

IAAC – Israeli Association for Automatic Control  
IEEE Israel – Aerospace and Electronic Systems Chapter

Guest tutorial workshop on  
**Advanced Nonlinear Filtering:**  
An Overview and New Results

Speaker: **Fred Daum**  
Raytheon Company  
Woburn, MA

Monday, October 11, 2010  
Daniel Hotel, Herzlia

**Program (tentative):**

9:00 to 10:30	Overview of nonlinear filters	EKF, UKF, particle filters, exact recursive filters, non-recursive filters, various bells & whistles.
10:30 to 11:00	coffee	
11:00 to 12:00	Exact particle flow filters: theory	Detailed derivation of new (2010) theory
12:00 to 14:00	lunch	
14:00 to 15:30	Exact particle flow: numerical results	Accuracy & computational complexity with a wide range of parameters varied
15:30 to 16:00	coffee	
16:00 to 17:00	Fresh perspective on research in nonlinear filters	Fast PDE solvers, quasi-Monte Carlo, GPUs, sparse tensor products, adjoint method, information based computational complexity theory, meshfree PDE solvers, concentration of measure

## About the Speaker

Fred Daum is a senior principal Fellow at Raytheon, an IEEE Fellow, a Distinguished Lecturer of the IEEE-AES Society, and a graduate of Harvard University. Fred was awarded the Tom Phillips prize for technical excellence, in recognition of his ability to make complex radar systems work in the real world.

He developed, analyzed and tested the real time algorithms for essentially all the large long range phased array radars built by the USA in the last four decades, including: Cobra Dane, PAVE PAWS, Cobra Judy, BMEWS, THAAD, ROTH, UWR, and SBX, as well as several shipboard firecontrol systems and air traffic control systems. These real time algorithms include: extended Kalman filters, radar waveform scheduling, Bayesian discrimination, data association, discrimination of satellites from missiles, calibration of tropospheric and ionospheric refraction, and target object mapping. Fred's exact fixed finite dimensional nonlinear filter theory generalizes the Kalman and Beneš filters.

He has published nearly one hundred technical papers, and he has given invited lectures at MIT, Harvard, Yale, Caltech, Brown, Georgia Tech., Univ. of Connecticut, Univ. of Minnesota, Melbourne Univ., Univ. of New South Wales, Univ. of Canterbury, Univ. of Illinois at Chicago, Washington Univ. at St Louis, McMaster Univ. and Northeastern

## Content Outline

First, we describe many nonlinear filters, including: extended Kalman (EKF), unscented Kalman (UKF), particle filters (PF), exact recursive (Beneš, Daum, Yau, Wonham), non-recursive and semi-recursive. We discuss practical engineering bells & whistles for EKFs (good coordinate systems, iteration, second order corrections, Battin's trick, preferred order of scalar updates, quasi-decoupling of covariance matrices, and tuning process noise). We explain the curse of dimensionality for particle filters with a simple non-pathological example, as well as geometrical intuition (including a quiz). We survey the state-of-the-art in particle filters, with diverse attempts to mitigate the curse of dimensionality. We emphasize more or less subtle but important details that are often suppressed in such papers.

Second, we derive a completely new theory of exact particle flow for particle filters. This filter is a radical departure from other particle filters: (1) we never resample, (2) we never use a proposal density; (3) we never use MCMC methods; (4) we compute the log of the unnormalized density rather than the density itself. This theory fixes the well known problem of particle degeneracy by implementing Bayes' rule by particle flow rather than as a point wise multiplication. The basic idea is to compute the exact flow of particles

corresponding to Bayes' rule using a log-homotopy and the Fokker-Planck equation. We analyze ten methods to solve the fundamental PDE in this theory. The key issues are computational complexity, how to formulate the problem to obtain a unique solution of the PDE and stability of the particle flow and the filter itself.

Third, we show many numerical results of the exact particle flow filter, both filter accuracy and computational complexity compared with EKFs, UKFs and other PFs. We show such results for a wide variation of parameters: dimension of the state vector of the plant ( $d = 1$  to  $30$ ), initial uncertainty of the state, process noise, measurement noise, stability of the plant, data rate, for various nonlinearities (cubic, quadratic, sinusoidal, ballistic trajectories, Euler's equations of rotation) as well as linear systems. All of these examples are not sparse; that is, the effective dimension of the problem is equal to the dimension of the state vector of the plant. We give simple intuitive explanations of the trends in these plots. The new theory is many orders of magnitude faster than standard particle filters (for a given accuracy), and it is several orders of magnitude more accurate than the EKF for difficult nonlinear problems, including unstable plants and problems with multimodal densities.

Finally, we explore future research in nonlinear filters, and we venture to predict which directions will be fruitful and which will not.

## **Intended Audience**

This seminar is for normal engineers & statisticians who do not have log-homotopy for breakfast. But mathematicians who would like to learn a new fast method to multiply functions in high dimensions, or who like to play with first order linear highly underdetermined PDEs are also extremely welcome.