ISRAELI ASSOCIATION FOR AUTOMATIC CONTROL

Workshop on Model Predictive Control for Constrained Linear and Hybrid Systems

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with keynote lecture by

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Model Predictive Control (MPC) is currently the accepted standard in the process industries for handling complex constrained multivariable control problems. Here at each sampling time, starting at the current state, an open-loop optimal control problem is solved over a finite horizon. The optimal command signal is applied to the process only during the following sampling interval. At the next time step a new optimal control problem based on new measurements of the state of the system is solved over a shifted horizon. MPC popularity stems from the fact that the optimal solution relies on a dynamic model of the process, respects all input and output constraints, and minimizes a performance figure, which would be very hard to accomplish in any other way. Over the last 25 years MPC has become to dominate the process industry, where it has been employed for thousands of problems.

Worldwide, over the last six years, the research on MPC has mostly focused on two subjects: (1) online optimization vs offline state-feedback solutions and (2) hybrid systems.

(1) One limitation of MPC is that running the optimization algorithm on-line at each time step requires substantial time and computational resources, which are generally available in a large ``slow" chemical production facility, but may not be available for the control of a system installed in an automobile that must be inexpensive and where the sampling time is in the range of milliseconds.
In order to push MPC to those applications where on-line optimization cannot be afforded, for controlling systems with constraints on input and state variables, we developed a theoretical framework based on multiparametric programming to find the explicit, state-feedback form of MPC.

By looking at the MPC problem as a multiparametric program, We have showed that MPC is equivalent to a piecewise affine state feedback law, and we developed algorithms to compute this control law. As a consequence, MPC control can be implemented as a look-up table of linear control gains. Unlike traditional gain scheduling techniques, the ***explicit MPC*** allows to automatically synthesize a gain schedule to optimally control systems under constraints, thus avoiding ad-hoc control schemes.

(2) Often MPC is used for the regulatory control of large multivariable linear systems with constraints, where the objective function is not related to an economical objective, but is simply chosen in a mathematically convenient way, namely quadratic in the states and inputs, to yield a ``good" closed loop response. Besides state, input and other operational constraints, many practical control problems are dominated by characteristics like switches between different operating regimes, and the interaction of continuous-time and discrete event systems. Those systems are usually referred to as ***Hybrid systems*** - loosely defined as systems comprised of continuous and discrete/switched components. Over the last few years this system class has attracted much attention and various tools have emerged for studying and affecting its behavior.

At present no methodology is available to design controllers for such systems in a systematic manner. The most common approach resorts to using tools developed for unconstrained linear systems, patched with a collection of heuristic rules. We have introduced a design theory for controllers for constrained and hybrid dynamical systems based on MPC. It leads to algorithms which systematically solve control synthesis problems for classes of systems, where there are few, or no tools, currently available.

The main intent of this workshop is to cover in a comprehensive, detailed and exhaustive way the two main recent developments on MPC discussed above. Moreover we will illustrate the merits of the technique on a range of examples including traction control (with Ford U.S.) and formation flight control (with the Honeywell Laboratories, Minneapolis).

Workshop Program:

08:30 – 09:00	Registration
09:00 – 09:10	Opening remarks (Prof. Per-Olof Gutman, Dr. Hector Rotstein)
09:10 – 10:00	Advanced Process Control Applications using MPC, Prof. Daniel Lewin, Technion
10:00 – 10:50	Basics on Optimization
10:50 – 11:20	Coffee break
11:20 – 12:10	Multiparametric Programming
12:10 – 13:00	MPC for Linear Systems (I)
13:00 – 14:20	Lunch Break
14:20 – 14:50	MPC for Linear Systems (II)
14:50 – 15:40	MPC for Hybrid Systems
15:40 – 16:10	Coffee break
16:10 – 17:30	Tools and Applications

Outline

***Part One: Basics on Optimization**
In this part we introduce the main concepts of convex and discrete optimization. Our intent is to provide only the necessary background for the understanding the succeeding parts. We introduce the main concepts and definitions of mathematical programming theory and the most widely used criteria for optimality.

***Part two: Multiparametric Programming**
In this part we will introduce the theory of multi-parametric programming. In our framework, parametric programming is the main technique used to study and compute state feedback optimal control laws.

***Part three: MPC for Linear Systems**
We focus on linear systems with polyhedral constraints on inputs and states. We study finite time and infinite time optimal control problems with cost functions based on two, one and infinity norms. We demonstrate that the solution to all these optimal control problems can be expressed as a piecewise affine state feedback law. Moreover, the optimal control law is continuous and the value function is convex and continuous. The results form a natural extension of the theory of the Linear Quadratic Regulator to constrained linear systems. They also have important consequences for the implementation of MPC. Precomputing off-line the explicit piecewise affine feedback policy reduces the on-line computation for the MPC to a function evaluation, therefore avoiding the on-line solution of a mathematical. We also address the robustness of the optimal control laws.

***Part Four: MPC for Hybrid Systems**
We give an introduction to the different formalisms used to model hybrid systems focusing on computation-oriented models. We will focus on a new framework for modeling, analyzing and controlling systems whose behavior is governed by interdependent physical laws, logic rules, and operating constraints, denoted as Mixed Logical Dynamical (MLD) systems. They are described by linear dynamic equations subject to linear inequalities involving real and integer variables. They have the advantage that all problems of system analysis (like controllability, observability, stability and verification) and all problems of synthesis (like controller design and filter design) can be readily expressed as mixed integer linear or quadratic programs, for which many commercial software packages exist.

We study finite time optimal control problems with cost functions based on on two, one and infinity norms. The optimal control law is shown to be, in general, piecewise affine over non-convex and disconnected sets. Along with the analysis of the solution properties we present algorithms that efficiently compute the optimal control law for all the considered cases.

***Part Five : Tools and Applications**
In the final part of the workshop we will discuss in detail some practical applications that have been tackled with these new tools. We will look at the traction control problem where the underlying hybrid model is piece-wise affine and various constraints must be obeyed. The synthesized controller is also piece-wise affine and can be implemented conveniently as a look-up table. The controller was tested successfully on a Ford Focus. Finally, we will briefly review applications in other areas and recent developments on decentralized MPC with its application a formation flight problem.

Lecturer's Biography

FRANCESCO BORRELLI was born in Milano, Italy in 1974. He received the "Laurea" degree in computer science engineering in 1998 from the University of Naples "Federico II", Italy. In 2002 he received the Ph.D. from the Automatic Control Laboratory at ETH Zurich, Switzerland, advised by Prof. Manfred Morari. In 2003 he received the ETH Medal for the best Ph.D. dissertation. He has been a research assistant at the Automatic Control Laboratory of the ETH Zurich and a contract assistant professor at the Aerospace and Mechanics Department at the University of Minnesota, USA. Currently he is an assistant professor at the "Universita' del Sannio", Benevento, Italy.

Francesco Borrelli is a consultant for Ford Research Laboratories (Dearborn, USA) and for Honeywell Laboratories (Minneapolis,USA). He is the author of the book "Constrained Optimal Control of Linear and Hybrid Systems" published by Springer Verlag. He is the winner of the "Innovation Prize 2004" from the ElectroSwiss Foundation. His research interests include hybrid constrained optimal control, model predictive control, robust control, parametric programming, singularly perturbed systems, automotive applications of automatic control, formation flight control and decentralized control for large scale systems.