Dynamics and control of (bio-)robotic locomotion: Nonlinear, nonholonomic and hybrid mechanical systems

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Background and motivation

Locomotion: ability of creature/robot/vehicle to propel itself

= internal motion/actuation + physical interaction with environment

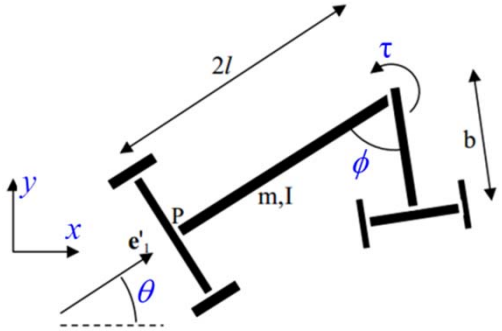
Research Goal: analyze simple models of locomotion
as dynamics and control systems

Theoretical challenges:

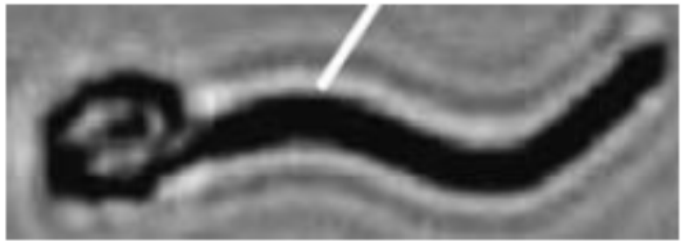
mechanical modeling, nonlinear dynamics, (non-)holonomic constraints, underactuation, geometric mechanics and symmetries, hybrid dynamics (contact transitions, impacts), orbital stability, optimization, control authority & sensing, control design

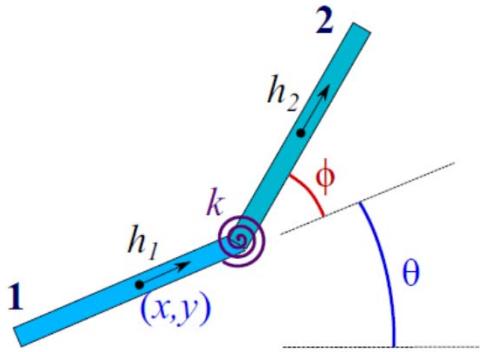
Outline

1. The Twistcar toy (Ofir Chakon, MSc work)



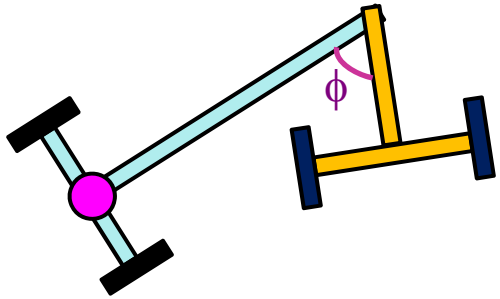
2. Microswimmers (Emiliya Gutman, PhD work)





3. Others: slippage effects on legged locomotion
senior project - reverse a truck & trailer

Twistcar - introduction



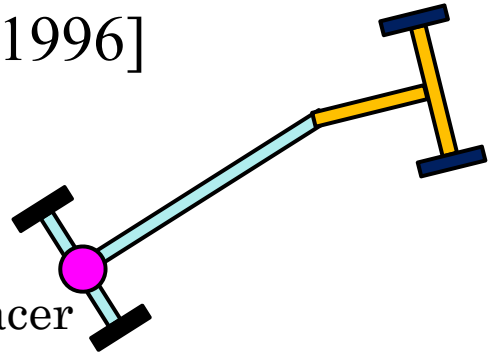
Planar two-link model

Previous works – "Roller Racer"

[Tsakiris 1995, Bullo & Zefran 2002, Bloch et al 1996]

ϕ oscillates about π

Roller Racer

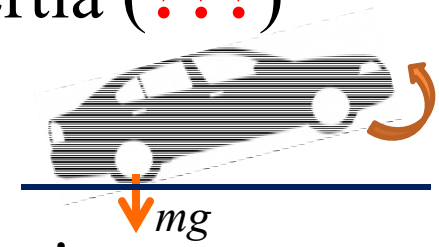


Assumptions made:

kinematic control of $\phi(t)$

steering link has zero mass and nonzero inertia (**???**)

body = **point mass** at back axle (**tipover!?**)



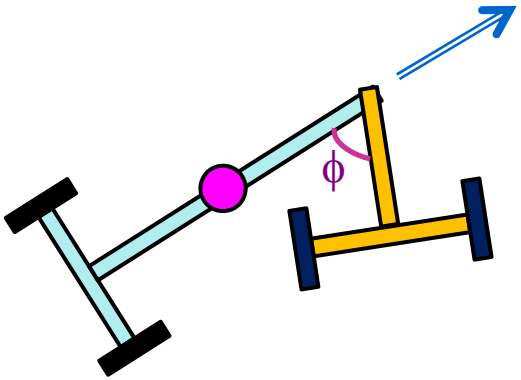
Used geometric mechanics, studied periodic gaits

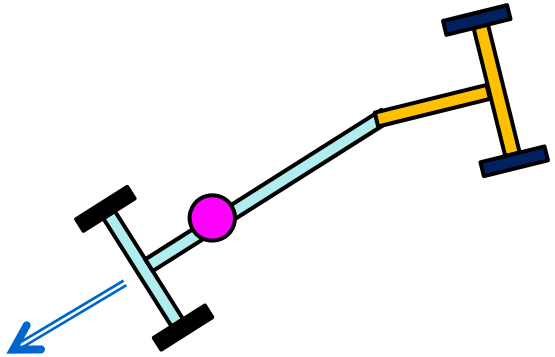
Twistcar – our goal

Our research goal:

Study both angle and torque input (harmonic)

For both Twistcar and Roller-Racer configurations





Analyze influence of parameters on dynamic behavior

Assume steering link w\ zero mass+inertia, point mass body

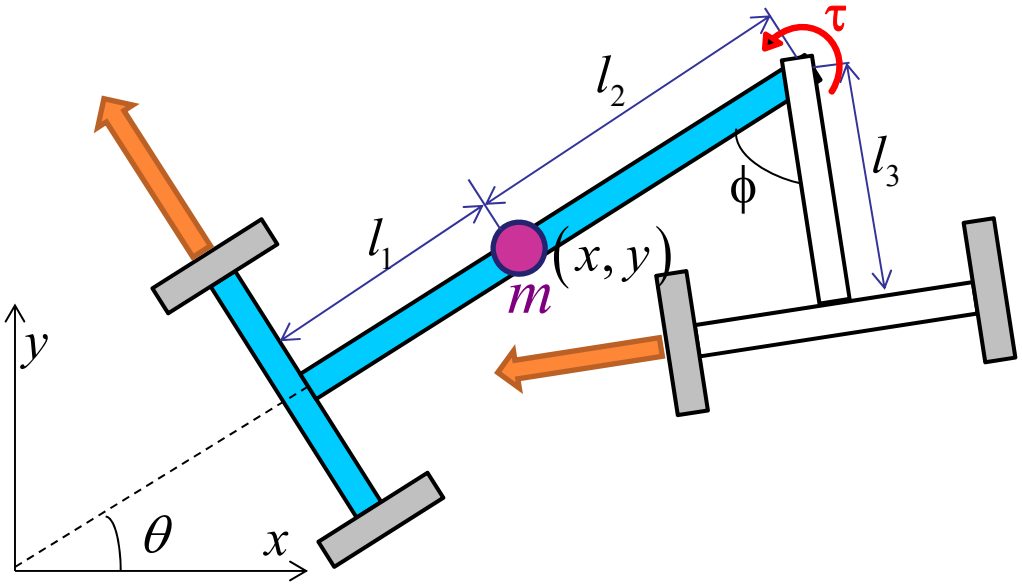
Methods: asymptotic analysis, numerics, simple experiments

Dynamic formulation

Coordinates: $\mathbf{q}=(x,y,\theta,\phi)$, input: $\tau(t)$ or $\phi(t)$

No-skid of wheels $\Longrightarrow$ two nonholonomic constraints

$$\mathbf{W}(\mathbf{q})\dot{\mathbf{q}}=0 \text{ , where } \mathbf{W}(\mathbf{q})=\begin{pmatrix} \sin(\theta) & -\cos(\theta) & l_1 & 0 \\ \sin(\theta+\phi) & -\cos(\theta+\phi) & l_3-l_2\cos(\phi) & l_3 \end{pmatrix}$$



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Constrained dynamic equations: $\boxed{\mathbf{M}(\mathbf{q})\ddot{\mathbf{q}}+\cancel{\mathbf{B}(\mathbf{q},\dot{\mathbf{q}})}=\mathbf{E}\tau+\mathbf{W}(\mathbf{q})^T\boldsymbol{\lambda}}$

$$\text{where } \mathbf{M}(\mathbf{q})=\begin{pmatrix} m & 0 & 0 & 0 \\ 0 & m & 0 & 0 \\ 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 \end{pmatrix},\ \mathbf{B}(\mathbf{q},\dot{\mathbf{q}})=0,\ \mathbf{E}=\begin{pmatrix} 0 \\ 0 \\ 0 \\ 1 \end{pmatrix},\ \boldsymbol{\lambda}=\begin{pmatrix} \lambda_1 \\ \lambda_2 \end{pmatrix} \text{ constraint forces}$$

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State-space formulation $\dot{\mathbf{x}}(t)=\mathbf{f}(\mathbf{x},\tau)$ where $\mathbf{x}=(\mathbf{q},\dot{\mathbf{q}})^T$

Differentiate the constraints: $\boxed{\dot{\mathbf{W}}\dot{\mathbf{q}}+\mathbf{W}\ddot{\mathbf{q}}=0}$

$$\Rightarrow \underbrace{\begin{pmatrix} \mathbf{M} & -\mathbf{W}^T \\ \mathbf{W} & 0 \end{pmatrix}}_{\mathbf{A}(\mathbf{q})_{6x6}} \begin{pmatrix} \ddot{\mathbf{q}} \\ \lambda \end{pmatrix} = \begin{pmatrix} \mathbf{E}\tau \\ -\dot{\mathbf{W}}\dot{\mathbf{q}} \end{pmatrix}$$

zero mass + inertia $\rightarrow$ $\mathbf{M}(\mathbf{q})$ is singular (avoided in previous works) but $\mathbf{A}(\mathbf{q})$ is not!!

Angle input: $\phi(t),\dot{\phi}(t),\ddot{\phi}(t)$ are known, $\tau(t)$ is eliminated

Simulation results 1 (movie)

1. Twistcar, steering angle input $\phi(t) = \phi_o \sin(\omega t)$

Forward motion, diverging oscillations of body orientation

Simulation results 2 (movie)

2. Twistcar, torque input $\tau(t) = \tau_o \sin(\omega t)$

Forward motion, decaying oscillations of body orientation
+ steering angle $\Longrightarrow$ straight line motion

Simulation results 3 (movie)

3. Roller-Racer, angle input $\phi(t) = \pi + \phi_o \sin(\omega t)$

Backward motion, diverging oscillations of body orientation

Simulation results 4 (movie)

4. Roller-Racer, angle input, different c.o.m. + lengths ratio

Reversed direction of motion,
diverging oscillations of body orientation

Simulation results 5 (movie)

5. Roller-Racer $\phi(0) = \pi$, torque input $\tau(t) = \tau_o \sin(\omega t)$

Steering angle converges to oscillations about $\phi=0$

$\Longrightarrow$ back to Twistcar configuration

Twistcar - Analysis

Reduced formulation:

1. Normalize time by $1/\omega$, length by l_1+l_2, mass by m

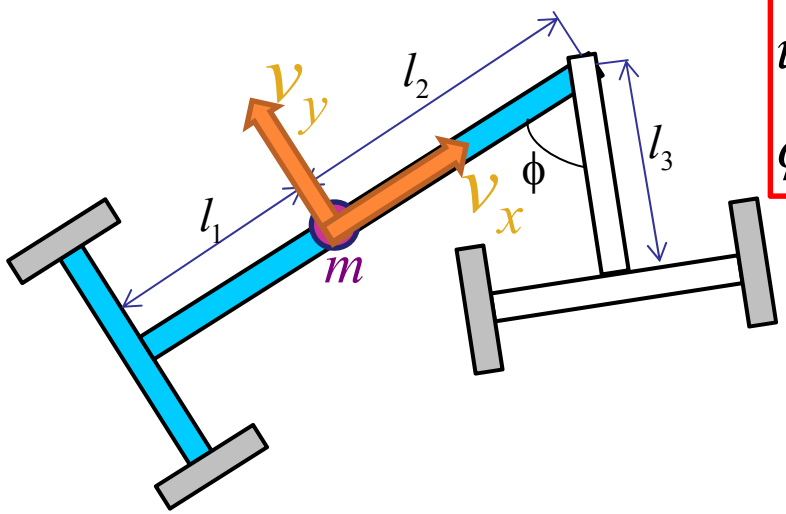
   two nondimensional parameters: $\alpha = \frac{l_3}{l_1+l_2}$, $\beta = \frac{l_1}{l_1+l_2}$

2. Body velocities $\mathbf{V} = (v_x, v_y, \dot{\theta}, \dot{\phi})^T$, constraints $\mathbf{W}(\mathbf{q})\dot{\mathbf{q}} = 0 \Rightarrow \tilde{\mathbf{W}}(\phi)\mathbf{V} = 0$

3. Constrained velocities $\mathbf{V} = \mathbf{w}_v(\phi)v + \mathbf{w}_u(\phi)u$, where $\tilde{\mathbf{W}}\mathbf{w}_u = \tilde{\mathbf{W}}\mathbf{w}_v = 0$

4. Reduced equations:

$$\boxed{\begin{aligned} \dot{v} &= f_v(\phi)v^2 + f_u(\phi)u^2 + f_{uv}(\phi)uv + f_\tau(\phi)\tau \\ \dot{u} &= g_v(\phi)v^2 + g_u(\phi)u^2 + g_{uv}(\phi)uv + g_\tau(\phi)\tau \\ \dot{\phi} &= h(\phi)u \end{aligned}}$$



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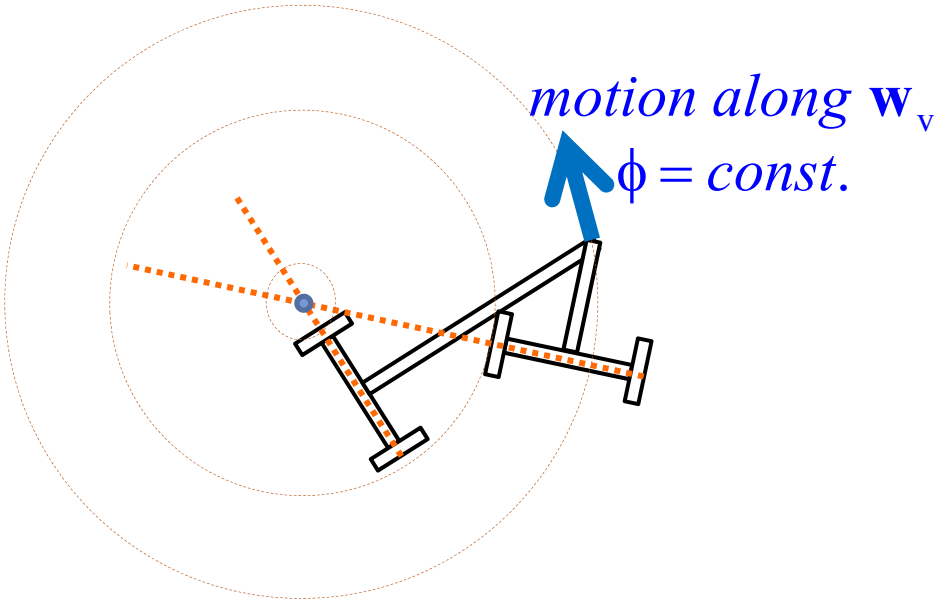

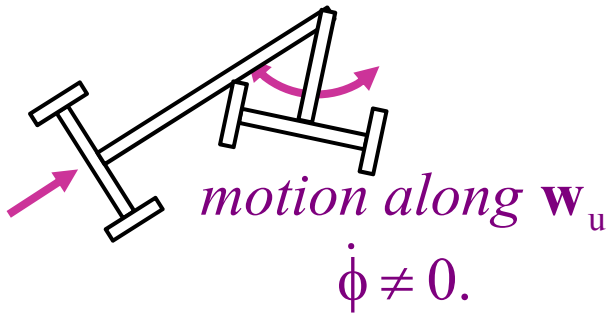

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5. Angle input: $\phi(t), \dot{\phi}(t), \ddot{\phi}(t) \rightarrow u(t), \dot{u}(t)$ known, eliminate τ

   $\Longrightarrow$ single linear time-varying ODE (integrable):

$$\boxed{\dot{v} = F\left(\phi(t), \dot{\phi}(t)\right)v + G\left(\phi(t), \dot{\phi}(t), \ddot{\phi}(t)\right)}$$

Perturbation expansion

Angle input: $\phi(t)=\varepsilon\sin t$, where $\varepsilon \ll 1$

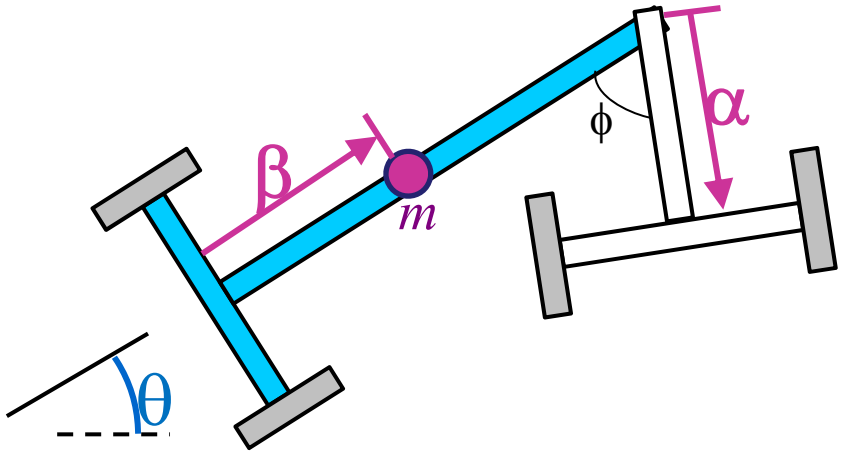
Assume a solution of the form $v(t)=v_0(t)+\varepsilon v_1(t)+\varepsilon^2 v_2(t)+\ldots$

Expand $\dot{v}=F(\phi,\dot{\phi})v+G(\phi,\dot{\phi},\ddot{\phi})$ as series in $\phi,\dot{\phi},\ddot{\phi}$, then of ε

Solution: $v(t)=v_0+\varepsilon^2\left(\boxed{\frac{\alpha\beta(\alpha+\beta)}{2(1-\alpha)^3}t}+A_1(\alpha,\beta)\sin(2t)+A_2(\alpha,\beta)v_0(1-\cos(2t))\right)+O(\varepsilon^3)$

Acceleration is monotonic in α,β, vanishes at $O(\varepsilon^2)$ for $\beta=0$.

Plug solution into $\dot{\phi}=h(\phi)u$, then $\mathbf{V}=\mathbf{w}_u(\phi)v+\mathbf{w}_v(\phi)u$, expand...



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Plug solution into $\dot{\phi}=h(\phi)u$, then $\mathbf{V}=\mathbf{w}_u(\phi)v+\mathbf{w}_v(\phi)u$, expand...

$$\theta(t)=\varepsilon\left(v_0(1-\cos t)-\frac{\alpha}{\alpha-1}\sin t\right)+\varepsilon^3\Big[\boxed{B_0(\alpha,\beta)t\cos t}+B_1(\alpha,\beta)\sin t$$
$$+B_2(\alpha,\beta)\sin(3t)+B_3(\alpha,\beta)v_0(8-9\cos(t)+\cos(3t))\Big]+O(\varepsilon^4)$$

$\Longrightarrow$ $O(\varepsilon^3)$ divergence of body orientation angle

Roller-Racer motion reversal

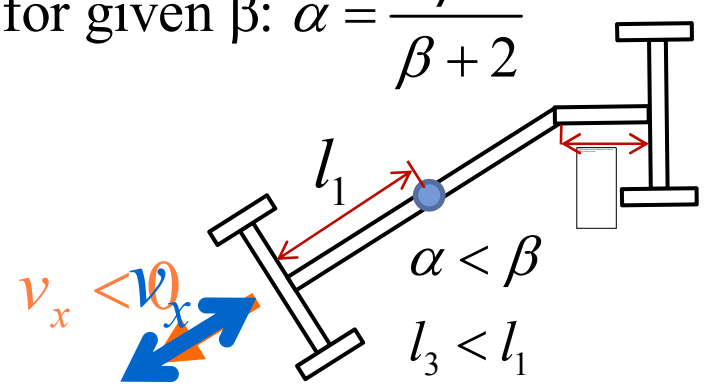
Angle input: $\phi(t)=\pi+\varepsilon\sin t$, where $\varepsilon \ll 1$. Expand, solve...

$$v_x(t) = -(\alpha+1)v_0 + \varepsilon^2\left[C_0(\alpha,\beta)v_0 + \boxed{\frac{\alpha\beta(\alpha-\beta)}{2(\alpha+1)^2}t}\right.$$
$$\left. + C_1(\alpha,\beta)\sin(2t) + C_2(\alpha,\beta)v_0\cos(2t)\right] + O(\varepsilon^3)$$

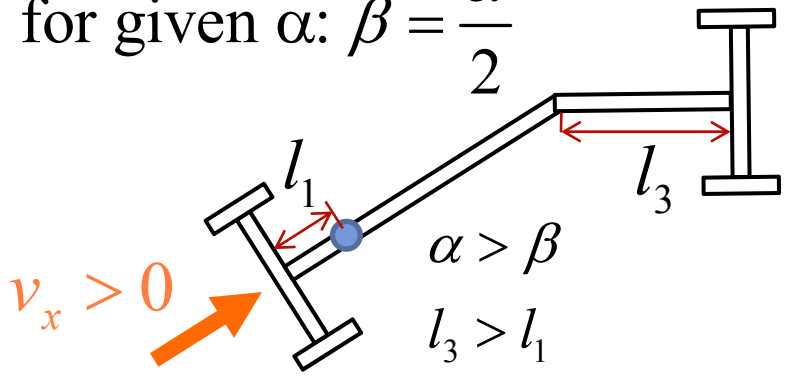
Direction of acceleration $\boldsymbol{a}$ depends on sgn(α−β):

Non-degenerate optimum of $|\boldsymbol{a}|$:

for given β: $\alpha = \dfrac{\beta}{\beta+2}$



for given α: $\beta = \dfrac{\alpha}{2}$



Torque input - analysis

Reduced state equations:

$$\begin{cases} \dot{v} = f_v(\phi)v^2 + f_u(\phi)u^2 + f_{uv}(\phi)uv + f_\tau(\phi)\tau \\ \dot{u} = g_v(\phi)v^2 + g_u(\phi)u^2 + g_{uv}(\phi)uv + g_\tau(\phi)\tau \\ \dot{\phi} = h(\phi)u \end{cases}$$

Perturbation expansion:

$$\begin{cases} v(t) = v_0(t) + \varepsilon v_1(t) + \varepsilon^2 v_2(t) + O(\varepsilon^3) \\ u(t) = \qquad \varepsilon u_1(t) + \varepsilon^2 u_2(t) + O(\varepsilon^3) \\ \phi(t) = \qquad \varepsilon \phi_1(t) + \varepsilon^2 \phi_2(t) + O(\varepsilon^3) \end{cases}$$

Zero-order dynamics: $\dot{v}_0(t) = 0 \quad \Rightarrow \quad v_0 = const.$

First-order dynamics:

$$\begin{bmatrix} \dot{v}_1 \\ \dot{u}_1 \\ \dot{\phi}_1 \end{bmatrix} = \begin{bmatrix} 0 & 0 & 0 \\ 0 & \dfrac{\alpha^2 + \alpha\beta - \alpha - \beta}{\alpha\beta} v_0 & \dfrac{1}{\alpha\beta} v_0^2 \\ 0 & -(\alpha - 1)^2 & 0 \end{bmatrix} \begin{bmatrix} v_1 \\ u_1 \\ \phi_1 \end{bmatrix} + \begin{bmatrix} 0 \\ -\dfrac{1}{\alpha^2\beta^2} \\ 0 \end{bmatrix} \sin(t)$$

Solution contains transient terms of $e^{\lambda_i t}$ + terms of $\sin(t+\psi)$

Negative real eigenvalues $\lambda_{1,2}$ and $\lambda_3=0 \Longrightarrow$ decay to steady-state

(neutral stability of free system due to energy conservation)

Torque input - solution

Steady-state solution:

$$v(t) = v_0 + \varepsilon^2 \Big[\frac{(1-\alpha)(\alpha+\beta)}{2\alpha\beta\left((\alpha-1)^2 v_0^2 + \alpha^2\right)\left((\alpha-1)^2 v_0^2 + \beta^2\right)} t$$

$$+ C_1(\alpha,\beta,v_0)\left(\cos(2t)-1\right) + C_2(\alpha,\beta,v_0)\sin(2t)\Big] + O(\varepsilon^3)$$

$$\theta(t) = \varepsilon \Bigg[-\frac{(\alpha-1)\left(\left(\alpha^2 v_0^2 - 2\alpha v_0^2 - \alpha\beta + v_0^2\right)(\alpha-1) + \alpha\left(\alpha^2 + \alpha\beta - \alpha - \beta\right)\right) v_0}{\alpha\beta\left((\alpha-1)^2 v_0^2 + \alpha^2\right)\left((\alpha-1)^2 v_0^2 + \beta^2\right)} (\cos(t)-1)$$

$$+ \frac{(\alpha-1)\left((\alpha-1)\left(\alpha^2 + \alpha\beta - \alpha - \beta\right) v_0^2 - \alpha\left(\alpha^2 v_0^2 - 2\alpha v_0^2 - \alpha\beta + v_0^2\right)\right)}{\alpha\beta\left((\alpha-1)^2 v_0^2 + \alpha^2\right)\left((\alpha-1)^2 v_0^2 + \beta^2\right)} \sin(t) \Bigg] + O(\varepsilon^2)$$

"Initial" speed v_0 should be updated - $\bar{v}(t)$ grows with time

Oscillation amplitude of θ(t) depends on v_0

$\rightarrow$ decays as $\frac{1}{\bar{v}(t)^2}$ $\Longrightarrow$ **straight line motion**

Roller-racer torque input

Torque $\tau(t)=\varepsilon\sin t$, expansion about $\phi=\pi$

First-order dynamics:

$$\begin{bmatrix} \dot{v}_1 \\ \dot{u}_1 \\ \dot{\phi}_1 \end{bmatrix} = \begin{bmatrix} 0 & 0 & 0 \\ 0 & \dfrac{\alpha^2-\alpha\beta+\alpha-\beta}{\alpha\beta}v_0 & -\dfrac{1}{\alpha\beta}v_0^2 \\ 0 & -(\alpha+1)^2 & 0 \end{bmatrix} \begin{bmatrix} v_1 \\ u_1 \\ \phi_1 \end{bmatrix} + \begin{bmatrix} 0 \\ -\dfrac{1}{\alpha^2\beta^2} \\ 0 \end{bmatrix} \varepsilon\sin(t)$$

Characteristic polynomial of linearization matrix:

$$\lambda^3 - \frac{\alpha^2+\alpha\beta-\alpha-\beta}{\alpha\beta}v_0\lambda^2 - \frac{(\alpha+1)^2}{\alpha\beta}v_0^2\lambda = 0$$

$\Longrightarrow$ $\phi=\pi$ is **unstable** for any sign of v_0

Preliminary experiments (movies)

VEX robotics kit.

Steering angle input by servo motor, triangular wave.

1. Twistcar
2. Roller-Racer backwards
3. Roller-Racer reversal

Future extensions

Improved experiments - measurements, torque control

Effects of wheels slippage → **hybrid dynamics**

stick-slip transitions (3D model, effect of c.o.m. height)

Control – path stabilization (hybrid control law?)

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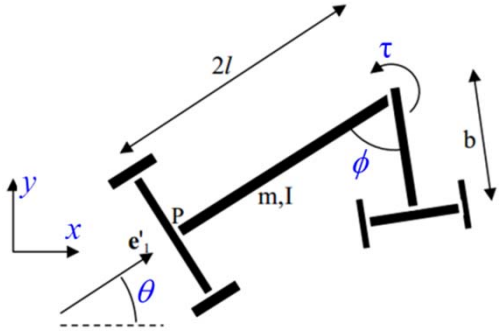

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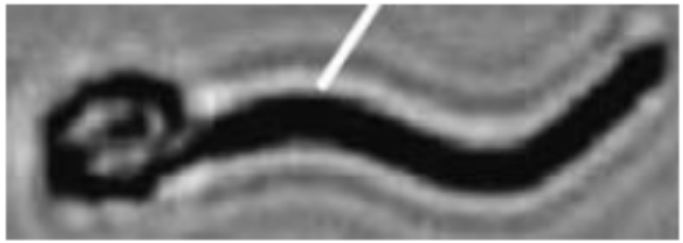
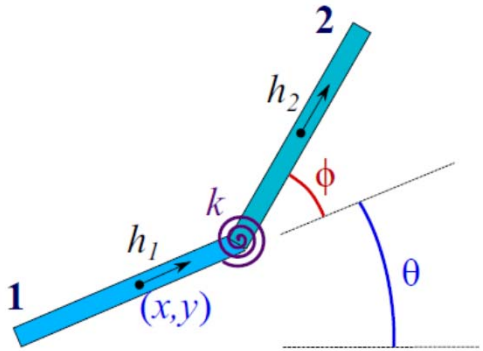




3. Others: slippage effects on legged locomotion
senior project - reverse a truck & trailer

Microswimmers – intro

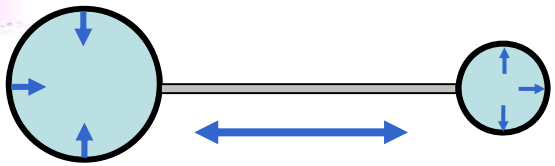
Swimming microorganisms [Taylor, 1951]

Low Reynolds num. hydrodynamics – viscosity ≫ inertia

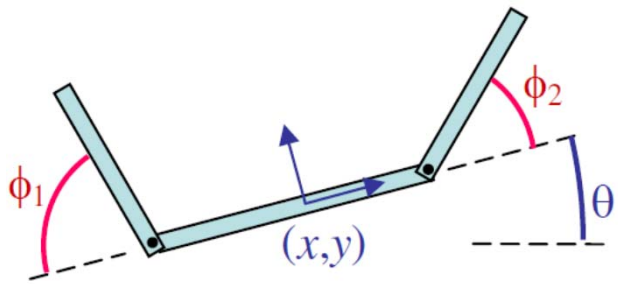
Simplistic theoretical models:

3-spheres, pushmepullyou, Purcell's 3-link swimmer...

Studied nonlinear controllability, geometric mechanics



Avron, 2007



Purcell, 1977

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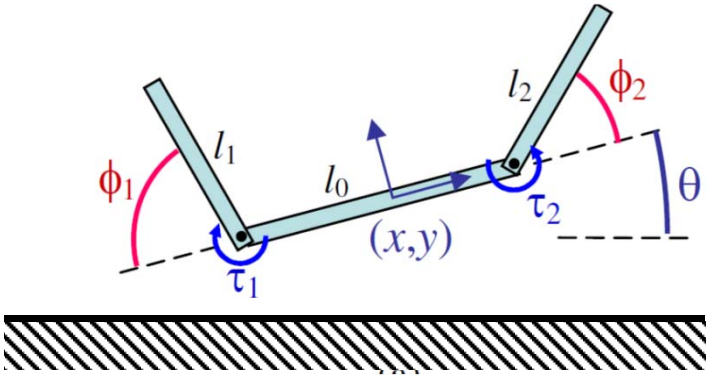
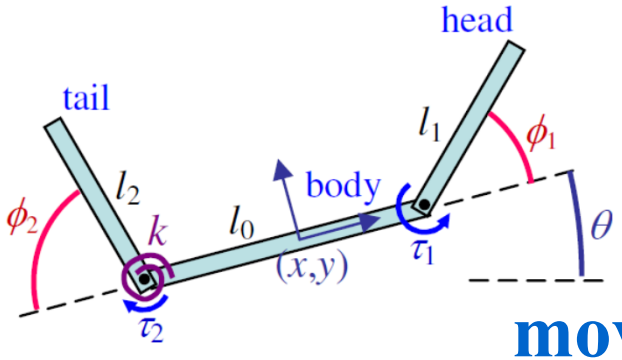
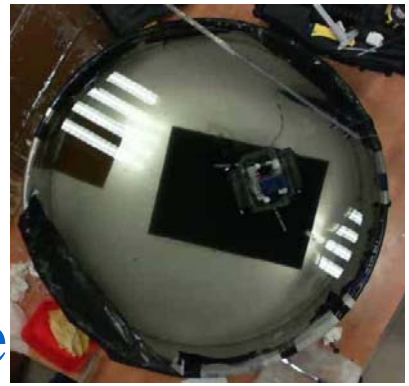
All assume: unbounded fluid, kinematic shape control

My works: wall effects, torque input, elastic tail, robot





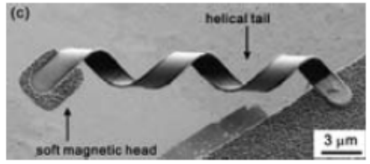
movie



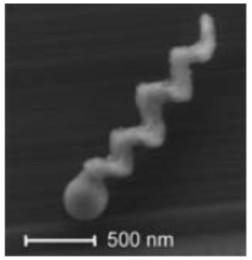
Magnetic microswimmers

For biomedical applications, magnetic actuation

Helical rigid tail, rotating magnetic field:

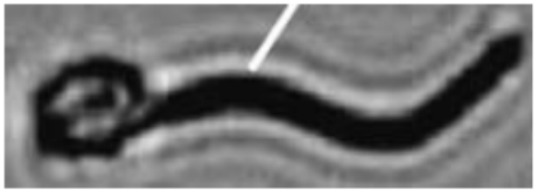


Zhang et al 2009



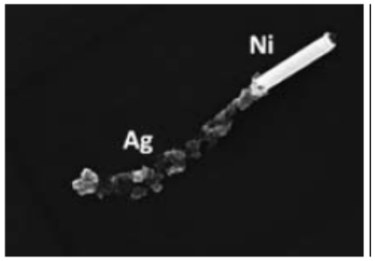
Ghosh and Fischer 2009

Flexible tail, rotating/oscillating magnetic field:



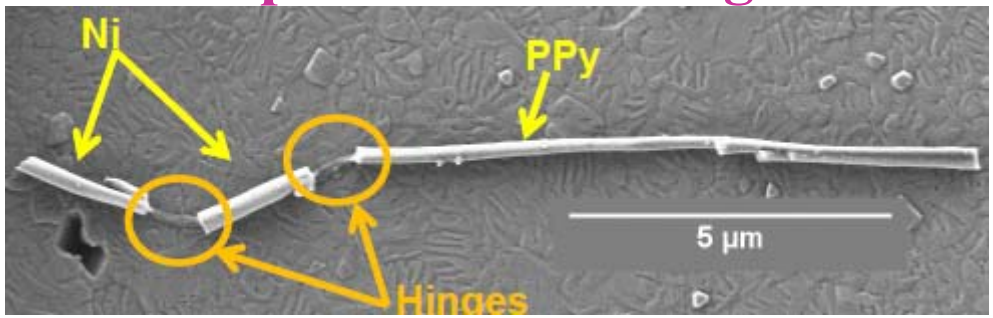
Dreyfus et al 2005

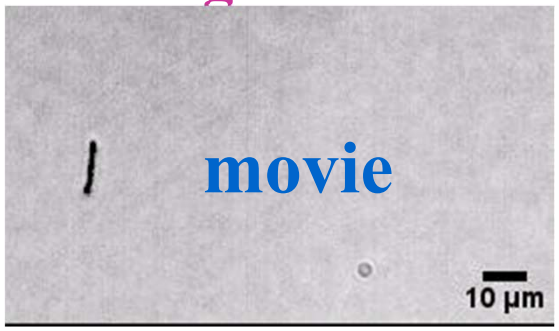
Gao et al 2010



Pak et al 2011

Nelson's planar undulating 2-link and 3-link magnetic swimmer





Optimal frequency ω for speed *V* or scaled *V*/ω ~ displacement *X*

Magnetic microswimmers

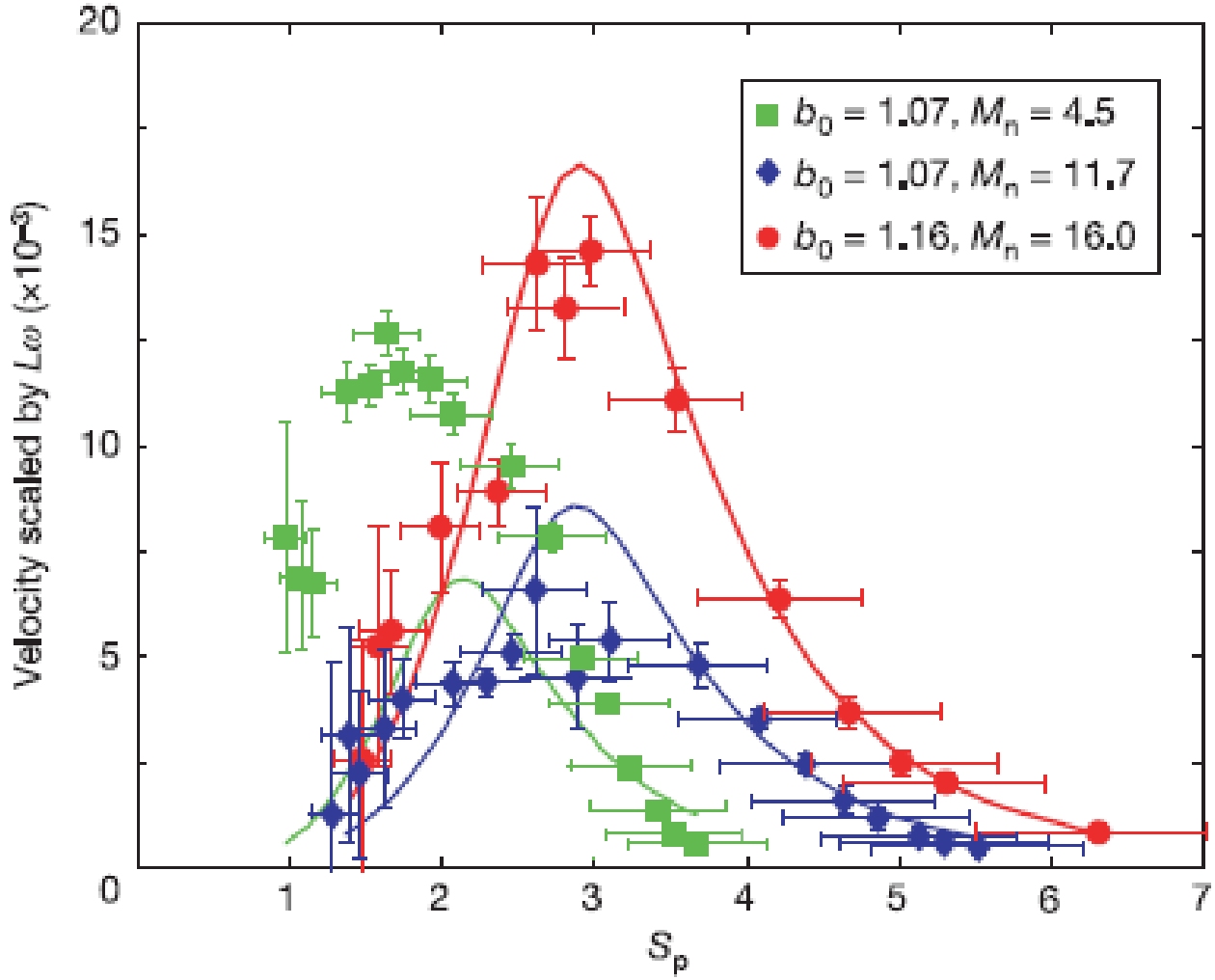
Fc

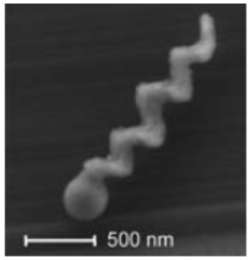
H

rc

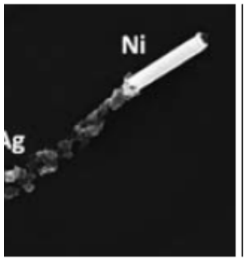
F

N



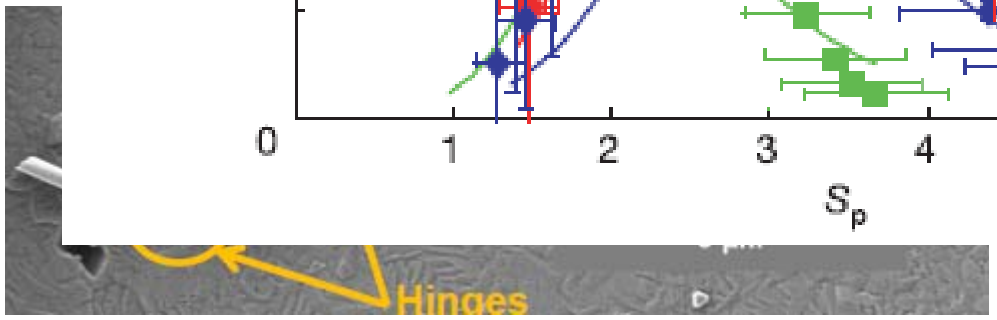


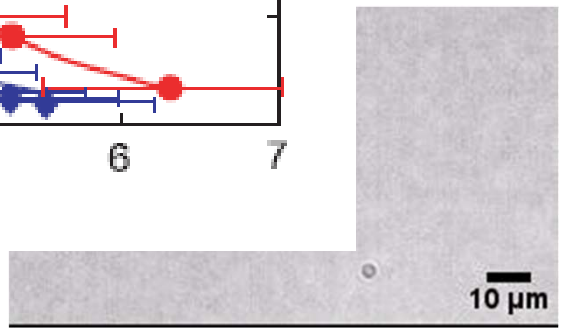
d Fischer 2009



t al 2011

c swimmer





Optimal frequency ω for speed *V* or scaled *V*/ω ~ displacement *X*

Planar undulating two-link model

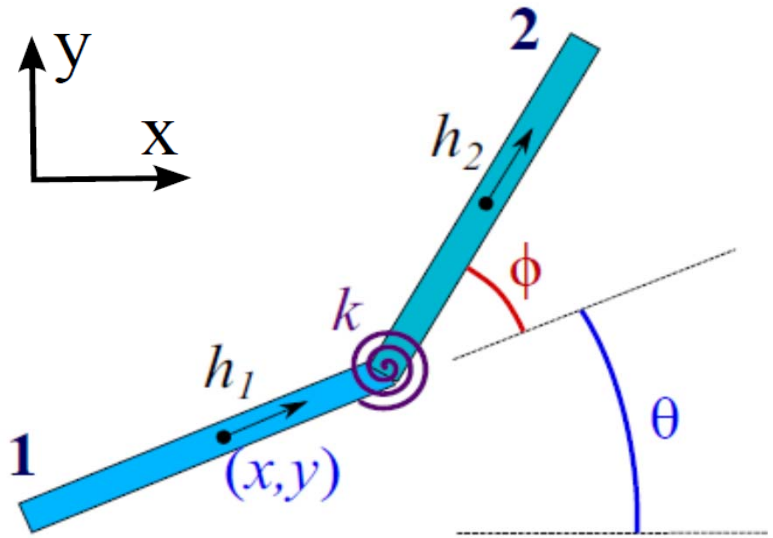
Coordinates of the swimmer: $\mathbf{q}=\left(\mathbf{q}_b^T, \phi\right)^T$

where $\mathbf{q}_b=\left(x, y, \theta\right)^T$ – body position,

ϕ – swimmer's shape

Passive torsion spring at the joint: $\tau = -k\phi$

(τ - torque, k - torsional stiffness)



Two magnetic dipoles: h_1, h_2 aligned with the links' longitudinal axes $\mathbf{t}_i$

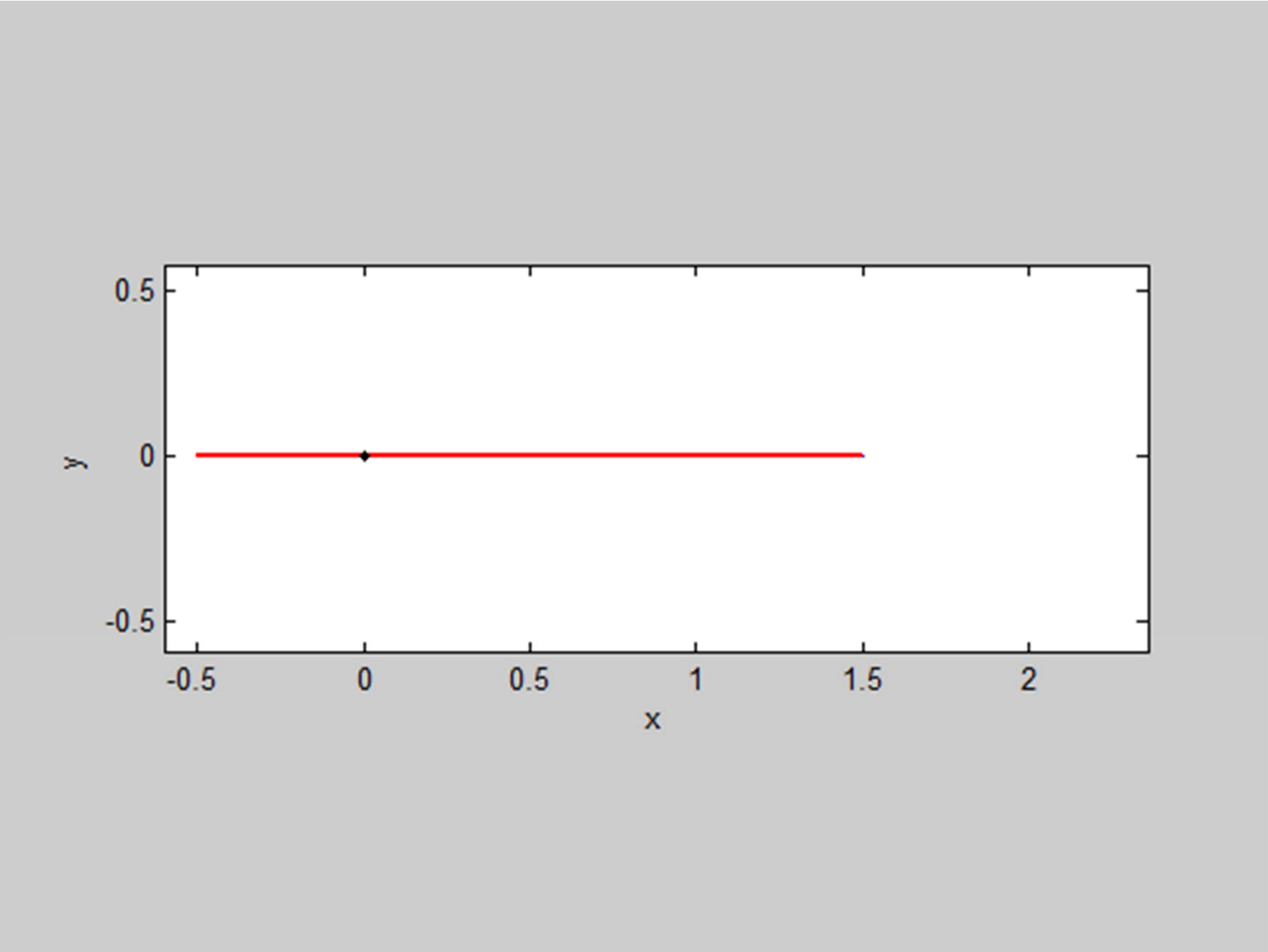
External magnetic field: $\mathbf{B}(t) = B_x\left[1, \ \varepsilon \sin\left(\omega t\right)\right]^T$

Aligning magnetic torque on the ith link: $\mathbf{M}_i = h_i \mathbf{t}_i \times \mathbf{B}$

Stokes Equations (Re=0): $\nabla \cdot \mathbf{u} = 0, \ -\nabla p + \mu \nabla^2 \mathbf{u} = 0$

Linear relation between viscous forces and velocities (resistance)

Dynamics – formulation (movie)



Analysis

Two characteristic time scales:

1. Visco-magnetic time – magnetic alignment of a single link

$$h_1 = 0, h_2 \neq 0, \mathbf{B} = \begin{bmatrix} B_x \\ 0 \end{bmatrix}, \phi = 0 \Rightarrow \dot{\theta} = -\frac{1}{t_m}\sin\theta \underset{small\,\theta}{\Rightarrow} \theta \approx \theta_0 e^{-t/t_m},\ t_m = \frac{4c_t l^3}{3B_x h_2}$$

2. Visco-elastic time – alignment of elastic joint

$$h_1 = 0, h_2 = 0, \mathbf{B} = \begin{bmatrix} 0 \\ 0 \end{bmatrix}, k > 0 \Rightarrow \dot{\phi} = -\frac{1}{2}\frac{1}{t_k}\frac{3+\cos\phi}{3-\cos\phi}\phi \underset{small\,\phi}{\Rightarrow} \phi \approx \phi_0 e^{-t/t_k},\ t_k = \frac{c_t l^3}{12k}$$

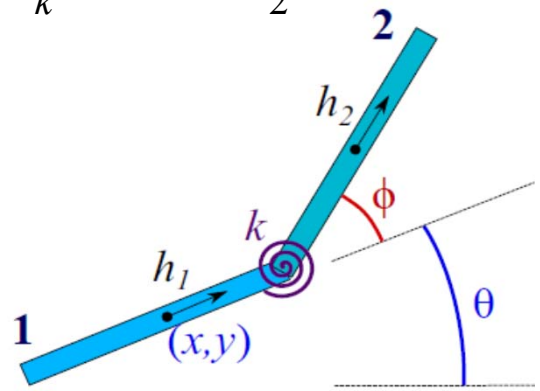
Scale time by t_m,
define two non-dimensional parameters: $\alpha = \frac{t_m}{t_k},\ \beta = \frac{h_1}{h_2}$

Three cases:

I. $h_1 = 0, h_2 \neq 0, k \neq 0 \Rightarrow \beta = 0$

II. $h_1 \neq 0, h_2 \neq 0, k = 0 \Rightarrow \alpha = 0$

III. $h_1 \neq 0, h_2 \neq 0, k \neq 0 \Rightarrow \alpha \neq 0, \beta \neq 0$



Perturbation expansion

Assume small amplitude oscillations of $B_y(t)$, $\varepsilon \ll 1$

Expand $\mathbf{q}(t) = \varepsilon \mathbf{q}_{(1)}(t) + \varepsilon^2 \mathbf{q}_{(2)}(t) + \varepsilon^3 \mathbf{q}_{(3)}(t) + \ldots$

Dynamics depend only on angles θ,ϕ. First-order system:

$$\begin{bmatrix} \dot{\theta}_{(1)} \\ \dot{\phi}_{(1)} \end{bmatrix} = \begin{bmatrix} -5\beta + 3 & 0.5\alpha + 3 \\ 8\beta - 8 & -\alpha - 8 \end{bmatrix} \begin{bmatrix} \theta_{(1)} \\ \phi_{(1)} \end{bmatrix} + \begin{bmatrix} 5\beta - 3 \\ -8\beta + 8 \end{bmatrix} \sin(\omega t)$$

Solution is of the form: $C_1 e^{\lambda_1 t} + C_2 e^{\lambda_2 t} + C_3 \sin(\omega t + \varphi)$

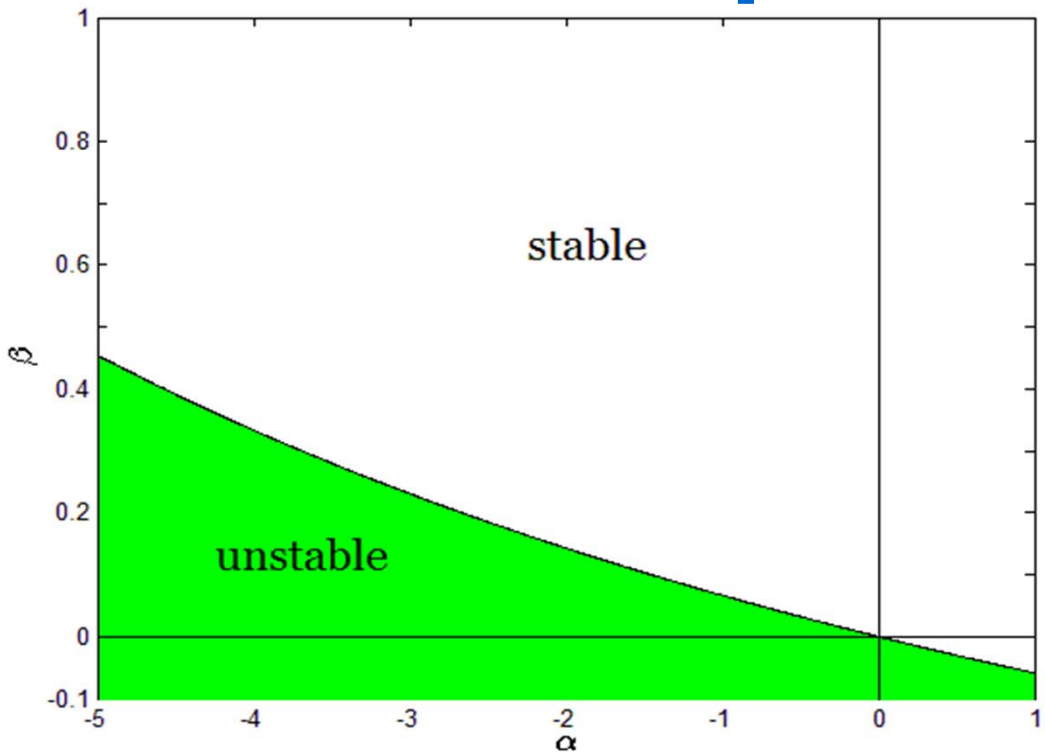
Stability conditions: $\mathbf{Re}(\lambda_1, \lambda_2) < 0 \rightarrow \alpha + 5\beta + 5 > 0$ and $\alpha + 16\beta + \alpha\beta > 0$

Case I: $\beta = 0 \Rightarrow \alpha > 0$; Case II: $\alpha = 0 \Rightarrow \beta > 0$

Case III: $\alpha, \beta \neq 0 \Rightarrow \alpha$ can be negative $\Rightarrow$ destabilizing torsion spring

β can be negative $\Rightarrow$ opposite links' magnetization

Perturbation expansion



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Leading-order solution

The second order equation for the forward motion $x(t)$:

$$\dot{x}_{(2)} = \frac{l}{2}\left[-(\beta+1)\theta_{(1)}^2 - \left(\frac{1}{4}\alpha+3\right)\phi_{(1)}^2 + (\beta-4)\theta_{(1)}\phi_{(1)} + \left((\beta+1)\theta_{(1)} + (-\beta+3)\phi_{(1)}\right)\sin(\omega t)\right]$$

The steady state solution is of the form:

$$x_{(2)}(t) = M(\omega,\alpha,\beta)\sin\left(2\omega t + \varphi(\omega,\alpha,\beta)\right) + \boxed{\tilde{V}(\omega,\alpha,\beta)t}$$ Net forward motion

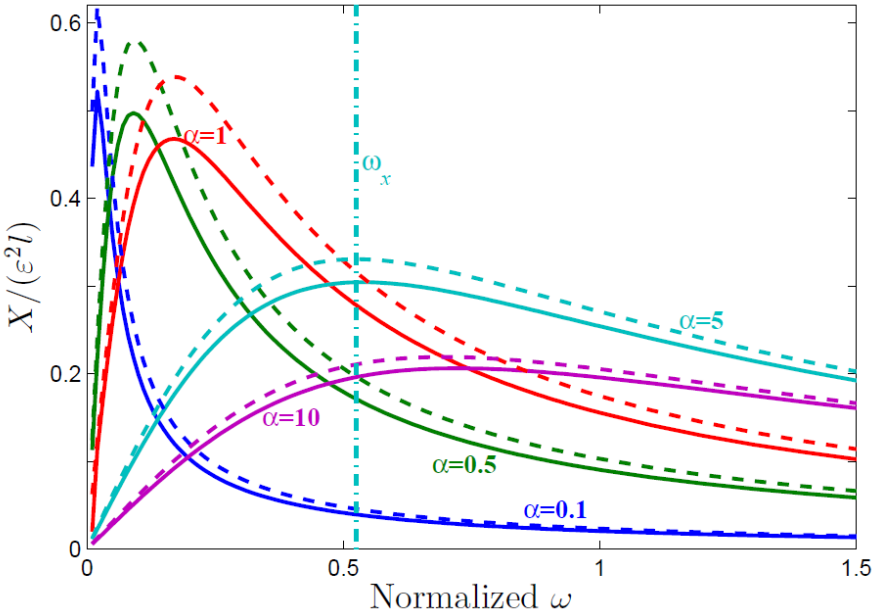
The average speed and the net displacement per cycle (leading-order):

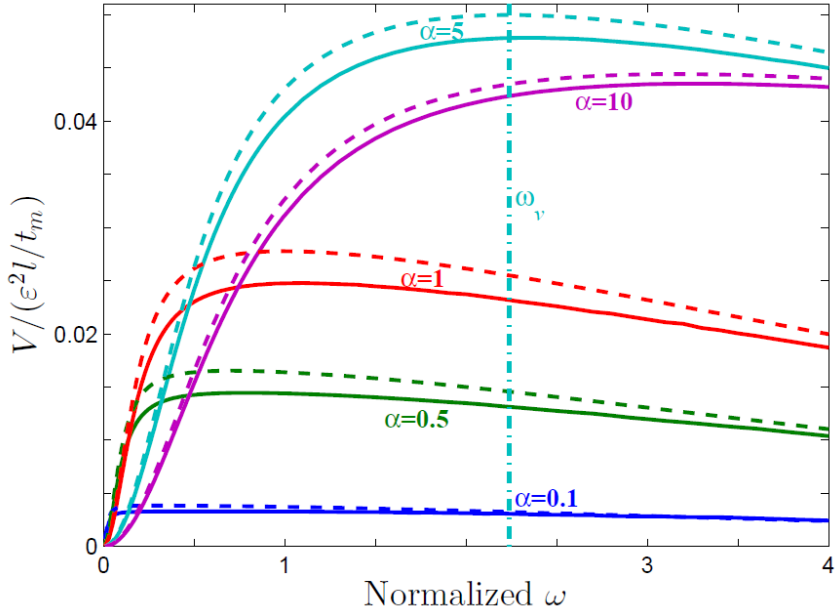
$$X = \varepsilon^2 l \frac{2\pi\omega(1-\beta)(\alpha+16\beta+\alpha\beta)}{\omega^4 + \left(\alpha^2+8\alpha\beta+8\alpha+25\beta^2+18\beta+25\right)\omega^2 + \alpha^2\beta^2 + 2\alpha^2\beta + \alpha^2 + 32\alpha\beta^2 + 32\alpha\beta + 256\beta^2}$$

$$V = \frac{X}{2\pi/\omega} = \varepsilon^2 l \frac{\omega^2(1-\beta)(\alpha+16\beta+\alpha\beta)}{\omega^4 + \left(\alpha^2+8\alpha\beta+8\alpha+25\beta^2+18\beta+25\right)\omega^2 + \alpha^2\beta^2 + 2\alpha^2\beta + \alpha^2 + 32\alpha\beta^2 + 32\alpha\beta + 256\beta^2}$$

Optimal actuation frequency

l=1, k=0.3, h_1=0, h_2=1, B_x=1, ε=0.4 and ω=1





Find optimal frequencies that maximize X and V:

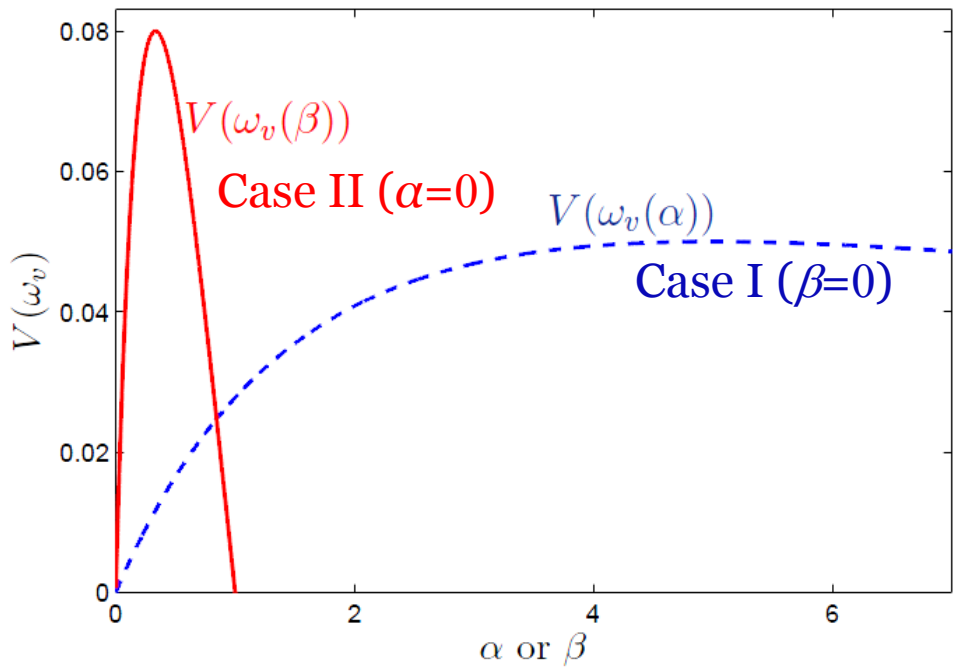
For X the optimal frequency ω_x is:

$$\omega_x = \frac{1}{\sqrt{6}}\left(-\left(\alpha^2+8\alpha\beta+8\alpha+25\beta^2+18\beta+25\right)^{\frac{1}{2}}+\left(\left(\alpha^2+8\alpha\beta+8\alpha+25\beta^2+18\beta+25\right)^2+12\left(\alpha^2\beta^2+2\alpha^2\beta+\alpha^2+32\alpha\beta^2+32\alpha\beta+256\beta^2\right)\right)^{\frac{1}{2}}\right)$$

For V the optimal frequency ω_v is: $\omega_v = \sqrt{\alpha+16\beta+\alpha\beta}$

Optimal swimmer's parameters

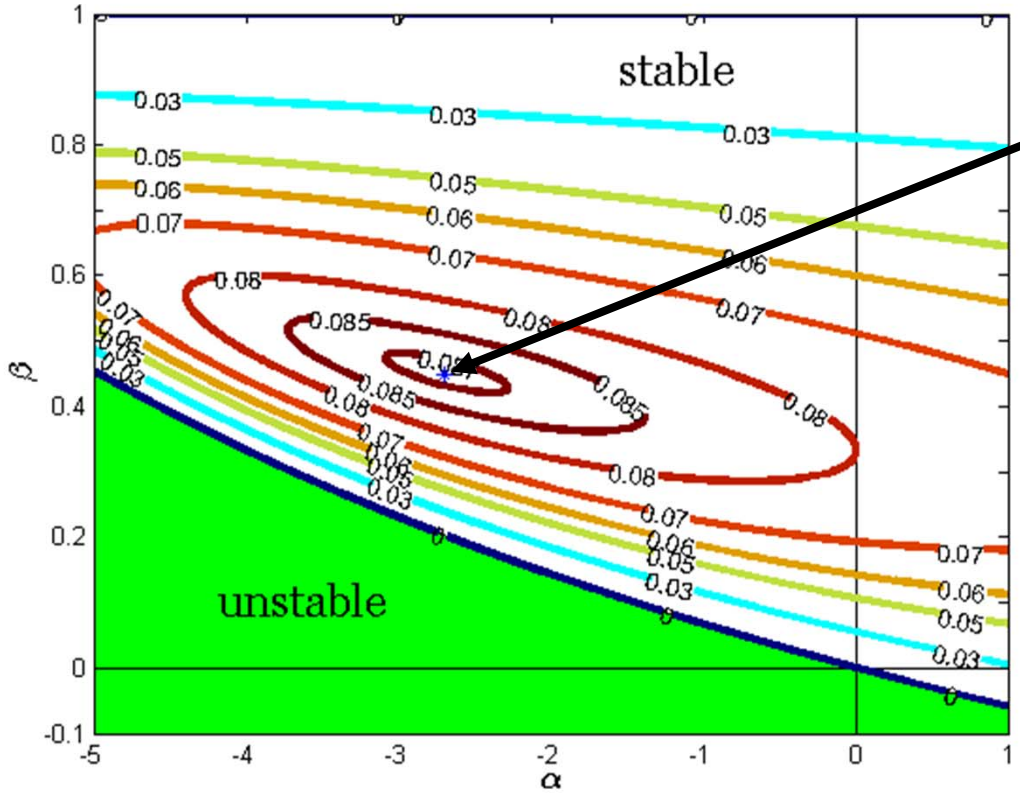
- Case I (β=0) the optimal value of V is: $V\left(\omega_v = \sqrt{5},\ \alpha_v = 5\right) = 0.05\varepsilon^2 \dfrac{l}{t_m}$
- Case II (α=0) the optimal value of V is: $V\left(\omega_v = \dfrac{4}{\sqrt{3}},\ \beta_v = \dfrac{1}{3}\right) = 0.08\varepsilon^2 \dfrac{l}{t_m}$



Optimal swimmer's parameters

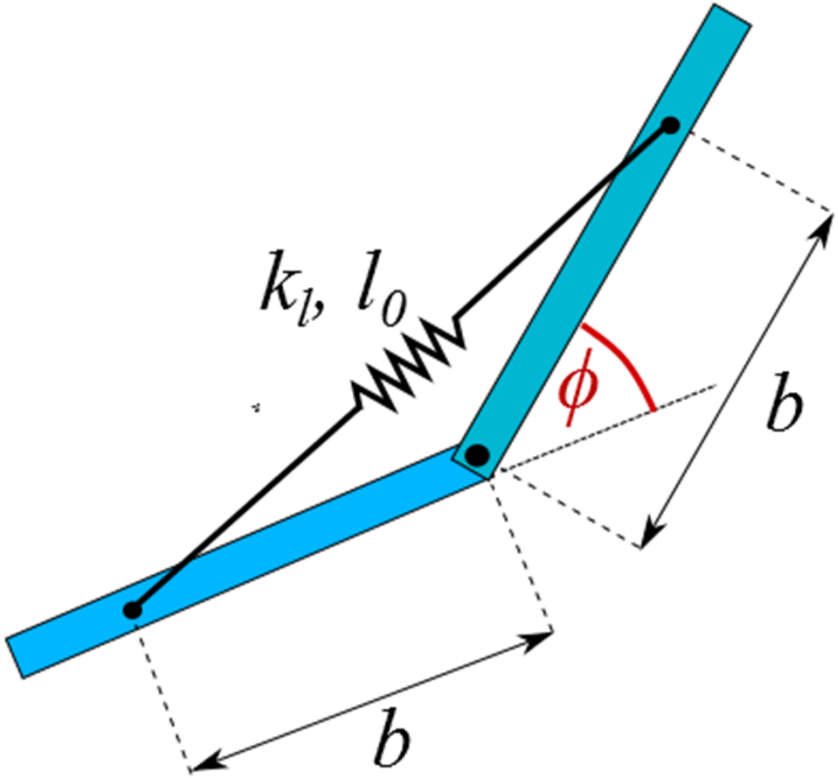
- For case III ($\alpha,\beta\neq 0$) the optimal value of V is:

$$V\left(\omega_v = 1.81922,\ \alpha_v = -2.7,\ \beta_v = 0.45197\right) = 0.0873\varepsilon^2 \frac{l}{t_m}$$



Optimum is obtained at $\alpha<0$
→ destabilizing spring

Destabilizing "torsion spring"



$$\tau(\phi) = -\underbrace{\boxed{\tfrac{1}{2}\tilde{k}b(l_0-2b)}}_{k}\phi+O(\phi^3)$$

l_0 <2b □ ϕ=0 is unstable (sring in tension)

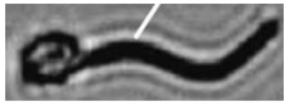
l_o>2b → ϕ =0 is stable (spring in compression)

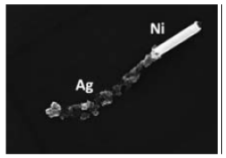
Can one manufacture this in a real micron-size swimmer??

Comparison to real swimmers

Microswimmer	L (μm)	$\varepsilon = b_y/b_x$	$\frac{X^*}{\varepsilon^2 L}$	f_x (Hz)	$\frac{V^*}{\varepsilon^2 L f_v}$	f_v (Hz)
Dreyfus *et al* [1]	24	10.3/8.9 =1.16	0.068	10	0.031	4
Pak *et al* [2]	5.8	10/9.5 =1.05	0.149	15	0.093	35
Our model	$2l$		0.31	$\omega_x \to 0$	0.15	$0.29/t_m$

[1] R. Dreyfus, J. Baudry, M. L. Roper, M. Fermigier, H. A. Stone, and J. Bibette, Nature 2005.
[2] O. S. Pak, W. Gao, J. Wang, and E. Lauga, Soft Matter 2011.





Reported experimental values of maximal displacement and speed are below the optimal values according to our theoretical model, yet they are in the same order of magnitude.

Current work with B. Nelson at ETH:

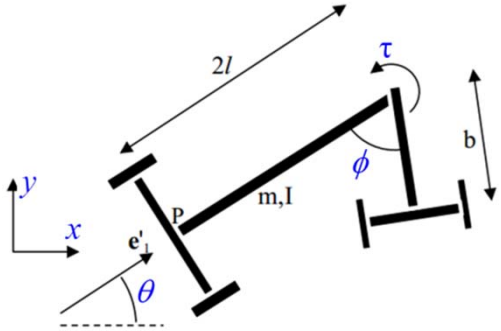
Match time scales t_m,t_k to experimental microswimmer prototype

Test the theoretical predictions of optimal frequency + parameters

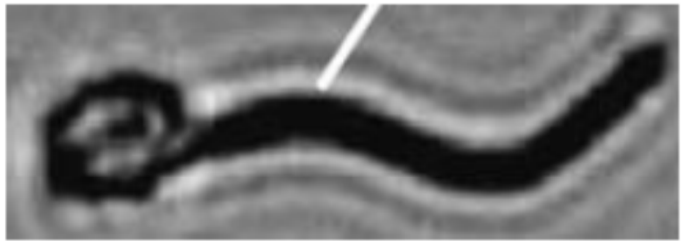
What's the relation of all this to CONTROL???

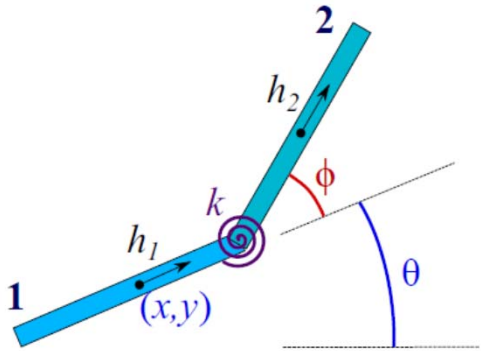
Outline

1. The Twistcar toy (Ofir Chakon, MSc work)



2. Microswimmers (Emiliya Gutman, PhD work)





3. Others: slippage effects on legged locomotion
senior project - reverse a truck & trailer

Slippage in legged locomotion

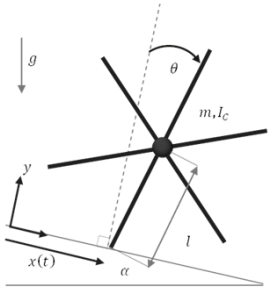
Benny Gamus, Moti Moravia (MSc works)

Dynamic legged locomotion – nonlinear hybrid system

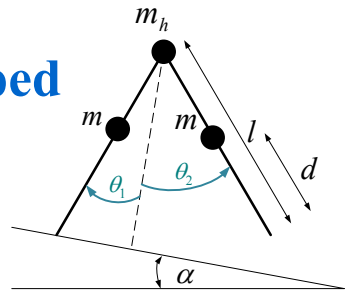
Control of legged robots – widely studied

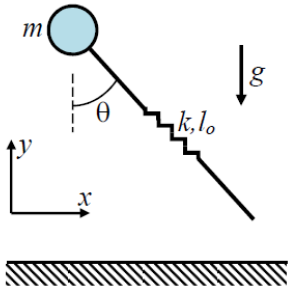
Passive dynamic walking – classical models (rigid links)

rimless wheel



compass biped





Animal locomotion - **SLIP model**
(**S**pring **L**oaded **I**nverted **P**endulum)

All models assume sticking contact at feet (high friction)

Our works: studied the effects of slippage under Coulomb friction model on performance
(stability, speed, energetic efficiency)

Reverse of LEGO truck & trailer

Senior thesis project of Iddo Perel and Hallel Bunis

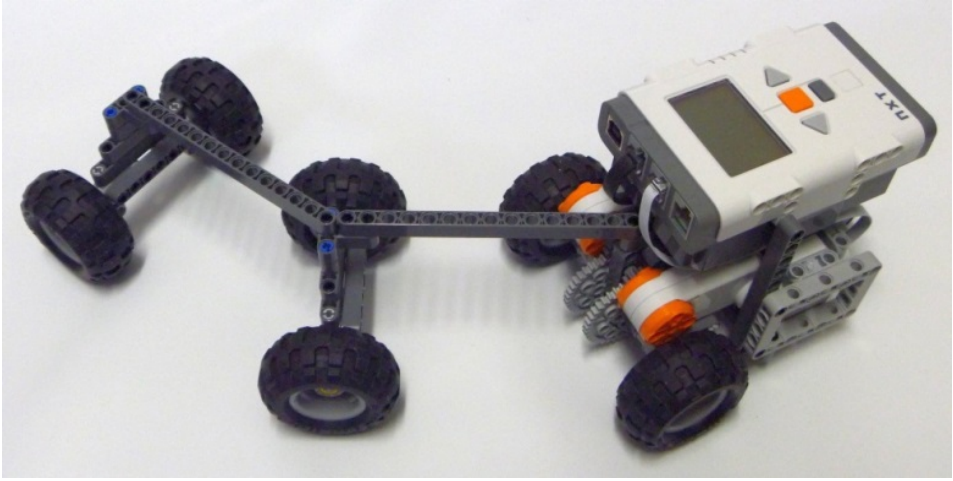
Stabilize truck & trailer(s) **in reverse**

Nonholonomic, driftless nonlinear control system

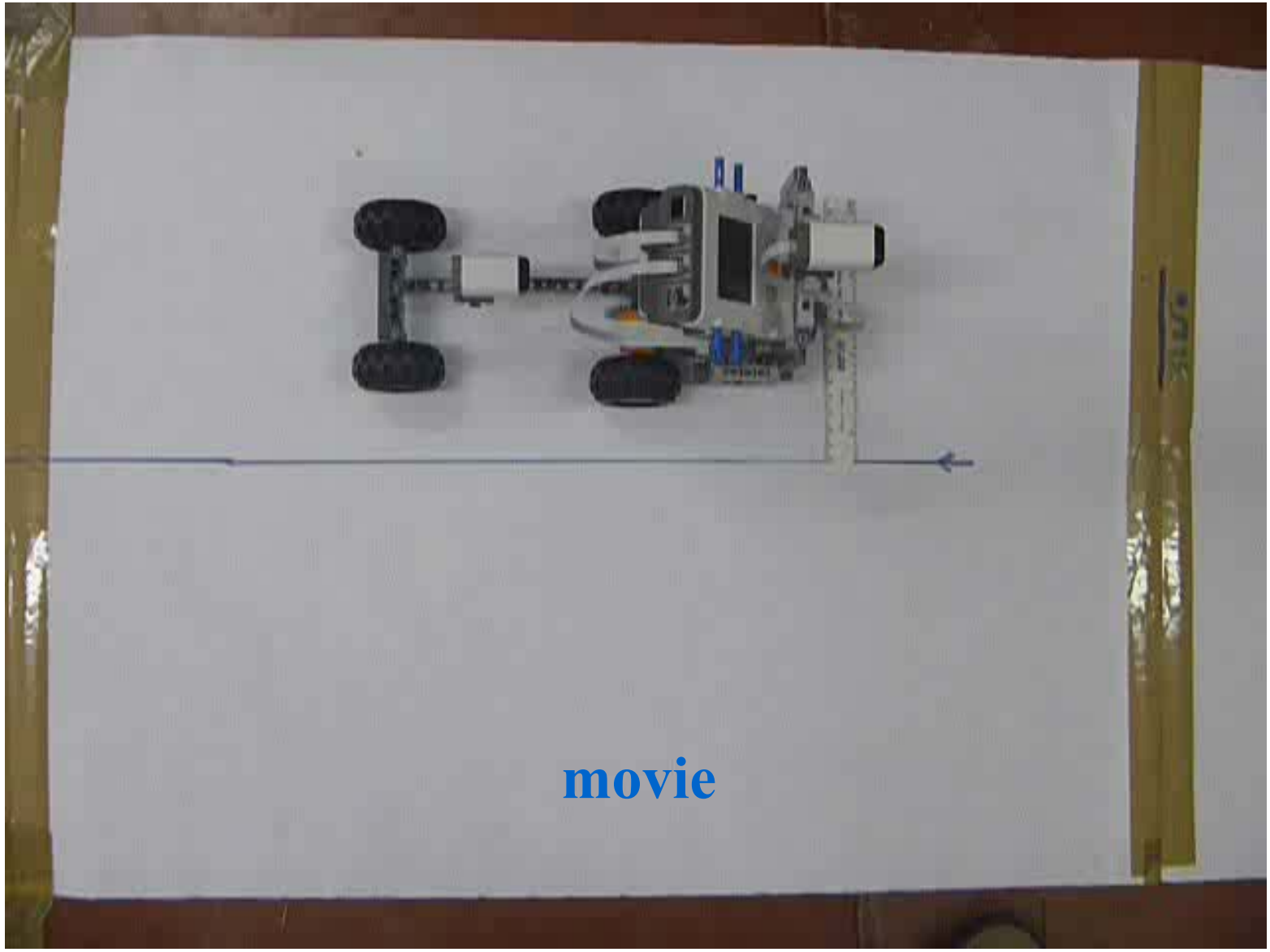
Linearization about straight line, control input is ω/v

State feedback / nested control loops

Implementation in LEGO NXT, controlled by Simulink



Reverse of LEGO truck & trailer



movie

Thank You

תודה רבה

Questions??