

- MIMO LTI processes with multiple DTs & I/O DTs
- Control of MIMO plants with multiple delays
 - GMDC - state of the art DTC
 - FASP- structure & properties
- FASP- potential implementation difficulties and solutions
- Quadruple-tank process with multiple delays (QTPwDT)
 - The QTPwDT setup
 - Properties of the QTPwDT
- Experimental studies & results
- Summary and conclusions

Processes with multiple DTs

$$P(s) = \left(p_{ij}(s) e^{-h_{ij}s} \right)$$

$P_{ij}(s)$ - a rational transfer function relating input j to output i
 h_{ij} - DT between input j and output i

- ✓ arise naturally in many areas of engineering and sciences.
(sensor networks, autonomous vehicles, biological systems, networked control systems, internet congestion control, farms of wind turbines and more)

Processes with multiple I/O DTs

$$P(s) = \Lambda_y(s)G(s)\Lambda_u(s)e^{-h_c s}$$

$G(s)$ - a rational transfer matrix

$$\Lambda_u(s) = \text{diag} \left\{ I_{m_0}, e^{-s \cdot h_{u,1}} \cdot I_{m_1}, \dots, e^{-s \cdot h_{u,q}} \cdot I_{m_q} \right\} ,$$

$$0 < h_{u,1} < \dots < h_{u,q} , \quad \sum m_j = n_u$$

$$\Lambda_y(s) = \text{diag} \left\{ I_{p_0}, e^{-s \cdot h_{y,1}} \cdot I_{p_1}, \dots, e^{-s \cdot h_{y,r}} \cdot I_{p_r} \right\} ,$$

$$0 < h_{y,1} < \dots < h_{y,r} , \quad \sum p_j = n_y$$

✘ less general than

$$P(s) = \left(p_{ij}(s) e^{-h_{ij}s} \right)$$

Multiple DTs & multiple I/O DTs

✓ Question: $P(s) = \left(p_{ij}(s) e^{-h_{ij}s} \right) \stackrel{?}{=} \Lambda_y(s) G(s) \Lambda_u(s)$

⇒ Answer: (Sanchez-Pena et al, 2009) - iff

$$h_{iq} - h_{jq} = h_{ik} - h_{jk}, \begin{cases} i = 1, \dots, m & j = 1, \dots, m \\ q = 1, \dots, n & k = 1, \dots, n \end{cases}$$

Partial extraction of DTs

- ✓ When it is impossible to extract all DTs to inputs and outputs we may factor (h_{ij}) as follows

$$(h_{ij})_{m \times n} = \underbrace{(h_{yi})_{m \times m}}_{\bar{\Lambda}_y} (\eta_{ij})_{m \times n} \underbrace{(h_{ui})_{n \times n}}_{\bar{\Lambda}_u}$$

- ✓ a partial extraction problem may be defined (German, 2010):

$$J(h_{u1}, \dots, h_{um}, h_{y1}, \dots, h_{yn}) = \sum_{i=1}^m \sum_{j=1}^n \eta_{ij}, \quad \eta_{ij} = h_{ij} - h_{ui} - h_{yj}$$

$$s.t. \quad -\eta_{ij} \leq 0; \quad -h_{ui} \leq 0; \quad -h_{yj} \leq 0$$

- ⇒ J could be minimized with additional constraints on the structure of (η_{ij})

Control of MIMO Plants with Multiple delays

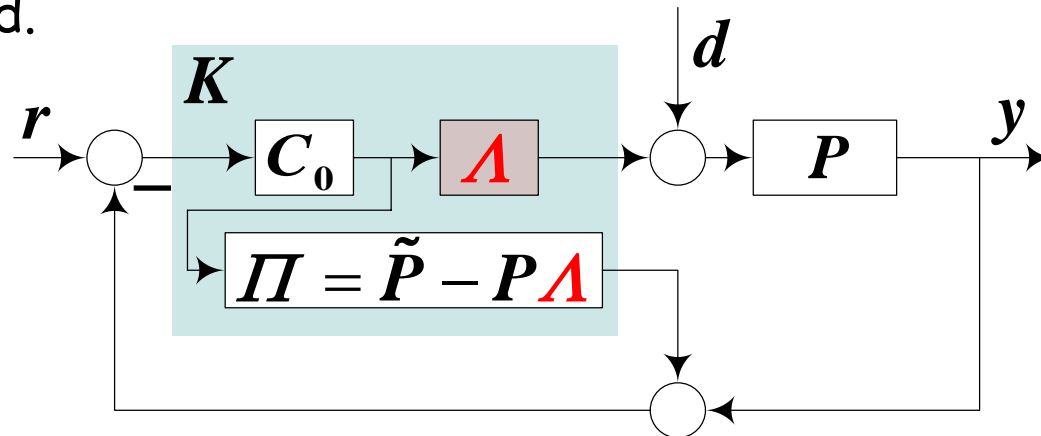
□ State of the art - the GMDC of Jerome & Ray (1986)

✓ Based on the Dynamic Resilience Theory (DRT) that considers the **DT decoupled response** as the best achievable response (in the limit)

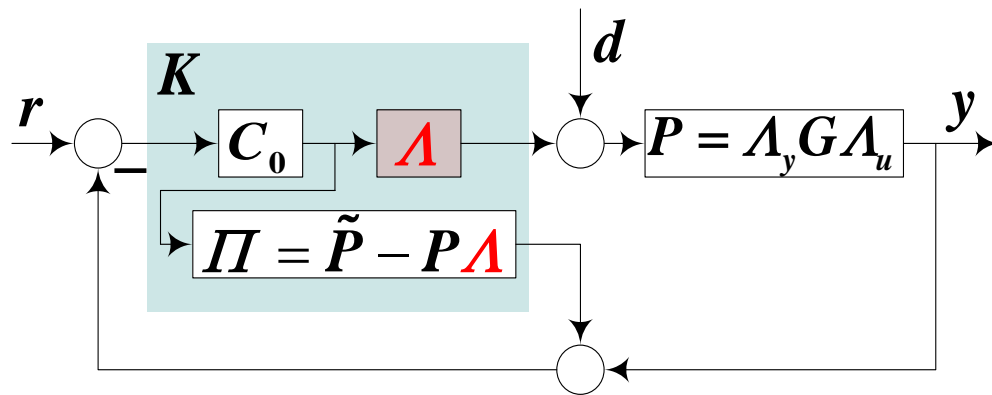
Λ - Matrix of artificially added delays if process fails the RT

C_0 - primary controller (typically a diagonal PI)

\tilde{P} - is the process (P or $P\Lambda$) with the smallest delay in each row subtracted.



The GMDC for I/O DTs

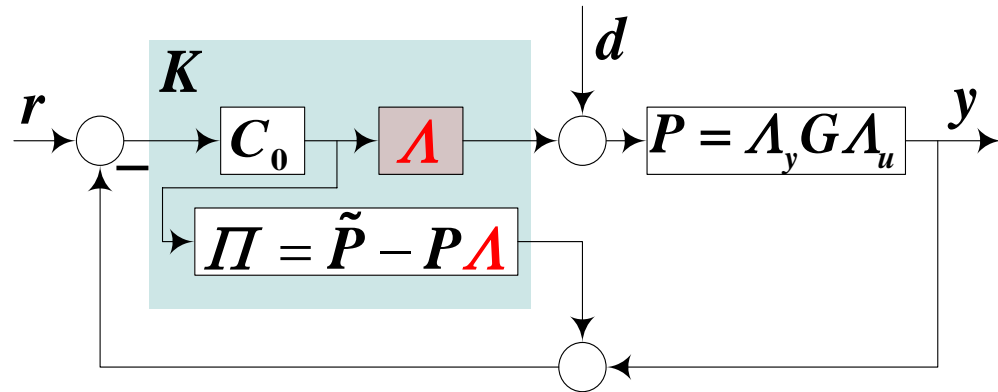


G - rational TF matrix
 Λ_u, Λ_y - diagonal delay I/O matrices
 Π - predictor
 K - overall GMDC

P passes the RT: $\tilde{P} = (\Lambda_y \Lambda_u)^{-1} G \Lambda_y$

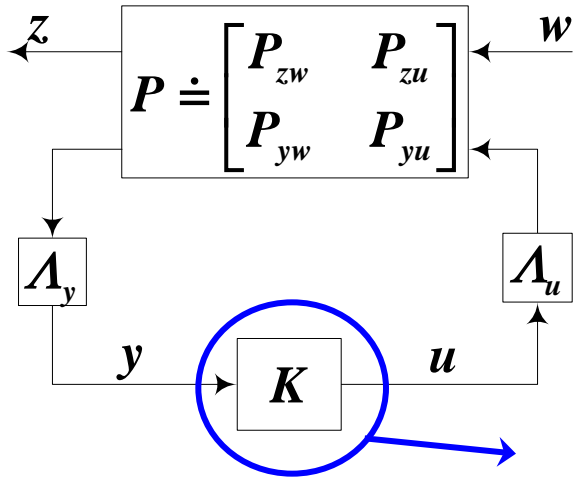
P doesn't pass the RT: $\tilde{P} = (\Lambda_y \Lambda_u \Lambda)^{-1} \Lambda_y G \Lambda_u \Lambda$

GMDC facts

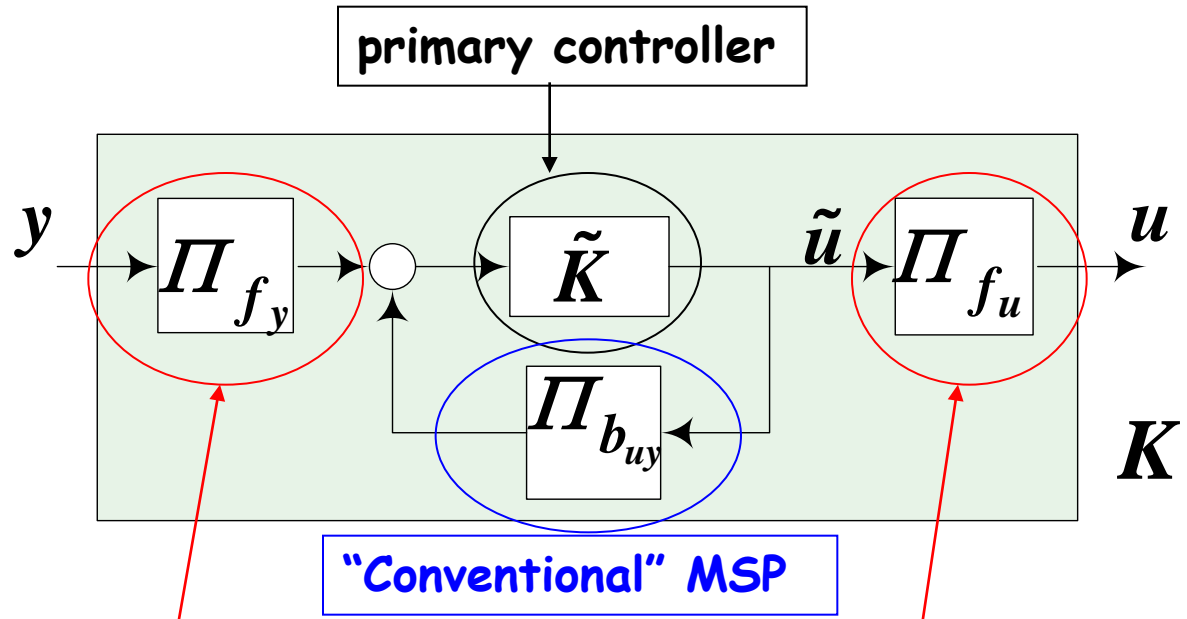


- ✘ \tilde{P} may be irrational \longrightarrow delays not eliminated from the characteristic equation
- ✘ Not applicable to unstable processes!
- ✘ No design/tuning method for C_0 provided!
- ☺ Applicable to the more general distribution of DTs

FASP - structure



$$K = \Pi_{f_u} \tilde{K} (I - \Pi_{b_{uy}} \tilde{K})^{-1} \Pi_{f_y}$$



inter-channel FF on measurements	inter-channel FF on controls
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$$\Pi_{b_{uy}} = \tilde{P}_{yu} - \Pi_{f_y} \Lambda_y P_{yu} \Lambda_u \Pi_{f_u}$$

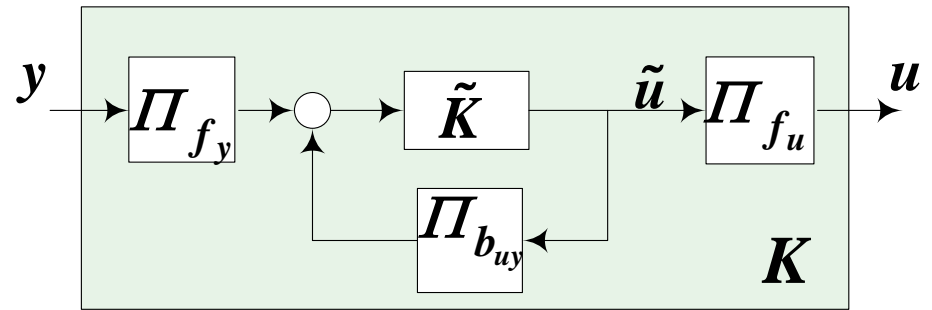
Role of FF compensators and predictor

- **FF on control** compensates for the coupling effects due to the longer delayed control channels on the outputs that are affected by the shorter delayed input channels.
 - **FF on measurement** compensates for longer delayed measurements of exogenous inputs that are sensed through the shorter delayed channels.
 - **Predictor** eliminates all DTs from the characteristic equation
- ✓ All three components consist of FIR blocks

FASP facts

- All H^2 controllers are FASPs
(Mirkin, Palmor & Shneiderman, 2009)
 - FASP is intrinsic to all H^∞ controllers
(Miensma & Mirkin, 2005)
- true extension of single delay MSP!
- Performance bounds of FASP are almost always better than those of GMDC.
(Shneiderman, Palmor & Mirkin, 2009)
 - FASP is performance dependent

FASP - potential implementation difficulties



□ Numerical instabilities

□ High dimensionality of rational components.

□ Realization of FIR blocks

Numerical instabilities

- ☹️ Both primary controller as well as predictor and FF compensators rely on matrix exponentials of Hamiltonian matrices \longrightarrow elements grow rapidly with large delays and render the implementation numerically unstable.
- 😊 It has been shown* that under mild conditions the overall H^2 controller can be realized with matrix exponentials of just Hurwitz matrices
- ☹️ However, the above increases significantly the dimensions of the primary controller

*(Mirkin, Palmor & Shneiderman, 2009)

High dimensionality of rational components

- Balanced truncation used to reduce dimensions of primary controllers simply and effectively. Criterion: similar singular values of full and reduced order primary controllers throughout bandwidth of optimal closed loop.
- Experience with the case study at hand shows that dimension reduction by at least a factor of 2 is possible without affecting control performance significantly.

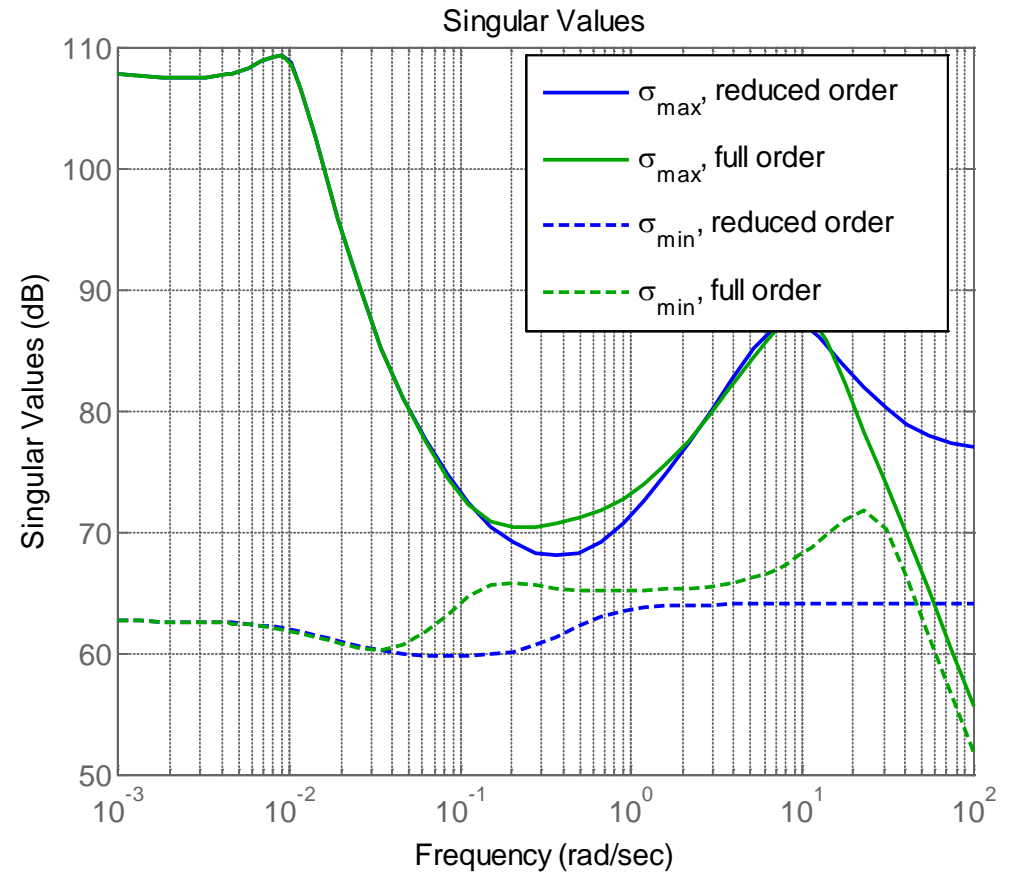
Example

Full order: 12

Reduced order: 6

Bandwidth: 1 rad/sec

Unstable pole left
untouched!.



FASP - FIR blocks

□ can be expressed as:

$$\Pi = \left[\begin{array}{c|c} A & B \\ \hline Ce^{-Ah} & 0 \end{array} \right] - \left[\begin{array}{c|c} A & B \\ \hline C & 0 \end{array} \right] e^{-sh} = C \cdot \left(\int_0^h e^{A(\theta-h)} e^{-\theta s} d\theta \right) \cdot B$$

- ✓ FIR (in $[0, h]$) - entire function- no poles
- ✓ Irrational

Realization of FIR blocks

- FIR block based upon **stable** system can be realized as a difference of two systems

$$\Pi_s = \left[\begin{array}{c|c} A & B \\ \hline Ce^{-Ah} & 0 \end{array} \right] - \left[\begin{array}{c|c} A & B \\ \hline C & 0 \end{array} \right] e^{-sh}$$

- FIR block based upon **unstable** system must be realized with no unstable poles

$$\Pi_u = C \cdot \left(\int_0^h e^{A(\theta-h)} e^{-\theta s} d\theta \right) \cdot B$$

→ approximations required

Approximations of Π_u

□ Numerical integrations (Lumped delay approximation)

$$\Pi_u = C \cdot \left(\int_0^h e^{A(\theta-h)} e^{-\theta s} d\theta \right) \cdot B \approx \frac{1}{\tau s + 1} \sum_{i=0}^n \eta_i C (I + \tau A) e^{A(\frac{i}{n}-1)h} B e^{-s\frac{h}{n}i}$$

n - number of divisions of DT

η_i - approximation dependent (rectangle rule, trapezoid...)

τ - time constant used to assure finite bandwidth of approximation*

*(Mirkin, 2004)

Approximations of Π_u

- ✓ Quality approximation measure (QAM)**

$$QAM = \left[\max_{\theta \in [0, h]} \frac{h^3}{12n^2} \frac{\bar{\sigma}(C(I + \tau A)e^{A(\theta-h)}A^2B)}{\bar{\sigma}(\Pi(0))} \right]^{-1}$$

- ⇒ QAM large- safe to use LDA. If small increase n or use Pade approximation (PA)

** (German, 2011)

Approximations of Π_u

- PADE approximation (AP) to cancel all unstable poles
- ✓ Delays in Π replaced by Pade approximation (of order n_p)
- ✓ Zeros (z_i) generated cancel unstable poles (p_i) within a distance ε
- ✓ Tradeoff between n_p and ε

Example 1 - large QAM ($> 10^3$)

DT: $h = 5[\text{sec}]$

LDA: $n = 10, \tau = 1[\text{sec}]$

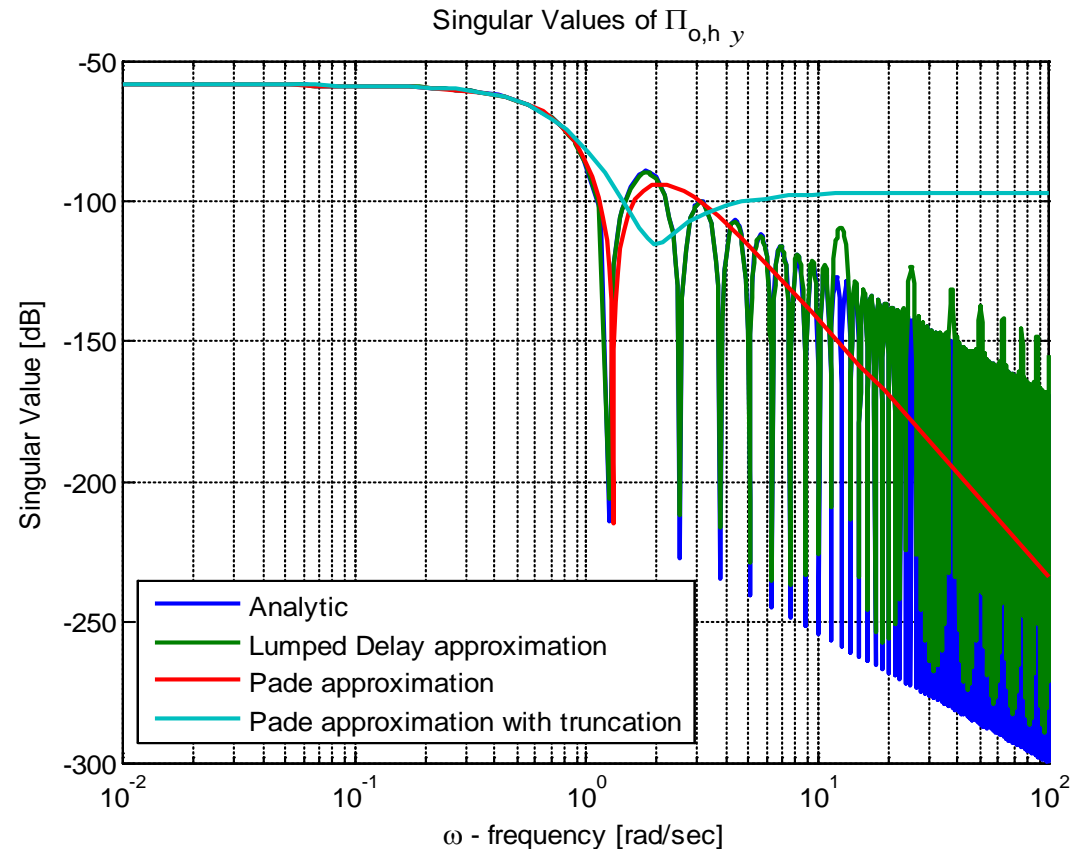
PA: $n_p = 4, \varepsilon = 10^{-2}$

Computation times:
Constant input for 10 sec
with sampling of 0.01sec

$t_{lda} = 0.0223[\text{sec}]$

$t_{pa} = 0.051[\text{sec}]$

$t_{pa, truncated} = 0.0384[\text{sec}]$



Example 2 - small QAM ($\epsilon = 10^{-3}$)

DT: $h = 10[\text{sec}]$

LDA: $n = 70, \tau = 1[\text{sec}]$

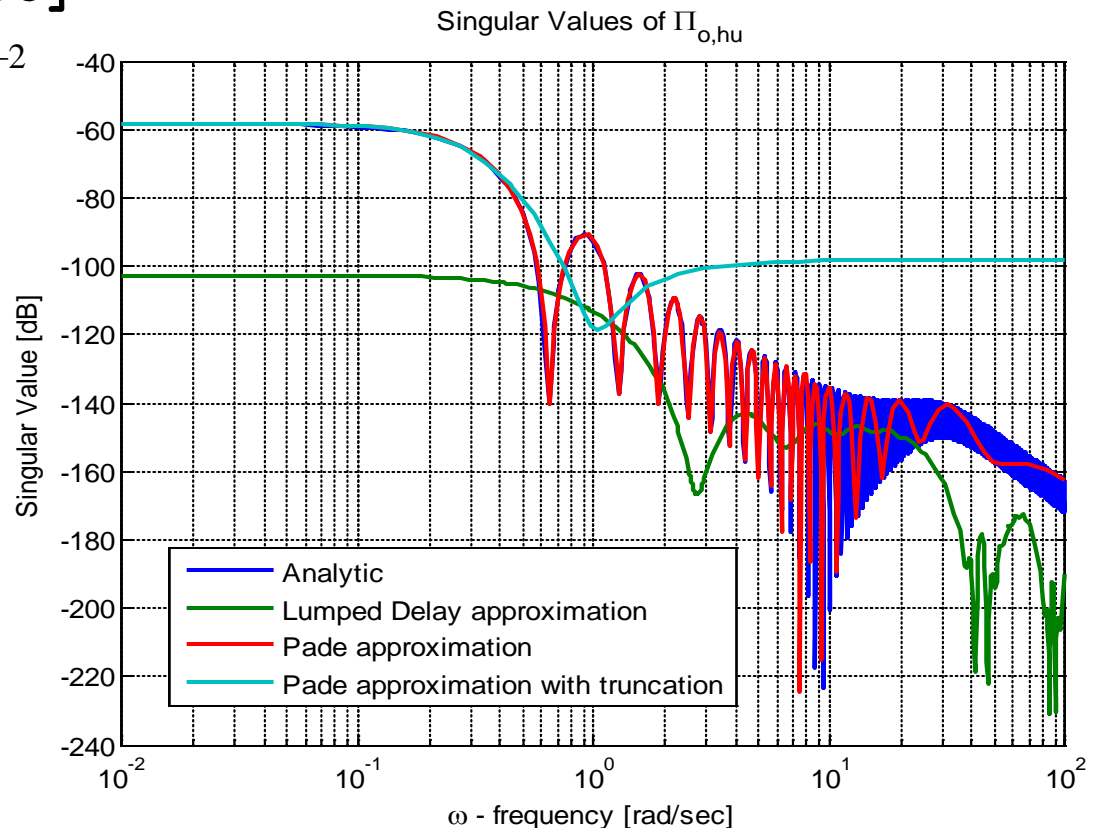
PA: $n_p = 40, \epsilon = 10^{-2}$

Computation times:
Constant input for 10 sec
with sampling of 0.01sec

$t_{lda} = 0.0439[\text{sec}]$

$t_{pa} = 0.0597[\text{sec}]$

$t_{pa, truncated} = 0.0445[\text{sec}]$



The original quadruple-tank process (QTP)

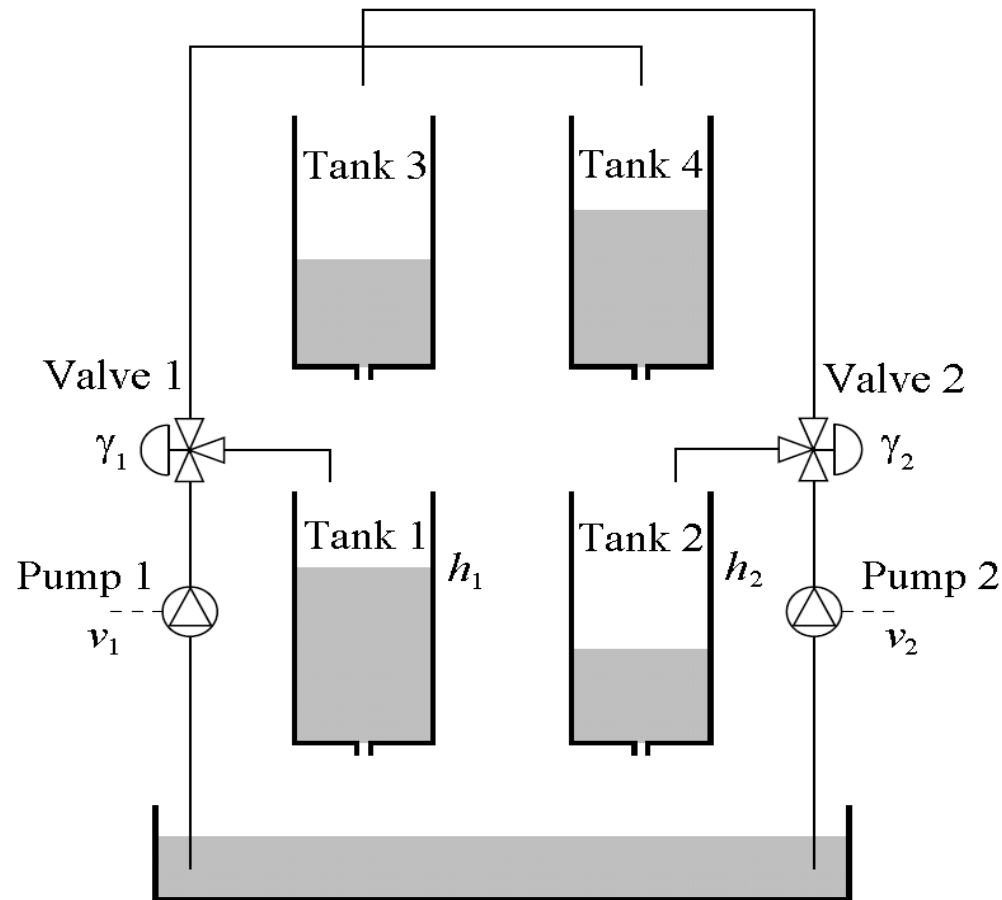
v_1, v_2 - voltages to pumps

$\gamma_1, \gamma_2 \in (0,1)$ - flow dividers

h_1, h_2 - outputs

NMP $0 < \gamma_1 + \gamma_2 < 1$

MP $1 < \gamma_1 + \gamma_2 < 2$



The quadruple-tank process with dead-times (QTPwDT)

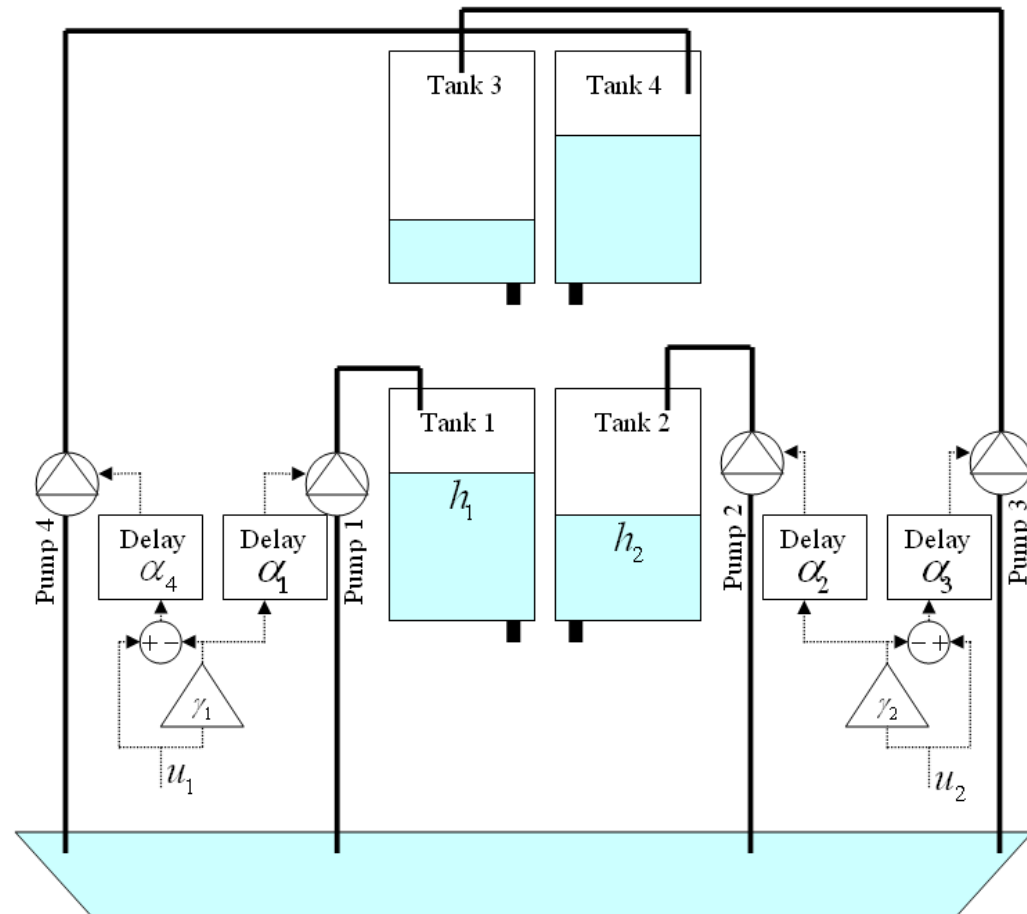
u_1, u_2 - voltages to pump

$\gamma_1, \gamma_2 \in (0,1)$ - flow dividers

h_1, h_2 - outputs

✓ Linearized plant:

$$P(s) = \begin{bmatrix} P_{11}(s)e^{-s\alpha_1} & P_{12}(s)e^{-s\alpha_3} \\ P_{21}(s)e^{-s\alpha_4} & P_{22}(s)e^{-s\alpha_2} \end{bmatrix}$$



Properties of QTPwDT*

✓ Depend on β and $G(0)$ where

$$\beta \triangleq \alpha_1 + \alpha_2 - \alpha_3 - \alpha_4$$

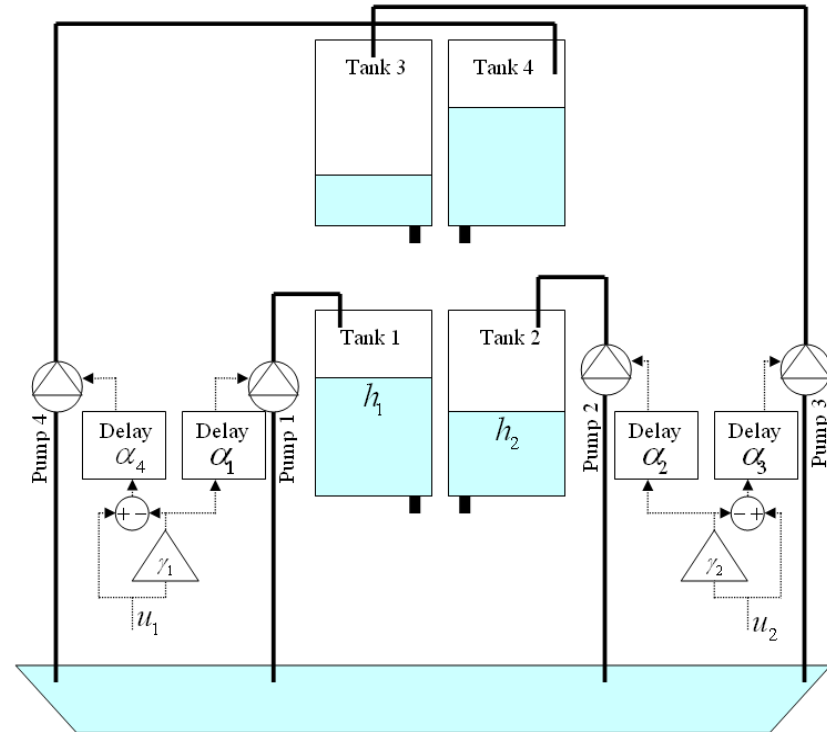
$$G(s) \triangleq \frac{-K \cdot (1 - \gamma_1) \cdot (1 - \gamma_2) / (\gamma_1 \gamma_2)}{(1 + s \cdot T_3) \cdot (1 + s \cdot T_4)} \cdot e^{s \cdot \beta}$$

$$K \triangleq k_3 k_4 / (k_1 k_2)$$

✓ if $\beta \leq 0$ and $G(0) < 1$ - no NMP zeros
 if $\beta \leq 0$ and $G(0) > 1$ - at least one NMP zero

✓ if $\beta > 0$ - infinite NMP zeros
 $G(0)$ determines the behavior of the dominant NMP zeros

* (Shneiderman & Palmor, 2010)



Experimental set-up

Flowmeters

Gear pumps:

Capacity - 2.3 [lit/min]

Voltage - 3:12 [V]

Gain - k_i [$\text{cm}^3/(\text{s.V})$]

Tanks

Height - 23 [cm]

Cross section - A_i [cm^2]

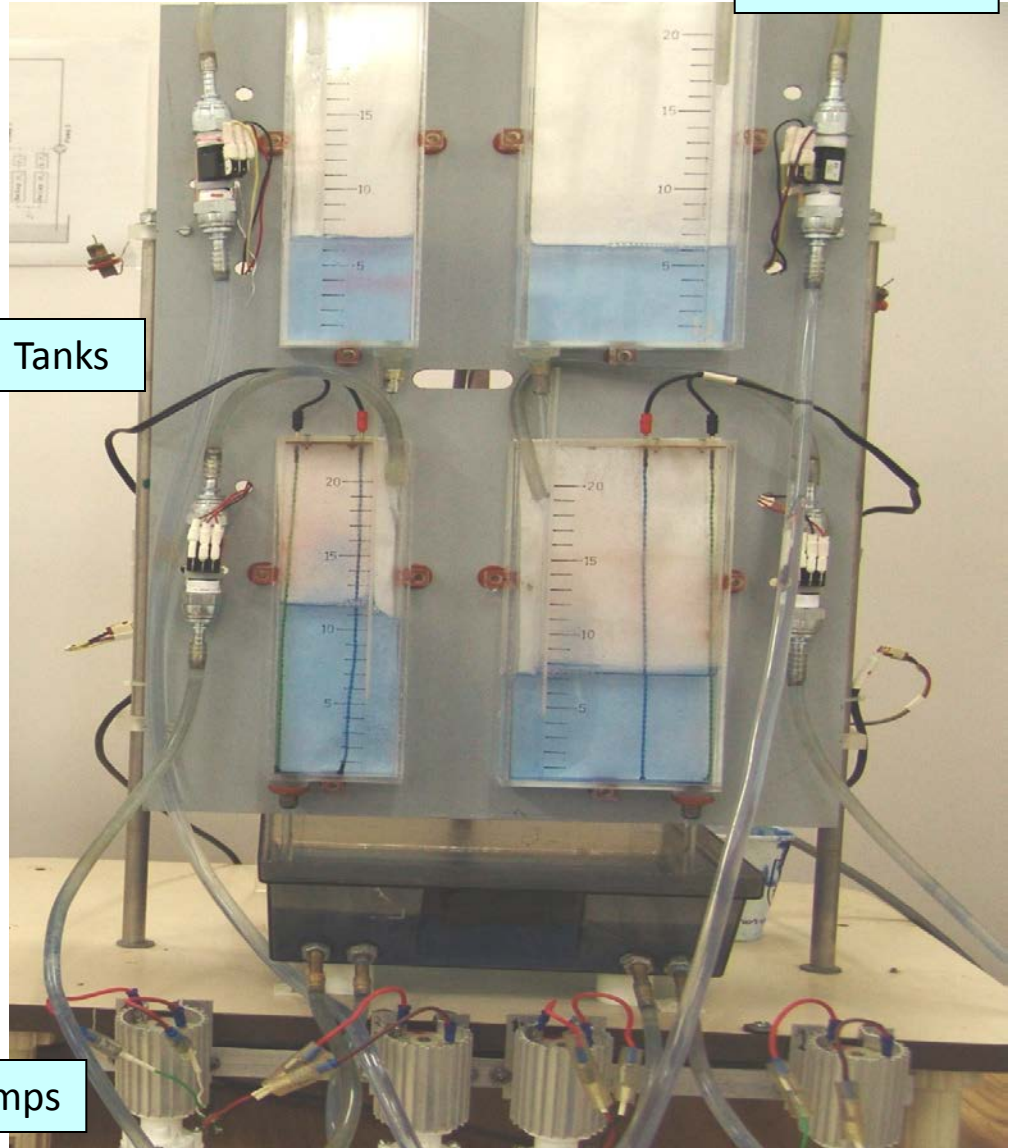
Cross section outlet: a_i [cm^2]

γ_i - voltage dividers

Servo control on each pump.

Tanks

Pumps



Experimental set-up



Modeling and data

Linearized model:

$$P(s) = \begin{bmatrix} \frac{\gamma_1 T_1 k_1 / A_1 \cdot e^{-s\alpha_1}}{(1 + sT_1)} & \frac{(1 - \gamma_2) T_1 k_3 / A_1 \cdot e^{-s\alpha_3}}{(1 + sT_3)(1 + sT_1)} \\ \frac{(1 - \gamma_1) T_2 k_4 / A_2 \cdot e^{-s\alpha_4}}{(1 + sT_4)(1 + sT_2)} & \frac{\gamma_2 T_2 k_2 / A_2 \cdot e^{-s\alpha_2}}{(1 + sT_2)} \end{bmatrix}$$

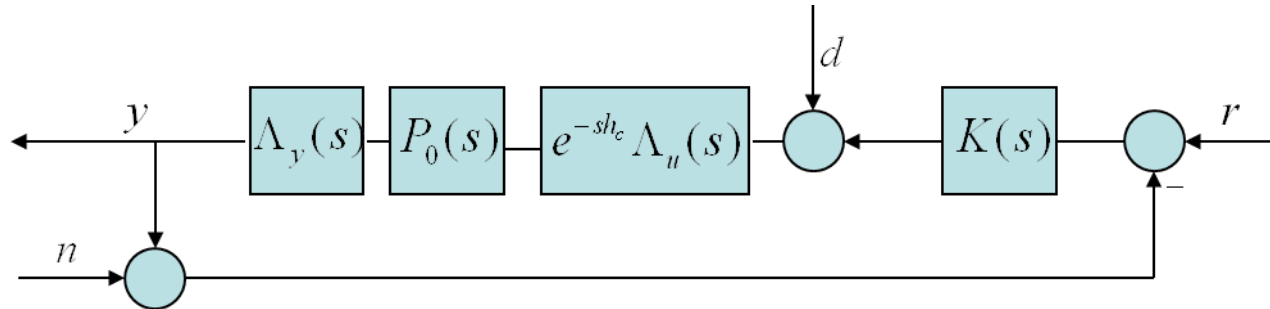
$$T_i = \frac{A_i}{a_i} \sqrt{\frac{2h_{i,ss}}{g}}$$

Parameter values of the laboratory QTPwDT.

i	A_i (cm ²)	a_i (cm ²)	k_i (cm ³ /(s · V))	k_{i2} (cm ³ /s)
1	12	0.26	3.808	2.712
2	20	0.26	3.831	2.006
3	12	0.21	3.904	3.069
4	20	0.21	3.752	3.499

Control Design

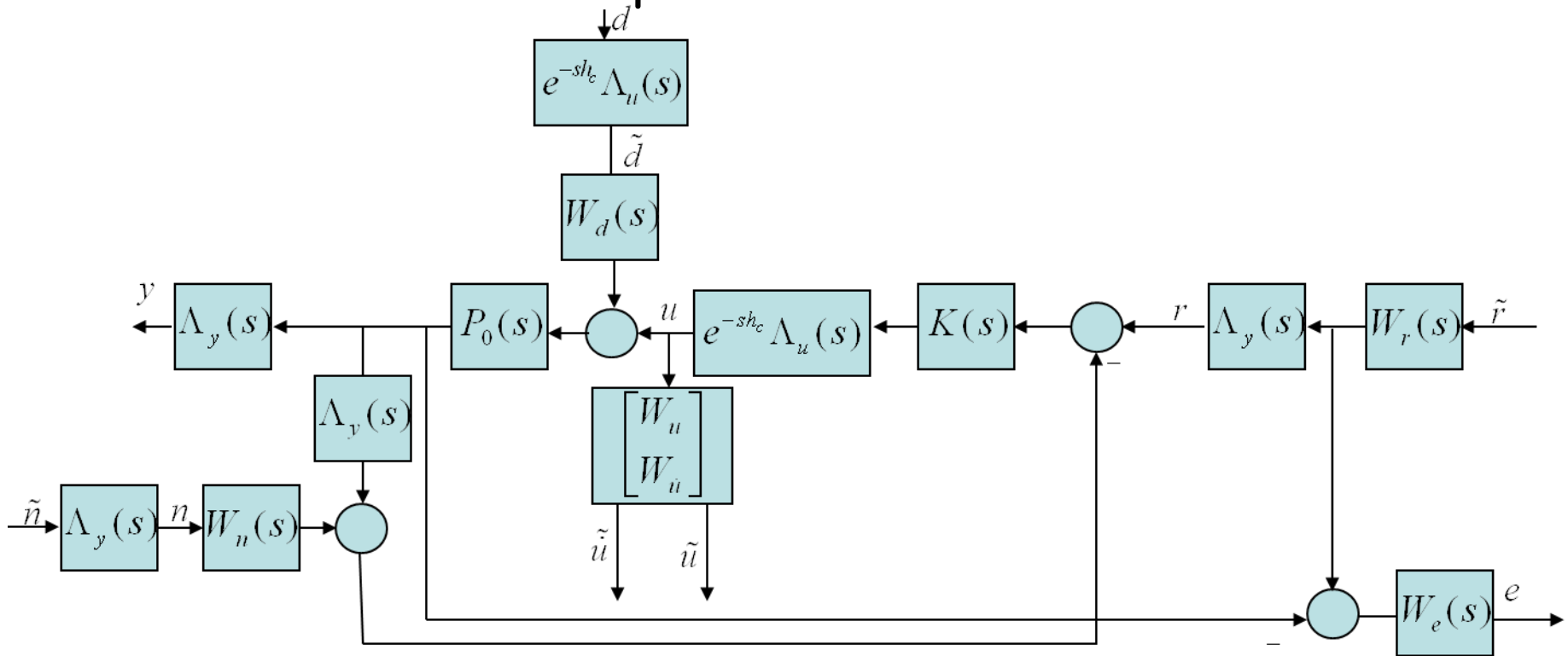
Standard configuration:



- ✓ 2x2 Process with adobe I/O delays + a common delay h_c
- ✓ $K(s)$ either FASP or GMDC
- ✓ Controllers implemented via Simulink on dSPACE card

FASP -H² Design

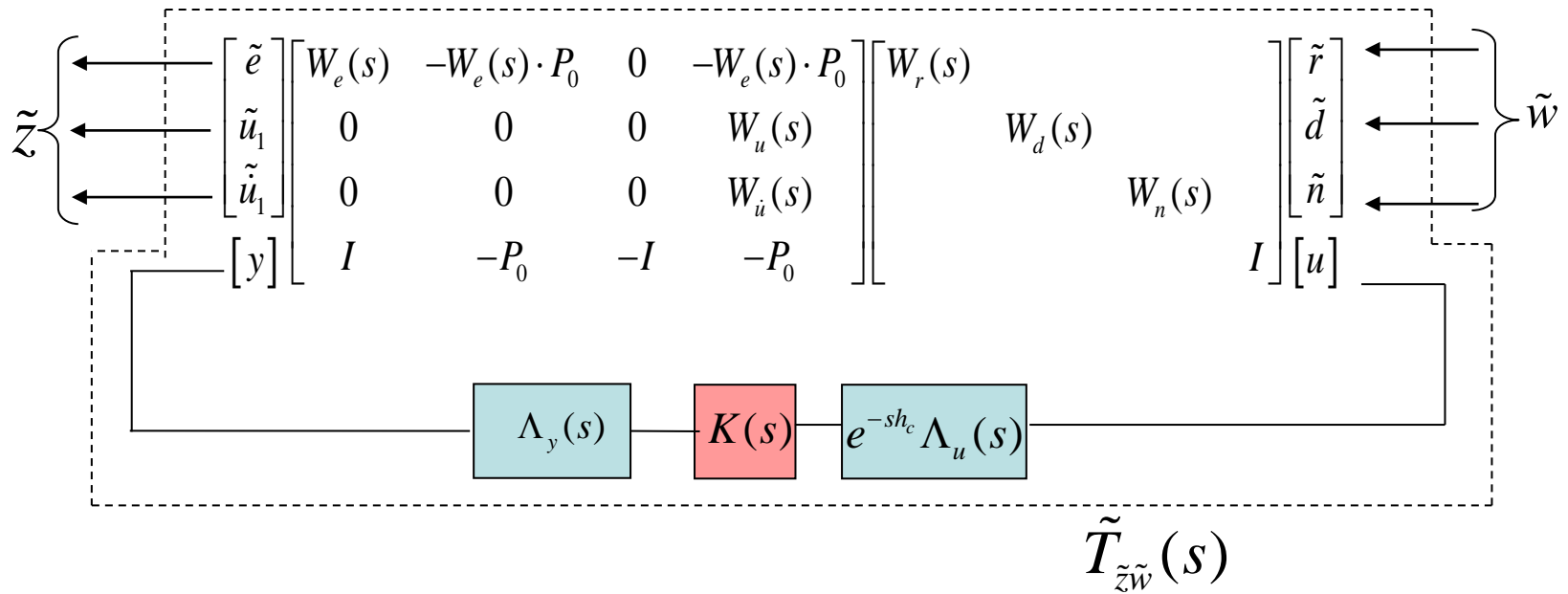
Generalized control problem:



weights added for performance and for guaranteeing solvability

FASP - H^2 Control Design

Generalized control problem:



Find internally stabilizing causal $K(s)$ that minimizes $\|\tilde{T}_{\tilde{z}\tilde{w}}\|_2$

GMDC Design

Primary controller - a diagonal PI

tuned to have either similar closed-loop bandwidth
or similar control effort as the corresponding FASP

Experimental cases

- 1) Cases with adobe I/O DTs + a common DT
 - a A MP case
 - b A NMP case
 - 2) Cases with partial DT extraction
- ✓ Implementation of FASP:
- Primary controller truncated
 - FF blocks approximated through PA
 - Predictor blocks approximate via LDA

Experimental results

Case 1a: MP case

$$\alpha_1 = 10, \alpha_2 = 25, \alpha_3 = 20, \alpha_4 = 15$$

$$\gamma_1 = 0.65, \gamma_2 = 0.6$$

$$h_c = 10, \Lambda_y = \begin{bmatrix} 1 \\ e^{-5s} \end{bmatrix}, \Lambda_u = \begin{bmatrix} 1 \\ e^{-10s} \end{bmatrix} \left. \begin{array}{l} \beta = 0 \\ |G(0)| = 0.294 \end{array} \right\}$$

$$\text{Fasp: } W_r(s) = W_d(s) = \frac{1}{s + 0.0005}; W_e(s) = 12000 \begin{bmatrix} \frac{10}{s+10} & 0 \\ 0 & \frac{1}{s+1} \end{bmatrix};$$

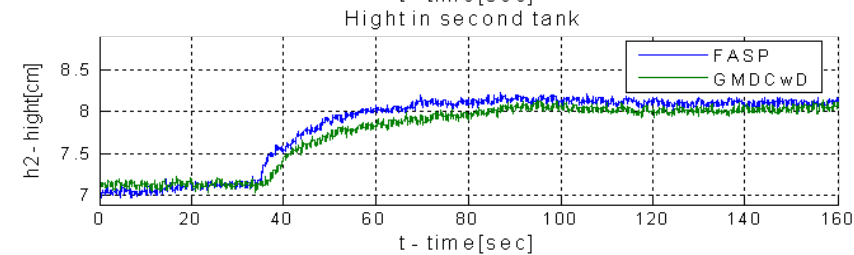
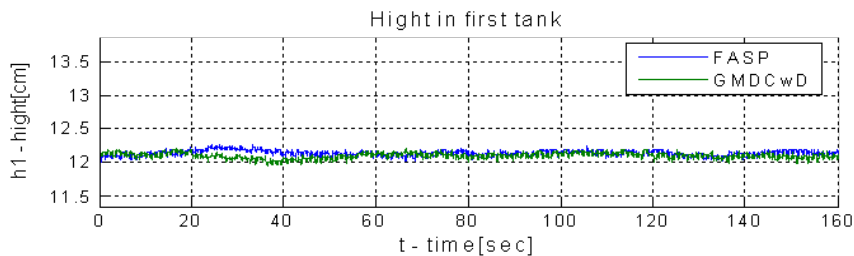
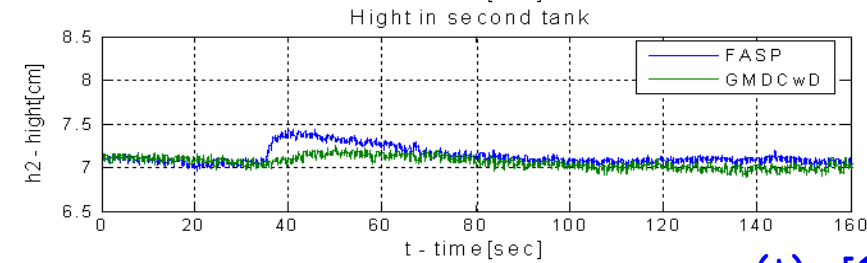
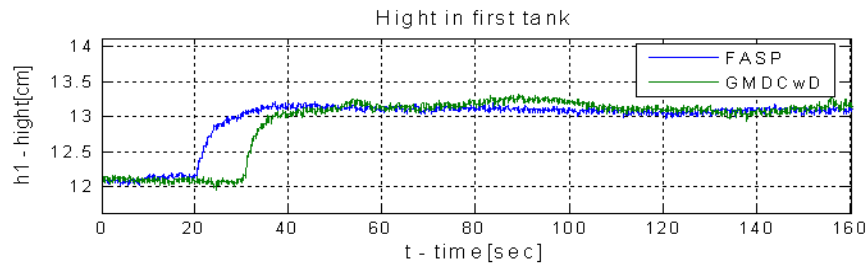
$$W_n(s) = 8; W_u(s) = 0.001; W_{\dot{u}}(s) = \frac{s}{0.005s + 1}$$

$$\text{GMDC: } D = \begin{bmatrix} e^{-10s} \\ 1 \end{bmatrix}$$

Case 1a - servo

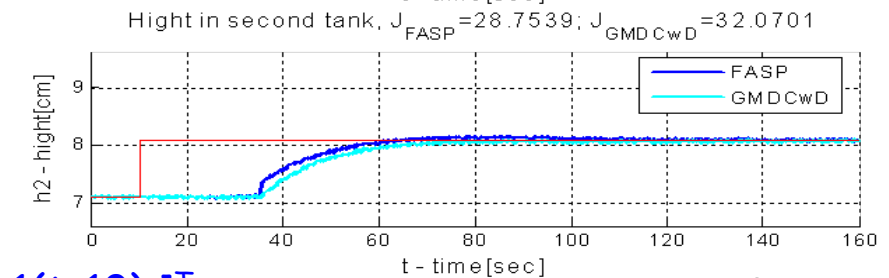
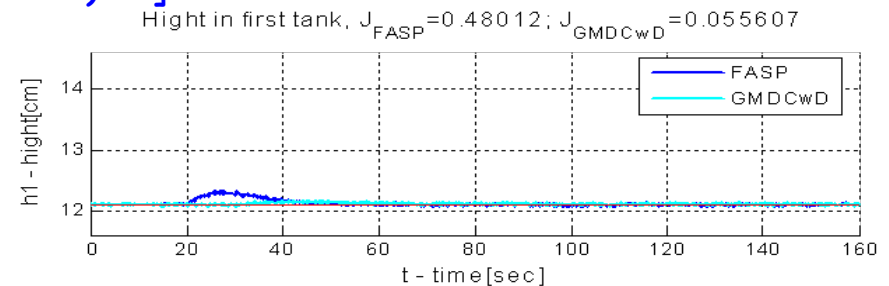
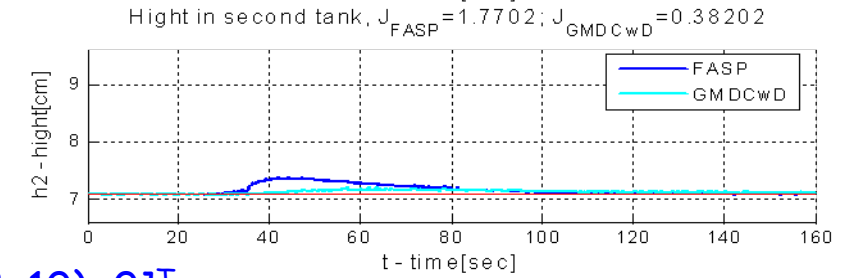
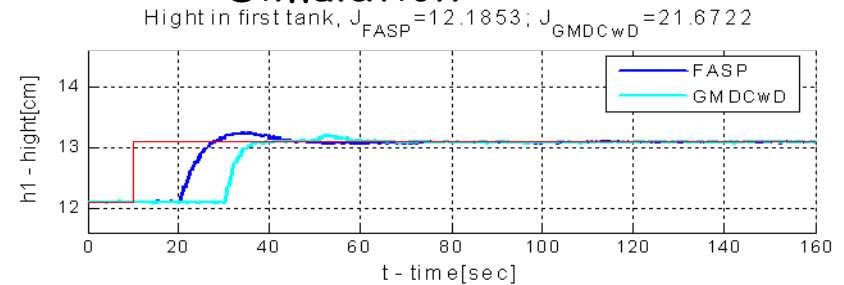
Experiment

Responses to Step Change in References



$$r(t) = [1(t-10) \ 0]^T$$

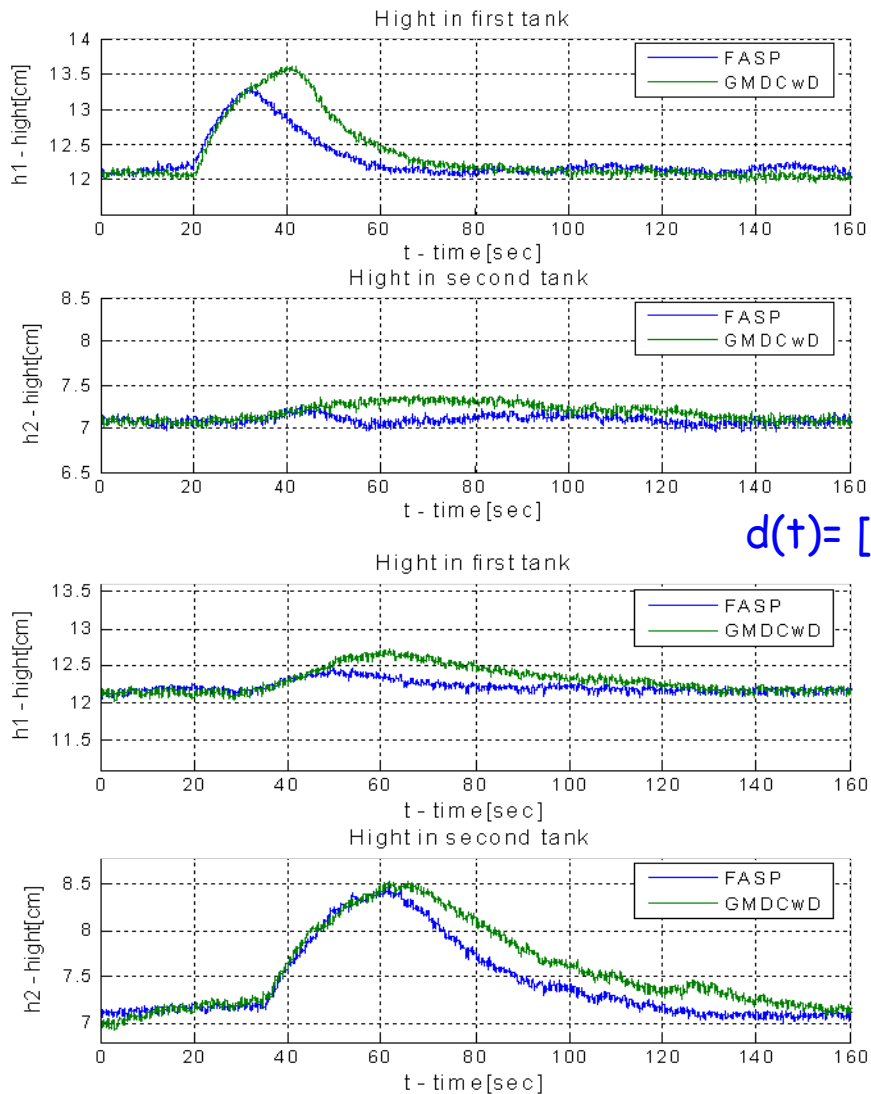
Simulation



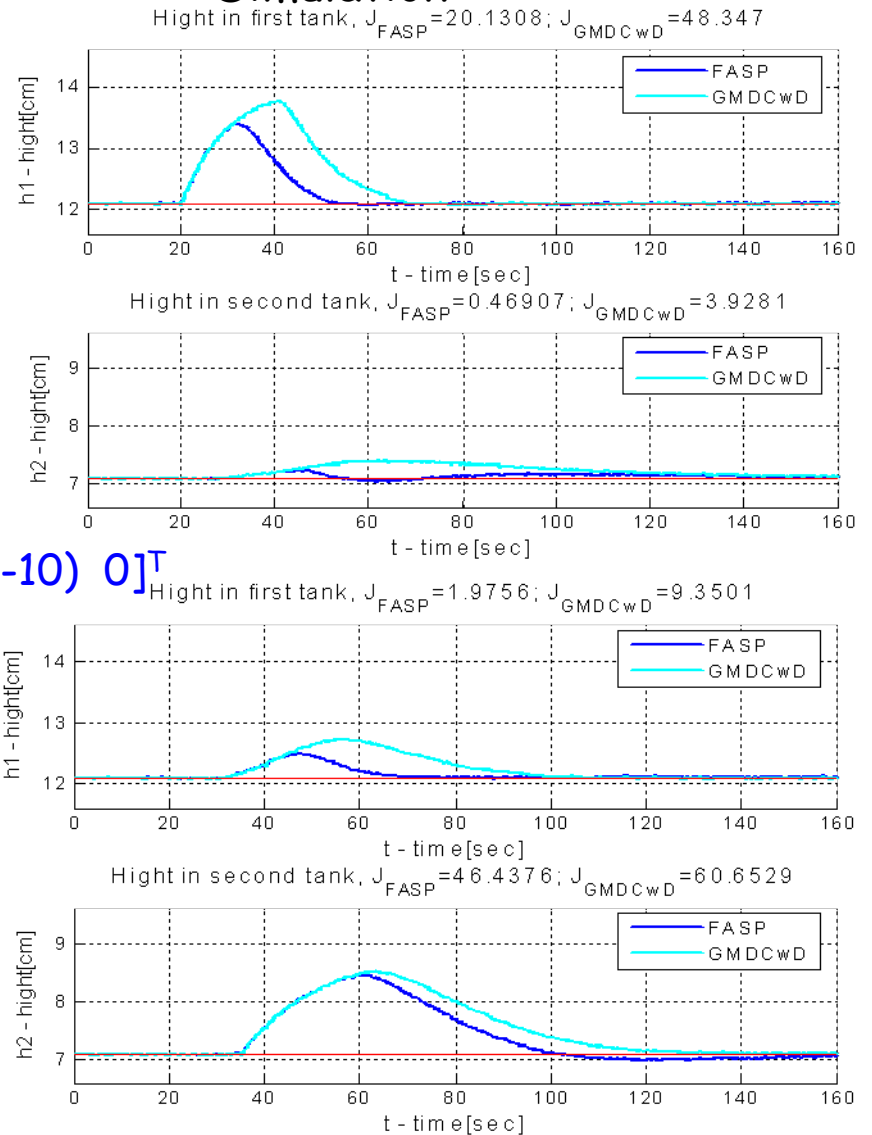
$$r(t) = [0 \ 1(t-10)]^T$$

Case 1a - Disturbance attenuation

Experiment



Simulation



$$d(t) = [1(t-10) \ 0]^T$$

$$d(t) = [0 \ 1(t-10)]^T$$

Experimental results

Case 1b: NMP case

$$\alpha_1 = 10, \alpha_2 = 25, \alpha_3 = 20, \alpha_4 = 15$$

$$\gamma_1 = 0.4, \gamma_2 = 0.35$$

$$h_c = 10, \Lambda_y = \begin{bmatrix} 1 & \\ & e^{-5s} \end{bmatrix}, \Lambda_u = \begin{bmatrix} 1 & \\ & e^{-10s} \end{bmatrix}$$

$$\left. \begin{array}{l} \beta = 0 \\ |G(0)| = 2.633 \end{array} \right\}$$

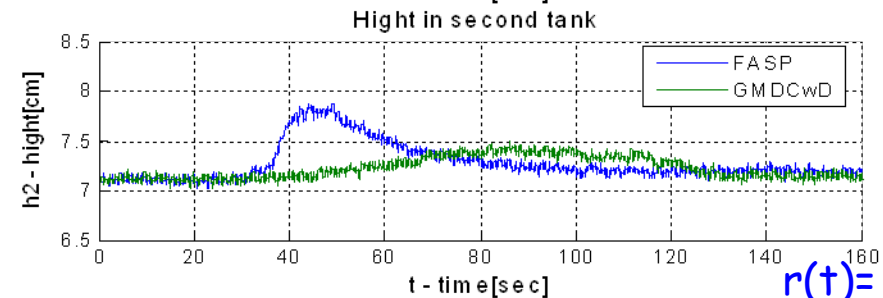
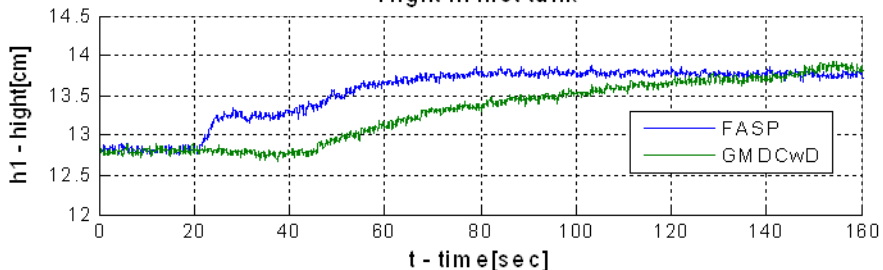
Fasp: $W_r(s) = W_d(s) = \frac{1}{s + 0.0005}; W_e(s) = \frac{20}{s + 1}; W_n(s) = 7.5;$

$$W_u(s) = 0.001; W_i(s) = \frac{s}{0.005s + 1}$$

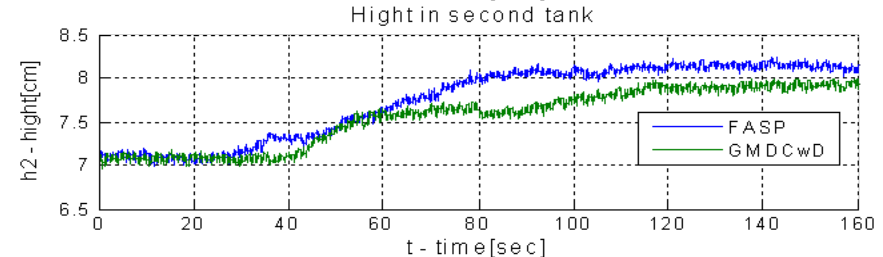
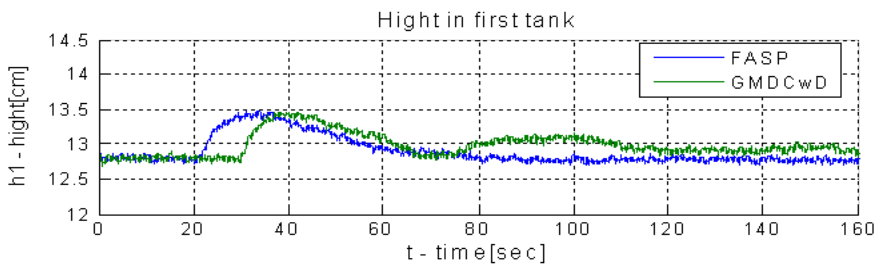
GMDC: $D = \begin{bmatrix} e^{-10s} & \\ & 1 \end{bmatrix}$

Case 1b - servo Responses to Step Change in References

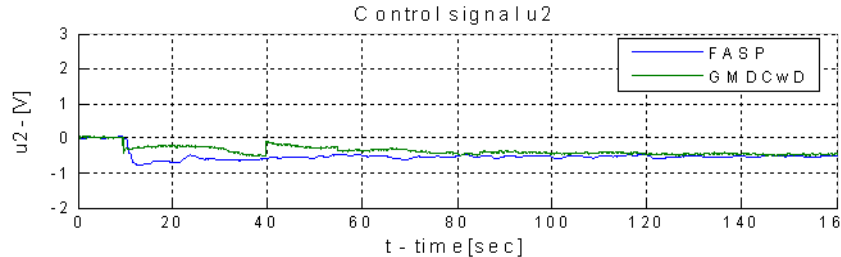
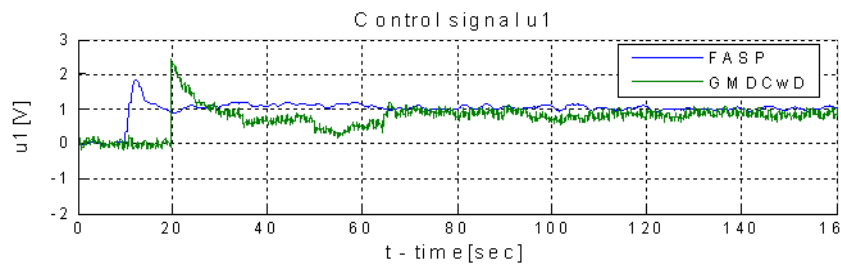
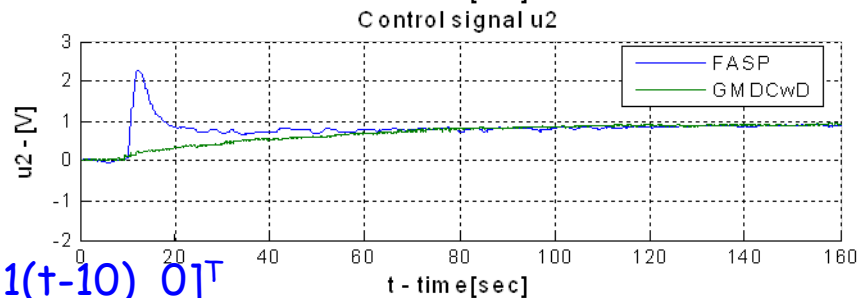
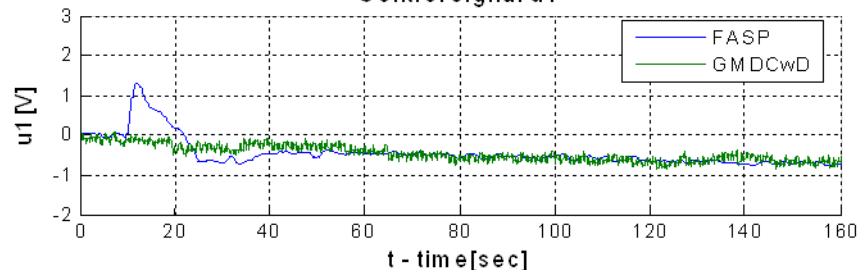
Liquid heights



$$r(t) = [1(t-10) \ 0]^T$$



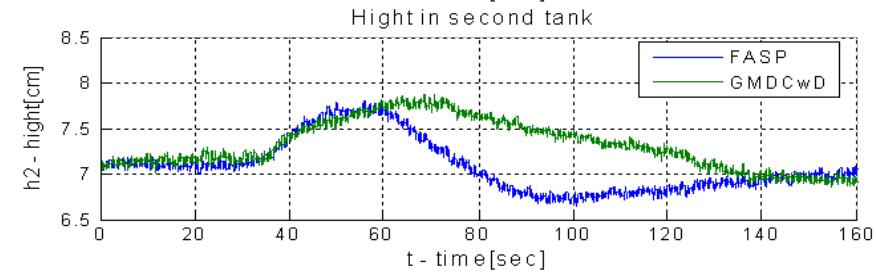
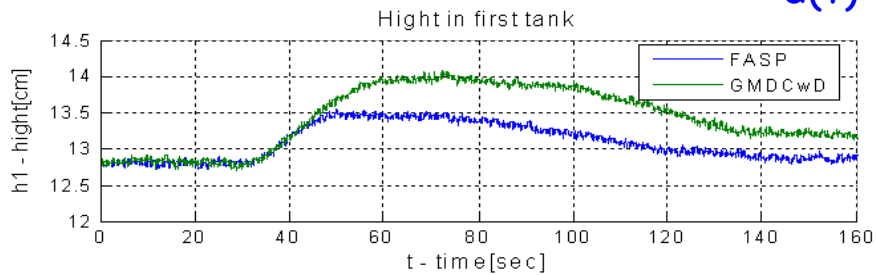
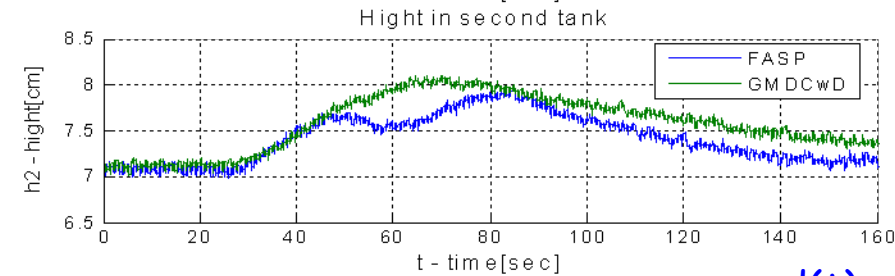
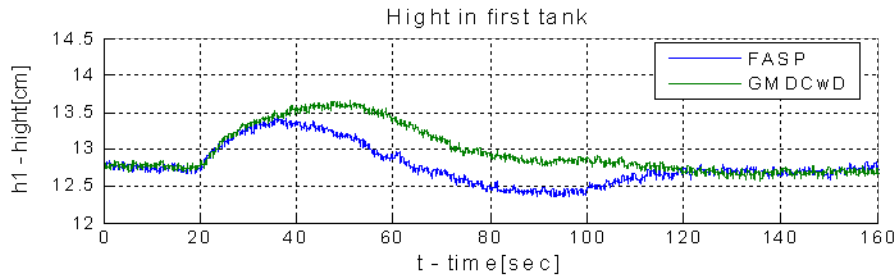
Control effort



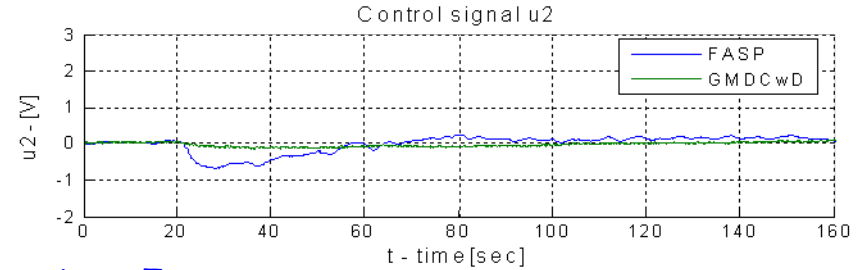
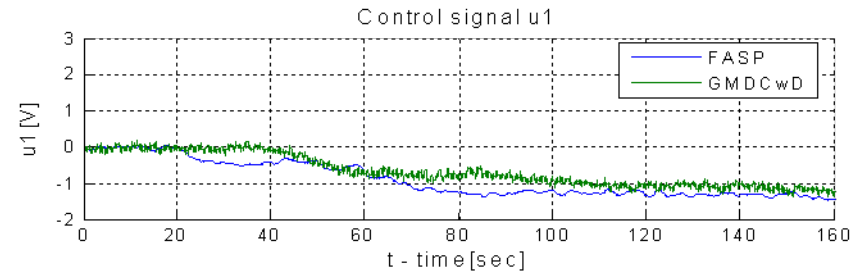
$$r(t) = [0 \ 1(t-10)]^T$$

Case 1b - Disturbance attenuation

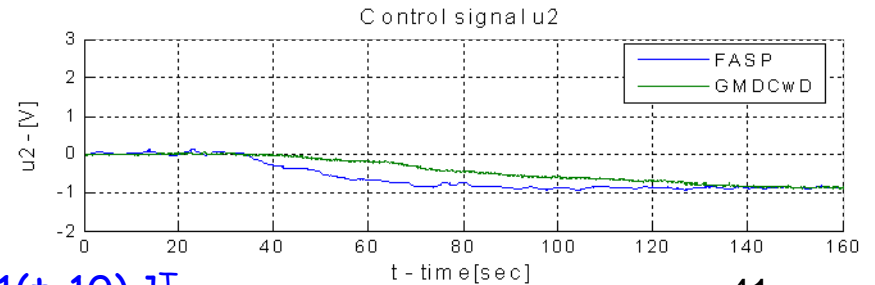
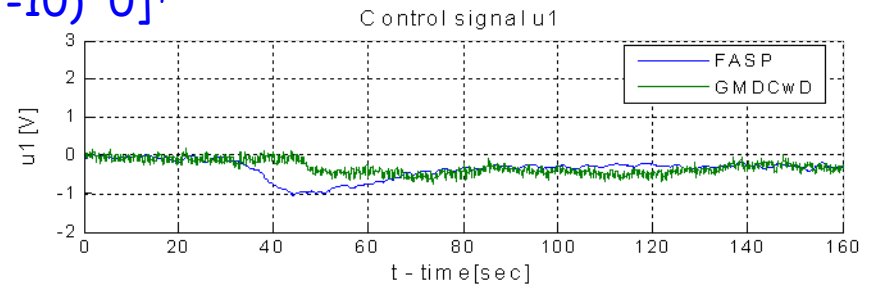
Liquid heights



Control effort



$$d(t) = [1(t-10) \ 0]^T$$



$$d(t) = [0 \ 1(t-10)]^T$$

Experimental results

2) 5 MP and 5 NMP Cases with full/partial DT extraction

$$\begin{aligned} 1 - \begin{bmatrix} 10 & 20 \\ 15 & 25 \end{bmatrix} &\Rightarrow \Lambda_u = \begin{bmatrix} 0 & \\ & 10 \end{bmatrix}, \Lambda_y = \begin{bmatrix} 0 & \\ & 5 \end{bmatrix}, h_c = 10, \quad \eta = 0 \quad \Rightarrow D = \begin{bmatrix} 10 & \\ & 0 \end{bmatrix} \\ 2 - \begin{bmatrix} 10 & 14 \\ 18 & 12 \end{bmatrix} &\Rightarrow \Lambda_u = \begin{bmatrix} 0 & \\ & 1 \end{bmatrix}, \Lambda_y = \begin{bmatrix} 0 & \\ & 1 \end{bmatrix}, h_c = 10, \quad \eta = \begin{bmatrix} 0 & 3 \\ 7 & 0 \end{bmatrix} \quad \Rightarrow D = \begin{bmatrix} 0 & \\ & 0 \end{bmatrix} \\ 3 - \begin{bmatrix} 10 & 18 \\ 22 & 20 \end{bmatrix} &\Rightarrow \Lambda_u = \begin{bmatrix} 0 & \\ & 5 \end{bmatrix}, \Lambda_y = \begin{bmatrix} 0 & \\ & 5 \end{bmatrix}, h_c = 10, \quad \eta = \begin{bmatrix} 0 & 3 \\ 7 & 0 \end{bmatrix} \quad \Rightarrow D = \begin{bmatrix} 0 & \\ & 0 \end{bmatrix} \\ 4 - \begin{bmatrix} 10 & 22 \\ 18 & 20 \end{bmatrix} &\Rightarrow \Lambda_u = \begin{bmatrix} 0 & \\ & 5 \end{bmatrix}, \Lambda_y = \begin{bmatrix} 0 & \\ & 5 \end{bmatrix}, h_c = 10, \quad \eta = \begin{bmatrix} 0 & 7 \\ 3 & 0 \end{bmatrix} \quad \Rightarrow D = \begin{bmatrix} 2 & \\ & 0 \end{bmatrix} \\ 5 - \begin{bmatrix} 5 & 17 \\ 13 & 20 \end{bmatrix} &\Rightarrow \Lambda_u = \begin{bmatrix} 0 & \\ & 9.5 \end{bmatrix}, \Lambda_y = \begin{bmatrix} 0 & \\ & 5.5 \end{bmatrix}, h_c = 5, \quad \eta = \begin{bmatrix} 0 & 2.5 \\ 2.5 & 0 \end{bmatrix} \quad \Rightarrow D = \begin{bmatrix} 7 & \\ & 0 \end{bmatrix} \end{aligned}$$

Experimental results - MP cases

DT distribution	Extraction (full/partial)	Similar BW or Control effort	Overall ISE for step in $r(t)$				Overall ISE for step in $d(t)$			
			$r(t) = \begin{bmatrix} 1(t-10) \\ 0 \end{bmatrix}$		$r(t) = \begin{bmatrix} 0 \\ 1(t-10) \end{bmatrix}$		$d(t) = \begin{bmatrix} 1(t-10) \\ 0 \end{bmatrix}$		$d(t) = \begin{bmatrix} 0 \\ 1(t-10) \end{bmatrix}$	
			FASP	GMDC	FASP	GMDC	FASP	GMDC	FASP	GMDC
$1 - \begin{bmatrix} 10 & 20 \\ 15 & 25 \end{bmatrix}$	Full	BW	<u>13.36</u>	22.07	<u>29.52</u>	32.14	<u>19.36</u>	52.3	<u>49.51</u>	70.01
$2 - \begin{bmatrix} 10 & 14 \\ 18 & 12 \end{bmatrix}$	Partial	BW	11.85	<u>11.56</u>	14.65	<u>13.83</u>	<u>18.67</u>	19.83	<u>17.05</u>	17.31
$3 - \begin{bmatrix} 10 & 18 \\ 22 & 20 \end{bmatrix}$	Partial	Control	12.22	<u>11.58</u>	<u>22.63</u>	22.77	<u>18.64</u>	21.44	<u>32.27</u>	37.06
$4 - \begin{bmatrix} 10 & 22 \\ 18 & 20 \end{bmatrix}$	Partial	Control	<u>12.85</u>	13.41	23.34	<u>22.91</u>	<u>20.33</u>	25.83	<u>32.78</u>	38.59
$5 - \begin{bmatrix} 5 & 17 \\ 13 & 20 \end{bmatrix}$	Partial	Control	<u>7.82</u>	13.95	<u>23.63</u>	28.66	<u>6.69</u>	25.68	<u>33.68</u>	55.88

Experimental results - NMP cases

DT distribution	Extract ion (full/partial)	Similar BW or Control effort	Overall ISE for step in $r(t)$				Overall ISE for step in $d(t)$			
			$r(t) = \begin{bmatrix} 1(t-10) \\ 0 \end{bmatrix}$		$r(t) = \begin{bmatrix} 0 \\ 1(t-10) \end{bmatrix}$		$d(t) = \begin{bmatrix} 1(t-10) \\ 0 \end{bmatrix}$		$d(t) = \begin{bmatrix} 0 \\ 1(t-10) \end{bmatrix}$	
			FASP	GMDC	FASP	GMDC	FASP	GMDC	FASP	GMDC
1- $\begin{bmatrix} 10 & 20 \\ 15 & 25 \end{bmatrix}$	Full	BW	<u>41.04</u>	66.16	<u>50</u>	60.89	<u>34.42</u>	85.76	<u>37.8</u>	112.8
2- $\begin{bmatrix} 10 & 14 \\ 18 & 12 \end{bmatrix}$	Partial	Control	<u>42.19</u>	52.9	<u>60.3</u>	68	<u>45.02</u>	71.84	<u>34.91</u>	55.98
3- $\begin{bmatrix} 10 & 18 \\ 22 & 20 \end{bmatrix}$	Partial	BW	<u>45.31</u>	61.65	<u>66.47</u>	73.82	<u>48.25</u>	85.38	<u>42.21</u>	73.19
4- $\begin{bmatrix} 10 & 22 \\ 18 & 20 \end{bmatrix}$	Partial	BW	<u>54.65</u>	90.61	<u>64.71</u>	71.47	<u>48.26</u>	86.48	<u>53.87</u>	112.36
5- $\begin{bmatrix} 5 & 17 \\ 13 & 20 \end{bmatrix}$	Partial	BW	<u>42.36</u>	89.41	<u>52.9</u>	70.52	<u>31.57</u>	81.22	<u>31.36</u>	110.89

Summary and Conclusions

- ✓ The FASP controller has been designed , implemented and applied to a laboratory QTPwDT setup.
- ✓ The study demonstrates:
 - ✓ FASP implementation is feasible
 - ✓ FASP's rational primary controller may be truncated significantly with out performance degradation.
 - ✓ FIR blocks may be approximated either via the LDA or via PADE approximation. The QAM defined.
 - ✓ FASP outperforms GMDC in both setpoint tracking and disturbance rejection. Holds even in the more general DTs cases particularly for disturbance rejection.

Thank you for your
attention!