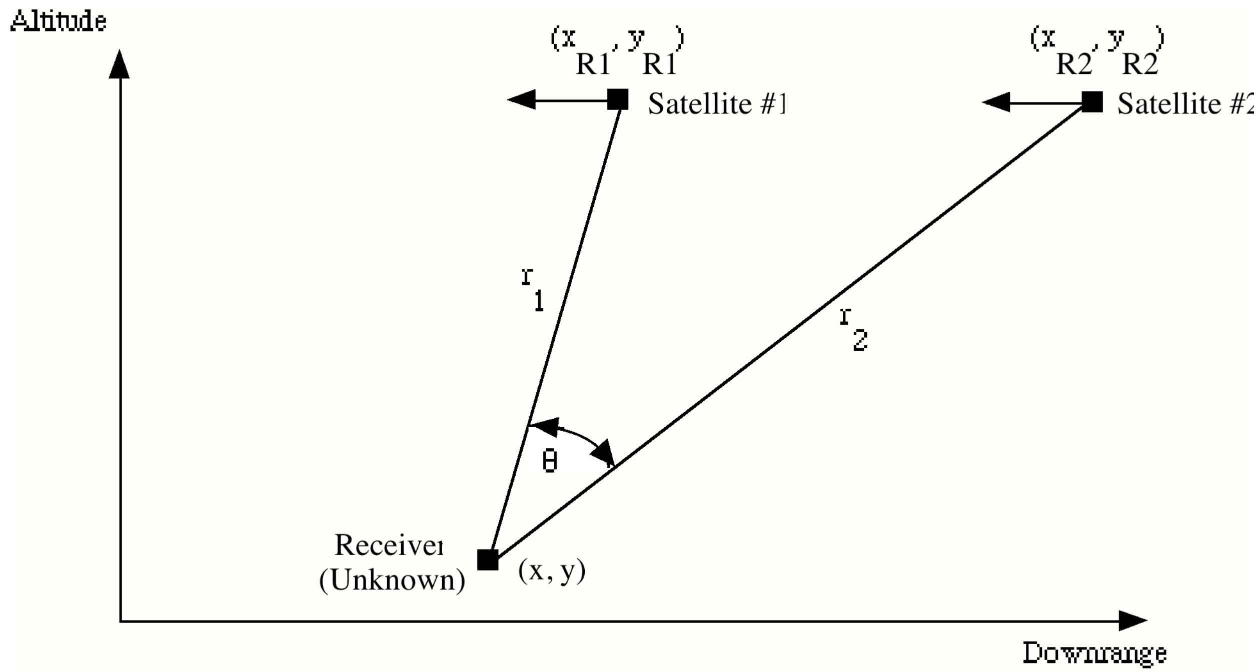
Biases

Biases
Overview

- Demonstration of problem
- Estimating satellite bias based on known receiver location
- Estimating receiver bias with unknown receiver location
    - 2 satellites
    - 3 satellites

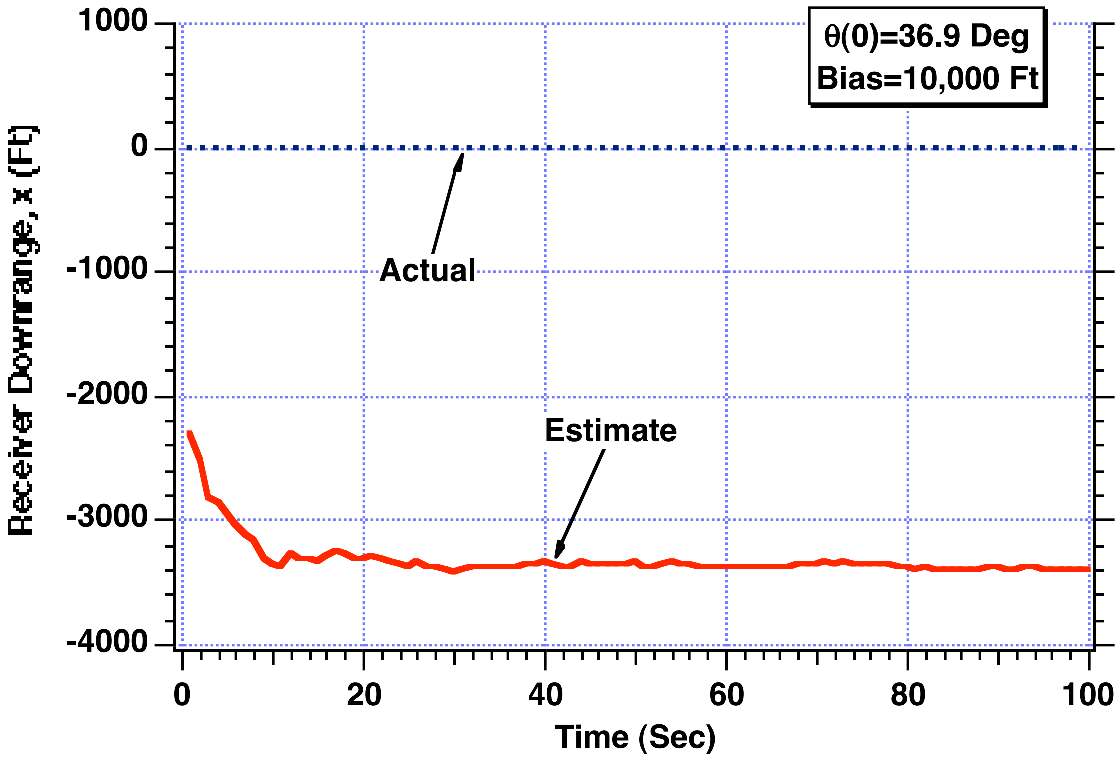
Demonstration of Problem

Satellite Geometry For Filtering Experiment

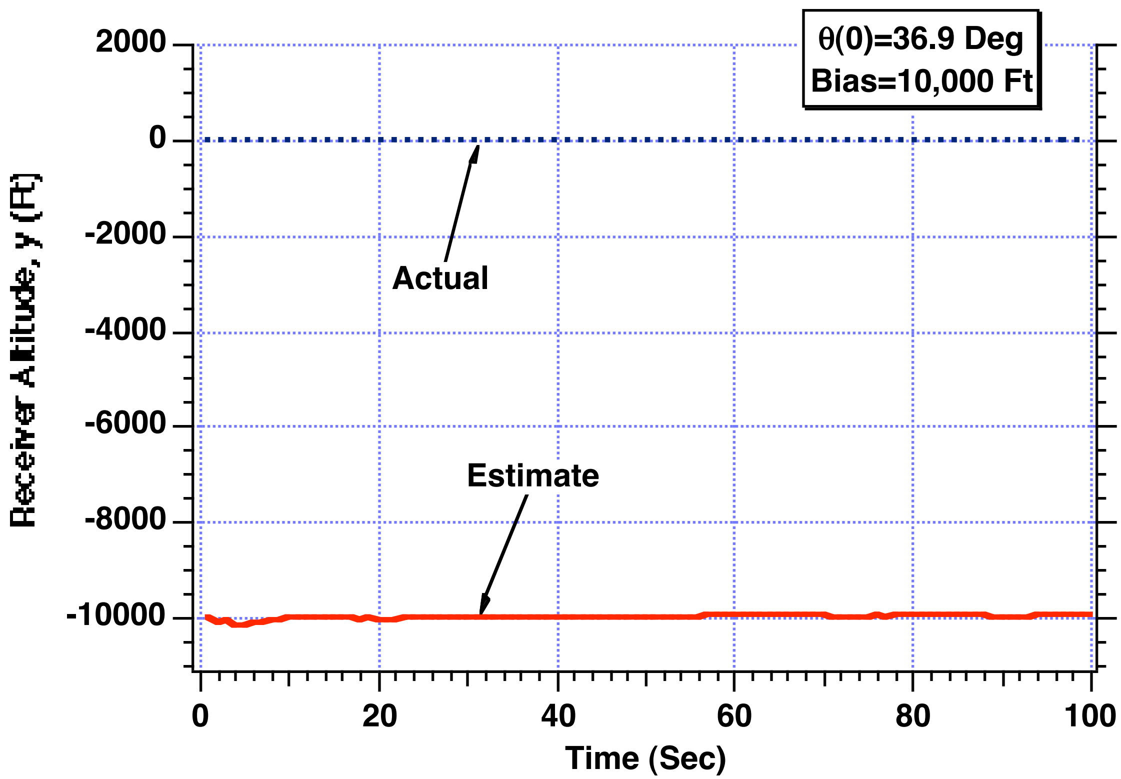


Range measurements have bias on them

Filter is Unable to Estimate Receiver Downrange When Range Bias is Present

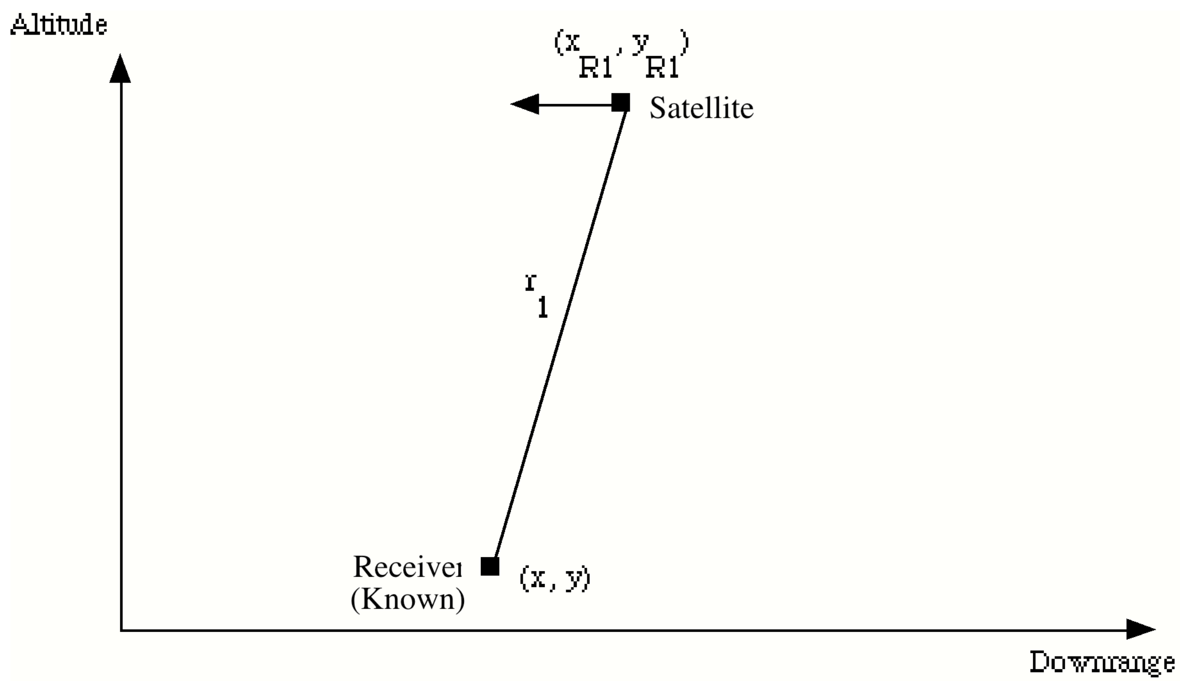


Filter is Unable to Estimate Receiver Altitude When Range Bias is Present



Estimating Satellite Bias With Known Receiver Location

Geometry for Attempting to Estimate Range Bias



Range measurement has bias on it

Estimating Bias on Range-1

Actual range from satellite to receiver

$$r_1 = \sqrt{(x_{R1} - x)^2 + (y_{R1} - y)^2}$$

Measured range

$$r_1^* = \sqrt{(x_{R1} - x)^2 + (y_{R1} - y)^2} + BIAS + v_{r1}$$

Discrete noise matrix is a scalar

$$\mathbf{R_k} = \sigma_{r1}^2$$

State space equation for bias

$$\dot{BIAS} = u_s$$

Systems dynamics matrix is zero therefore

$$\mathbf{\Phi_k} = 1$$

Continuous process noise matrix is a scalar

$$\mathbf{Q} = \Phi_s$$

Estimating Bias on Range-2

Formula for discrete process noise matrix

$$\mathbf{Q_k} = \int_0^{T_s} \boldsymbol{\Phi}(\tau)\mathbf{Q}\boldsymbol{\Phi}^T(\tau)\mathbf{d}\tau$$

Substitution yields

$$\mathbf{Q_k} = \int_0^{T_s} 1^* \, \Phi_s \, ^*1 \; d\tau = \Phi_s T_s$$

Or

$$\mathbf{Q_k} = \Phi_s T_s$$

Since measured range is

$$r_1^* = \sqrt{(x_{R1} - x)^2 + (y_{R1} - y)^2} + BIAS + v_{r1}$$

Linearized range measurement is

$$\Delta r_1^* = \frac{\partial r_1^*}{\partial BIAS} \Delta BIAS + v$$

Estimating Bias on Range-3

Partial derivative evaluated as

$$\frac{\partial r_1^*}{\partial BIAS} = 1$$

Therefore linearized measurement matrix is

$$\mathbf{H_k} = 1$$

Since fundamental matrix is unity projected bias is

$$\overline{BIAS}_k = \widehat{BIAS}_{k-1}$$

Projected range from satellite to receiver

$$\bar{r}_{1_k} = \sqrt{(x_{R1_k} - x_k)^2 + (y_{R1_k} - y_k)^2} + \overline{BIAS}_k$$

Residual and filter

$$RES_k = r^*_{1_k} - \bar{r}_{1_k}$$

$$\widehat{BIAS}_k = \overline{BIAS}_k + K_{1_k} RES_k$$

FORTRAN Extended Kalman Filter to Estimate Range Bias if Receiver Location is Known-1

```
GLOBAL DEFINE
        INCLUDE 'quickdraw.inc'
END
IMPLICIT REAL*8 (A-H)
IMPLICIT REAL*8 (O-Z)
REAL*8 M,MHT,KH,IKH
INTEGER ORDER
SIGNOISE=300.
BIAS=10000.
BIASH=0.
X=0.
Y=0.
XR1=1000000.
YR1=20000.*3280.
OPEN(2,STATUS='UNKNOWN',FILE='COVFIL')
OPEN(1,STATUS='UNKNOWN',FILE='DATFIL')
ORDER=1
TS=1.
TF=100.
PHIS=0.
Q=PHIS*TS
T=0.
S=0.
H=.01
P=10000.**2
RMAT=SIGNOISE**2
WHILE(T<=TF)
        XR1OLD=XR1
        XR1D=-14600.
        XR1=XR1+H*XR1D
        T=T+H
        XR1D=-14600.
        XR1=.5*(XR1OLD+XR1+H*XR1D)
        S=S+H
```

Integrating satellite equations

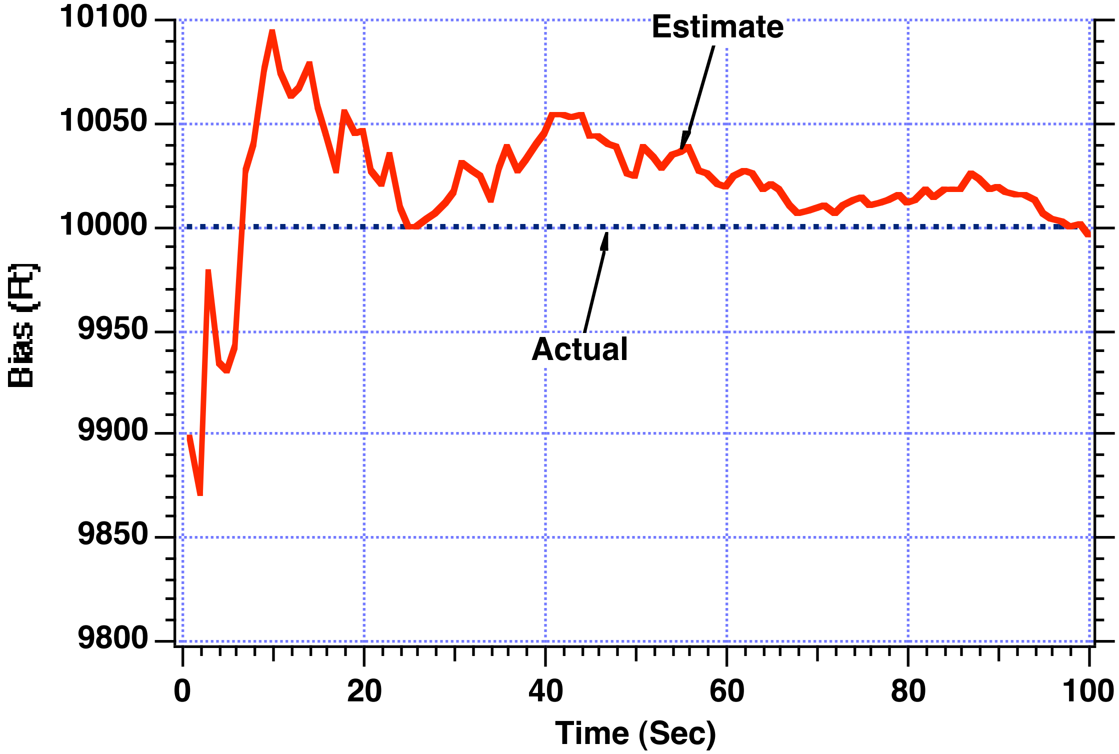
FORTRAN Extended Kalman Filter to Estimate Range Bias if Receiver Location is Known-2

```
          IF(S>=(TS-.00001))THEN
                    S=0.
                    BIASB=BIASH
                    R1B=SQRT((XR1-X)**2+(YR1-Y)**2)+BIASB
                    R1H=SQRT((XR1-X)**2+(YR1-Y)**2)+BIASH
                    HMAT=1.
                    M=P+Q
                    HMHT=HMAT*HMAT*M
                    HMHTR=HMHT+RMAT
                    HMHTRINV=1./HMHTR
                    MHT=M*HMAT
                    GAIN=MHT*HMHTRINV
                    KH=GAIN*HMAT
                    IKH=1.-KH
                    P=IKH*M
                    CALL GAUSS(R1NOISE,SIGNOISE)
                    R1=SQRT((XR1-X)**2+(YR1-Y)**2)+BIAS
                    RES1=R1+R1NOISE-R1H
                    BIASH=BIASB+GAIN*RES1
                    ERRBIAS=BIAS-BIASH
                    SP11=SQRT(P)
                    WRITE(9,*)T,BIAS,BIASH
                    WRITE(1,*)T,BIAS,BIASH
                    WRITE(2,*)T,ERRBIAS,SP11,-SP11
          ENDIF
END DO
PAUSE
CLOSE(1)
CLOSE(2)
END
```

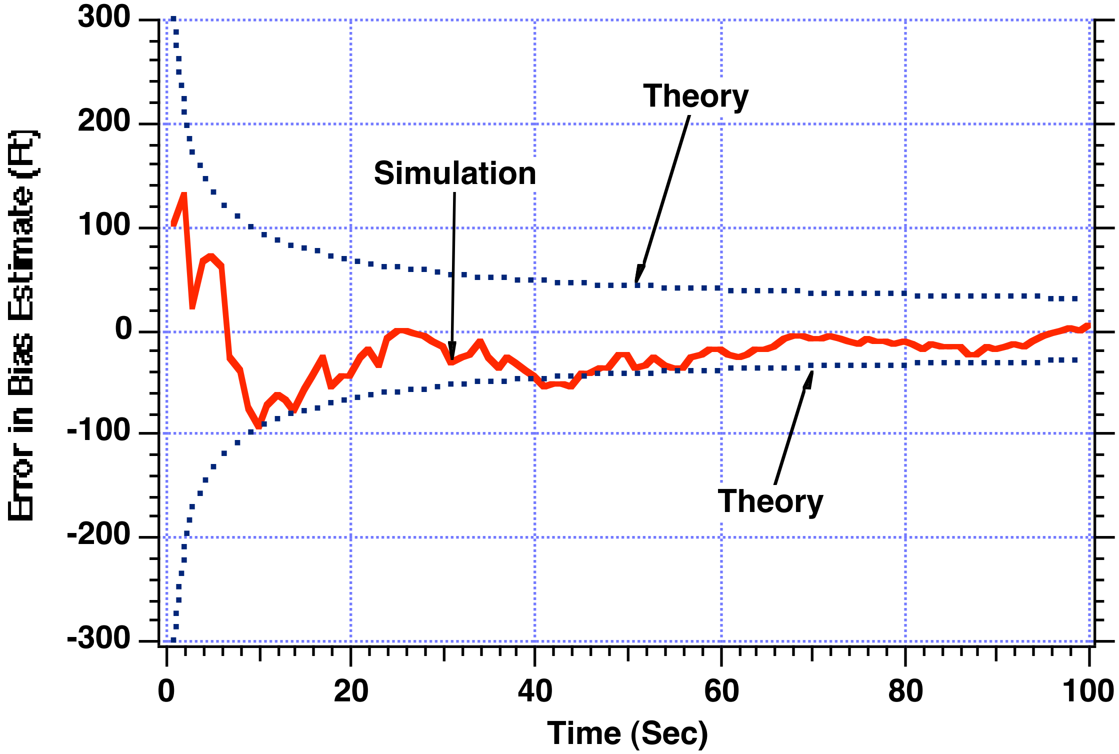
Scalar Riccati equations (M=P+Q through P=IKH*M)

Filter (CALL GAUSS through BIASH=BIASB+GAIN*RES1)

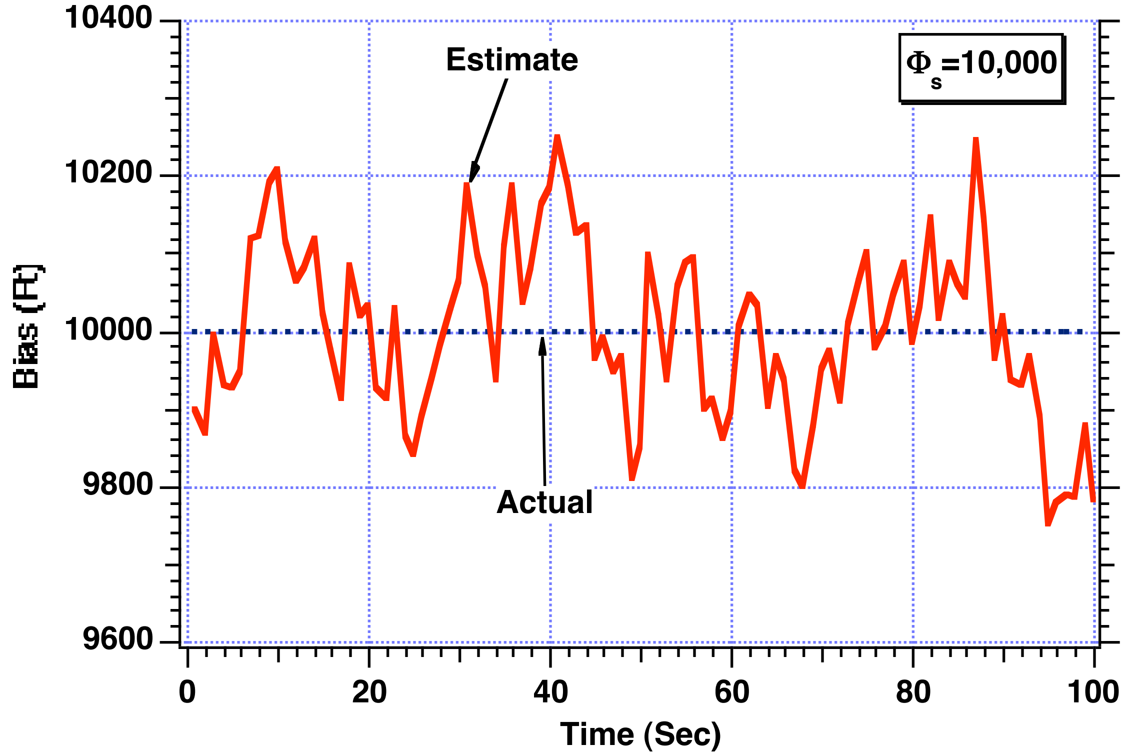
Bias Can be Estimated if Receiver Location is Known



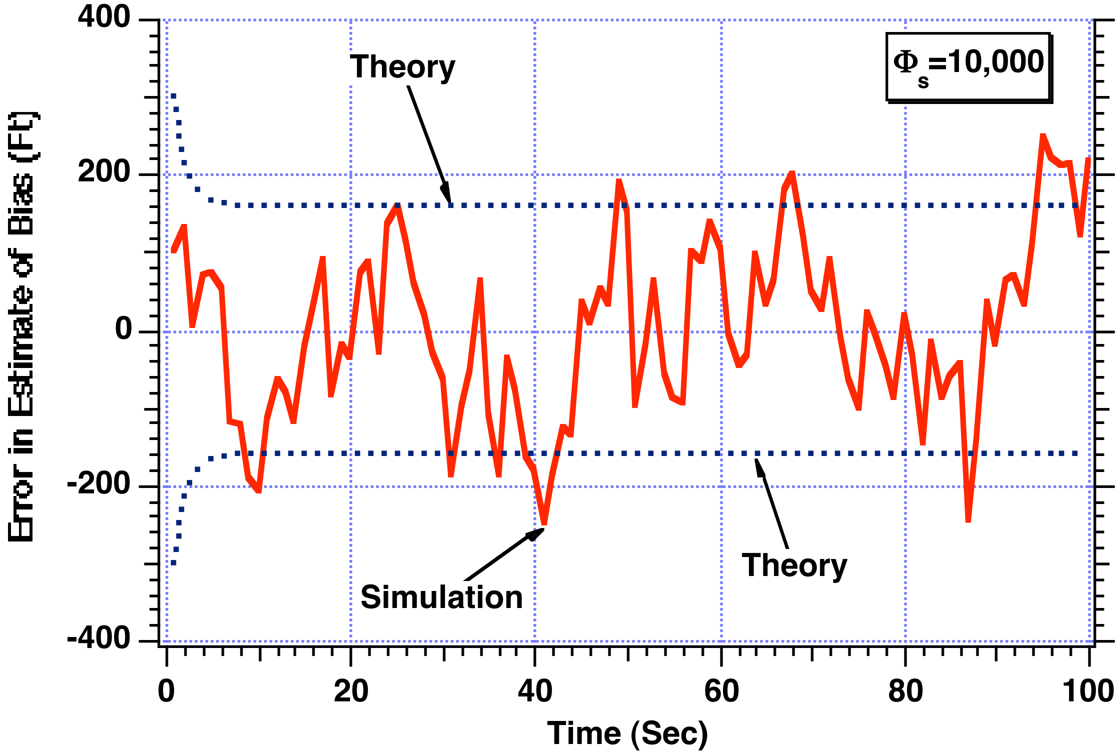
Extended Kalman Filter Appears to be Working Properly



Adding Process Noise Degrades Estimate of Bias

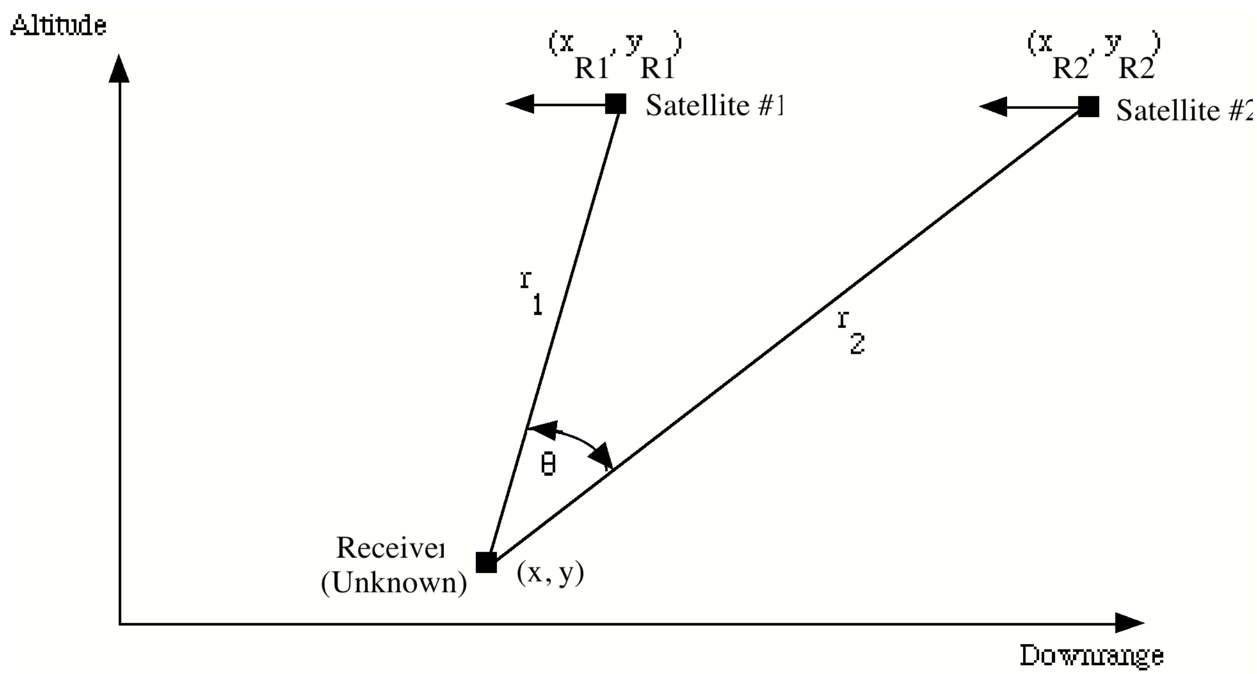


Adding Process Noise Causes Errors in Estimate of Bias to Approach a Steady State



Estimating Receiver Bias With Unknown Receiver Location and Two Satellites

Geometry for Estimating Receiver Location and Range Bias Due to the Receiver Based on Range Measurements From Two Satellites



Range measurements have bias on them

Designing Extended Kalman Filter-1

Model of real world for stationary receiver

$$\dot{x} = 0$$

$$\dot{y} = 0$$

$$\dot{BIAS} = u_s$$

State space model

$$\begin{bmatrix} \dot{x} \\ \dot{y} \\ \dot{BIAS} \end{bmatrix} = \begin{bmatrix} 0 & 0 & 0 \\ 0 & 0 & 0 \\ 0 & 0 & 0 \end{bmatrix} \begin{bmatrix} x \\ y \\ BIAS \end{bmatrix} + \begin{bmatrix} 0 \\ 0 \\ u_s \end{bmatrix}$$

Systems dynamics matrix is zero

$$\mathbf{F} = \begin{bmatrix} 0 & 0 & 0 \\ 0 & 0 & 0 \\ 0 & 0 & 0 \end{bmatrix}$$

Therefore fundamental matrix is identity matrix

$$\mathbf{\Phi_k} = \begin{bmatrix} 1 & 0 & 0 \\ 0 & 1 & 0 \\ 0 & 0 & 1 \end{bmatrix}$$

Designing Extended Kalman Filter-2

Continuous process noise matrix

$$\mathbf{Q} = \begin{bmatrix} 0 & 0 & 0 \\ 0 & 0 & 0 \\ 0 & 0 & \Phi_s \end{bmatrix}$$

Discrete process noise matrix

$$\mathbf{Q_k} = \int_0^{T_s} \mathbf{\Phi(\tau) Q \Phi^T(\tau) d\tau} \longrightarrow \mathbf{Q_k} = \begin{bmatrix} 0 & 0 & 0 \\ 0 & 0 & 0 \\ 0 & 0 & \Phi_s T_s \end{bmatrix}$$

True range from each satellite to receiver

$$r_1 = \sqrt{(x_{R1} - x)^2 + (y_{R1} - y)^2}$$

$$r_2 = \sqrt{(x_{R2} - x)^2 + (y_{R2} - y)^2}$$

Measured range from each satellite

$$r_1^* = \sqrt{(x_{R1} - x)^2 + (y_{R1} - y)^2} + BIAS + v_{r1}$$

$$r_2^* = \sqrt{(x_{R2} - x)^2 + (y_{R2} - y)^2} + BIAS + v_{r2}$$

Designing Extended Kalman Filter-3

Linearized measurement equation

$$\begin{bmatrix} \Delta r_1^* \\ \Delta r_2^* \end{bmatrix} = \begin{bmatrix} \frac{\partial r_1^*}{\partial x} & \frac{\partial r_1^*}{\partial y} & \frac{\partial r_1^*}{\partial BIAS} \\ \frac{\partial r_2^*}{\partial x} & \frac{\partial r_2^*}{\partial y} & \frac{\partial r_2^*}{\partial BIAS} \end{bmatrix} \begin{bmatrix} \Delta x \\ \Delta y \\ \Delta BIAS \end{bmatrix} + \begin{bmatrix} v_{r1} \\ v_{r2} \end{bmatrix}$$

Discrete measurement noise matrix

$$\mathbf{R_k} = \begin{bmatrix} \sigma_{r1}^2 & 0 \\ 0 & \sigma_{r2}^2 \end{bmatrix}$$

Linearized measurement matrix

$$\mathbf{H_k} = \begin{bmatrix} \frac{\partial r_1^*}{\partial x} & \frac{\partial r_1^*}{\partial y} & \frac{\partial r_1^*}{\partial BIAS} \\ \frac{\partial r_2^*}{\partial x} & \frac{\partial r_2^*}{\partial y} & \frac{\partial r_2^*}{\partial BIAS} \end{bmatrix}$$

Designing Extended Kalman Filter-4

Evaluating partial derivatives

$$r_1^* = \sqrt{(x_{R1} - x)^2 + (y_{R1} - y)^2} + BIAS + v_{r1} \longrightarrow$$

$$\frac{\partial r_1^*}{\partial x} = .5\left[(x_{R1}-x)^2+(y_{R1}-y)^2\right]^{-.5} 2(x_{R1}-x)(-1) = \frac{-(x_{R1}-x)}{r_1}$$

$$\frac{\partial r_1^*}{\partial y} = .5\left[(x_{R1}-x)^2+(y_{R1}-y)^2\right]^{-.5} 2(y_{R1}-y)(-1) = \frac{-(y_{R1}-y)}{r_1}$$

$$\frac{\partial r_1^*}{\partial BIAS} = 1$$

$$r_2^* = \sqrt{(x_{R2} - x)^2 + (y_{R2} - y)^2} + BIAS + v_{r2} \longrightarrow$$

$$\frac{\partial r_2^*}{\partial x} = .5\left[(x_{R1}-x)^2+(y_{R1}-y)^2\right]^{-.5} 2(x_{R2}-x)(-1) = \frac{-(x_{R2}-x)}{r_2}$$

$$\frac{\partial r_2^*}{\partial y} = .5\left[(x_{R1}-x)^2+(y_{R1}-y)^2\right]^{-.5} 2(y_{R2}-y)(-1) = \frac{-(y_{R2}-y)}{r_2}$$

$$\frac{\partial r_2^*}{\partial BIAS} = 1$$

Linearized measurement matrix

$$\mathbf{H_k} = \begin{bmatrix} \frac{-(x_{R1}-x)}{r_1} & \frac{-(y_{R1}-y)}{r_1} & 1 \\ \frac{-(x_{R2}-x)}{r_2} & \frac{-(y_{R2}-y)}{r_2} & 1 \end{bmatrix}$$

Designing Extended Kalman Filter-5

Projected state estimates

$$\bar{x}_k = \hat{x}_{k-1}$$

$$\bar{y}_k = \hat{y}_{k-1}$$

$$\overline{BIAS}_k = \widehat{BIAS}_{k-1}$$

Projected ranges from each satellite

$$\bar{r}_{1_k} = \sqrt{(x_{R1_k} - \bar{x}_k)^2 + (y_{R1_k} - \bar{y}_k)^2} + \overline{BIAS}_k$$

$$\bar{r}_{2_k} = \sqrt{(x_{R2_k} - \bar{x}_k)^2 + (y_{R2_k} - \bar{y}_k)^2} + \overline{BIAS}_k$$

Filtering equations

$$RES_{1_k} = r^*_{1_k} - \bar{r}_{1_k}$$

$$RES_{2_k} = r^*_{2_k} - \bar{r}_{2_k}$$

$$\hat{x}_k = \bar{x}_k + K_{11_k} RES_{1_k} + K_{12_k} RES_{2_k}$$

$$\hat{y}_k = \bar{y}_k + K_{21_k} RES_{1_k} + K_{22_k} RES_{2_k}$$

$$\widehat{BIAS}_k = \overline{BIAS}_k + K_{31_k} RES_{1_k} + K_{32_k} RES_{2_k}$$

MATLAB Version of Extended Kalman Filter-1

```
SIGNOISE=300.;
TF=100.;
X=0.;
Y=0.;
XH=1000.;
YH=2000.;
XR1=1000000.;
YR1=20000.*3280.;
XR2=50000000.;
YR2=20000.*3280.;
BIAS=10000.;
BIASH=0.;
PHIS=0.;
ORDER=3;
TS=1.;
T=0.;
S=0.;
H=.01;
PHI=zeros(ORDER,ORDER);
P=zeros(ORDER,ORDER);
IDNP=eye(ORDER);
Q=zeros(ORDER,ORDER);
Q(3,3)=PHIS*TS;
P(1,1)=1000.^2;
P(2,2)=2000.^2;
P(3,3)=500.^2;
RMAT(1,1)=SIGNOISE^2;
RMAT(1,2)=0.;
RMAT(2,1)=0.;
RMAT(2,2)=SIGNOISE^2;
count=0;
```

Actual and estimated receiver location (X, Y, XH, YH)

Actual and estimated range bias (BIAS, BIASH)

Initial covariance matrix (P(1,1), P(2,2), P(3,3))

Measurement noise matrix (RMAT)

MATLAB Version of Extended Kalman Filter-2

```
while T<=TF
        XR1OLD=XR1;
        XR2OLD=XR2;
        XR1D=-14600.;
        XR2D=-14600.;
        XR1=XR1+H*XR1D;
        XR2=XR2+H*XR2D;
        T=T+H;
        XR1D=-14600.;
        XR2D=-14600.;
        XR1=.5*(XR1OLD+XR1+H*XR1D);
        XR2=.5*(XR2OLD+XR2+H*XR2D);
        S=S+H;
        if S>=(TS-.00001)
                S=0.;
                R1H=sqrt((XR1-XH)^2+(YR1-YH)^2)+BIASH;
                R2H=sqrt((XR2-XH)^2+(YR2-YH)^2)+BIASH;
                HMAT(1,1)=-(XR1-XH)/sqrt((XR1-XH)^2+(YR1-YH)^2);
                HMAT(1,2)=-(YR1-YH)/sqrt((XR1-XH)^2+(YR1-YH)^2);
                HMAT(1,3)=1.;
                HMAT(2,1)=-(XR2-XH)/sqrt((XR2-XH)^2+(YR2-YH)^2);
                HMAT(2,2)=-(YR2-YH)/sqrt((XR2-XH)^2+(YR2-YH)^2);
                HMAT(2,3)=1.;
                HT=HMAT';
                PHI(1,1)=1.;
                PHI(2,2)=1.;
                PHI(3,3)=1.;
                PHIT=PHI';
                PHIP=PHI*P;
                PHIPPHIT=PHIP*PHIT;
                M=PHIPPHIT+Q;
                HM=HMAT*M;
                HMHT=HM*HT;
                HMHTR=HMHT+RMAT;
                HMHTRINV=inv(HMHTR);
                MHT=M*HT;
                GAIN=MHT*HMHTRINV;
                KH=GAIN*HMAT;
                IKH=IDNP-KH;
                P=IKH*M;
```

Integration of satellite equations with second-order Runge-Kutta technique (XR1OLD=XR1; through S=S+H;)

Linearized measurement matrix (R1H through HMAT(2,2))

Fundamental matrix (PHI(1,1) through PHI(3,3))

Riccati equations (PHIP=PHI*P; through P=IKH*M;)

MATLAB Version of Extended Kalman Filter-3

```
			R1NOISE=SIGNOISE*randn;
			R2NOISE=SIGNOISE*randn;
			R1=sqrt((XR1-X)^2+(YR1-Y)^2);
			R2=sqrt((XR2-X)^2+(YR2-Y)^2);
			RES1=R1+R1NOISE+BIAS-R1H;
			RES2=R2+R2NOISE+BIAS-R2H;
			XH=XH+GAIN(1,1)*RES1+GAIN(1,2)*RES2;
			YH=YH+GAIN(2,1)*RES1+GAIN(2,2)*RES2;
			BIASH=BIASH+GAIN(3,1)*RES1+GAIN(3,2)*RES2;
			ERRX=X-XH;
			SP11=sqrt(P(1,1));
			ERRY=Y-YH;
			SP22=sqrt(P(2,2));
			ERRBIAS=BIAS-BIASH;
			SP33=sqrt(P(3,3));
			R1T=sqrt((XR1-X)^2+(YR1-Y)^2);
			R2T=sqrt((XR2-X)^2+(YR2-Y)^2);
			TEMP=(XR1-X)*(XR2-X)+(YR1-Y)*(YR2-Y);
			TH=atan2(sqrt((R1T*R2T)^2-TEMP^2),TEMP);
			THET=57.3*TH;
			SP11P=-SP11;
			SP22P=-SP22;
			SP33P=-SP33;
			count=count+1;
			ArrayT(count)=T;
			ArrayX(count)=X;
			ArrayXH(count)=XH;
			ArrayY(count)=Y;
			ArrayYH(count)=YH;
			ArrayBIAS(count)=BIAS;
			ArrayBIASH(count)=BIASH;
			ArrayERRX(count)=ERRX;
			ArraySP11(count)=SP11;
			ArraySP11P(count)=SP11P;
			ArrayERRY(count)=ERRY;
			ArraySP22(count)=SP22;
			ArraySP22P(count)=SP22P;
			ArrayERRBIAS(count)=ERRBIAS;
			ArraySP33(count)=SP33;
			ArraySP33P(count)=SP33P;
		end
end
```

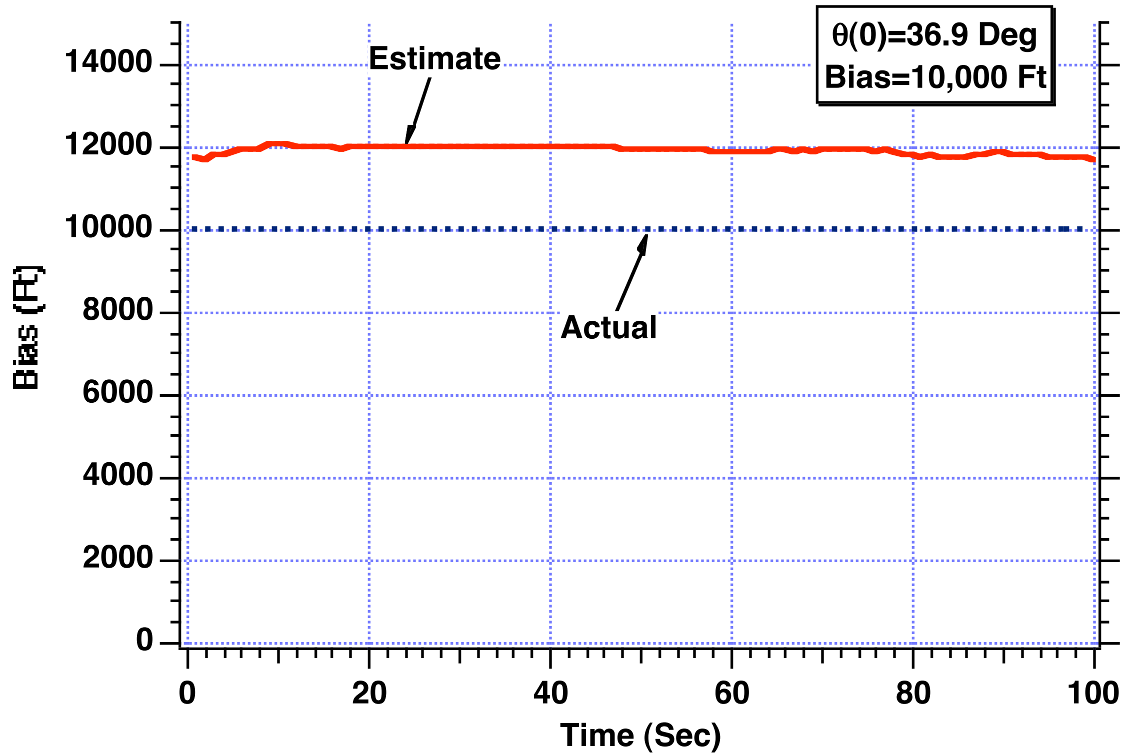
Filter

Actual and theoretical errors in estimates

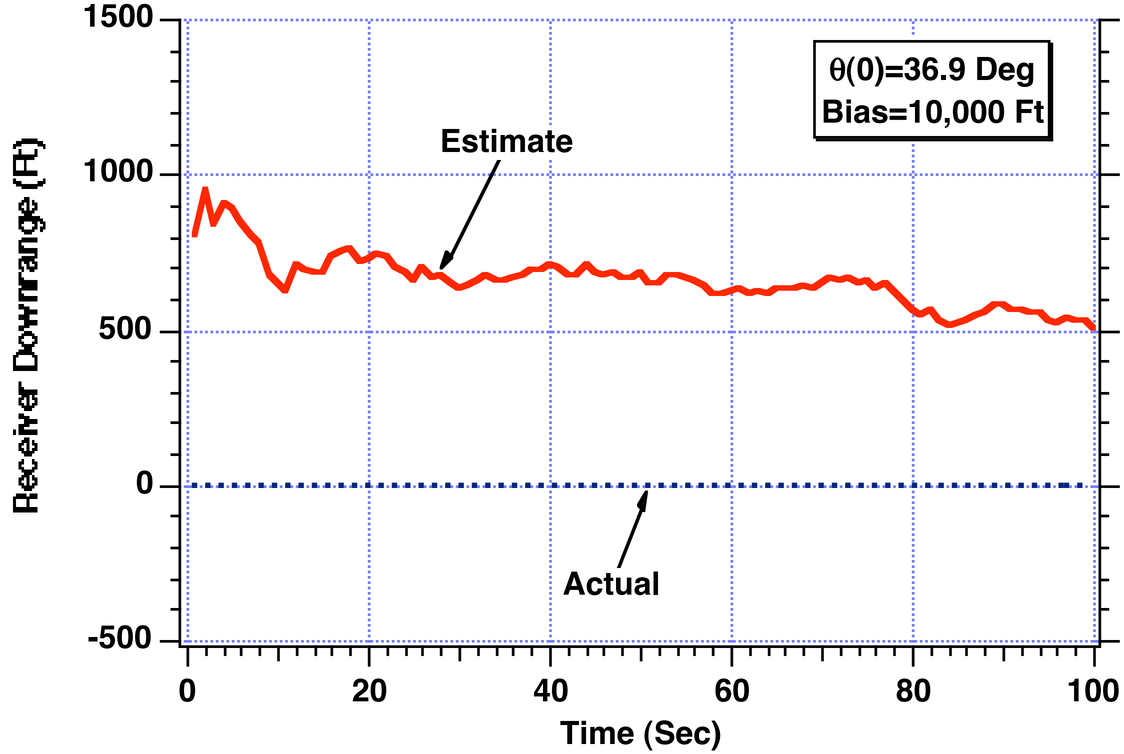
Angle between two range vectors

Save data as arrays for plotting and writing to files

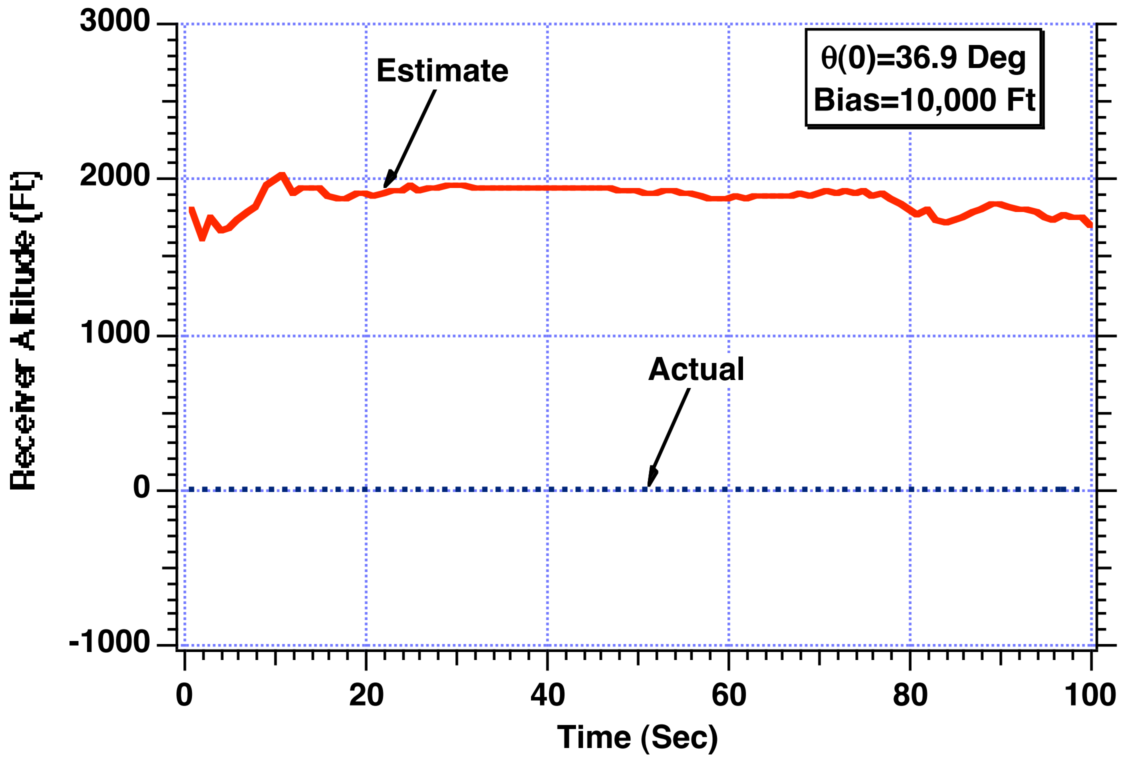
Extended Kalman Filter is Unable to Estimate Bias Based on Range Measurements From Two Satellites



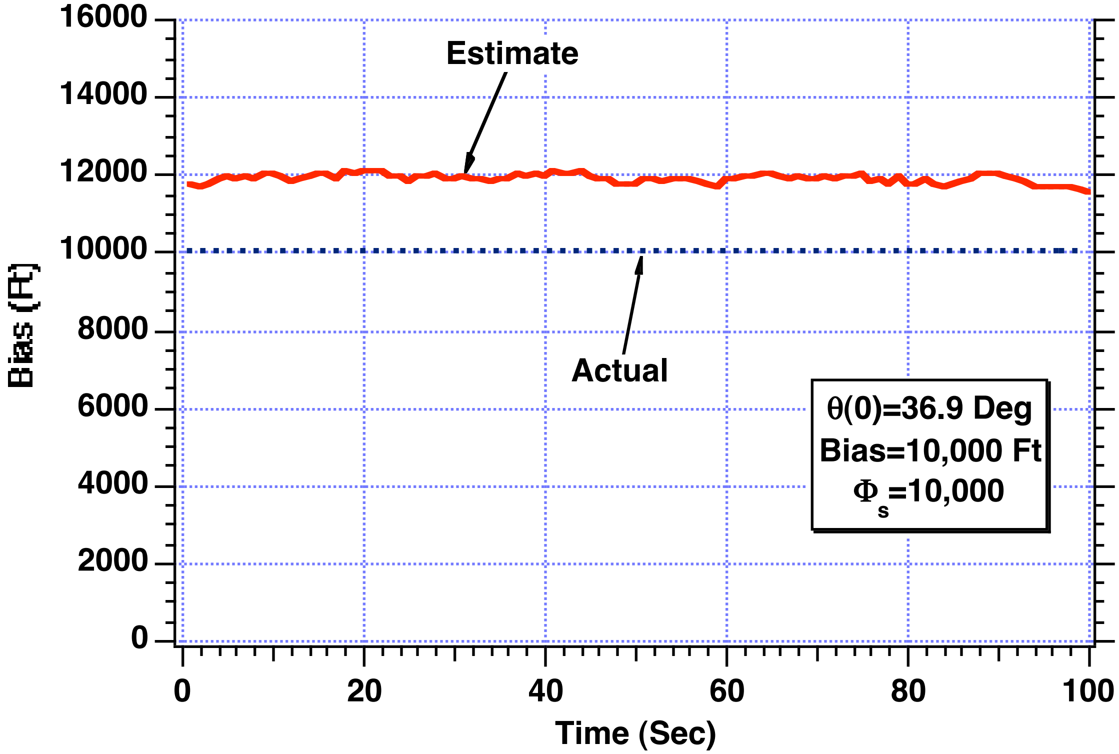
Extended Kalman Filter is Unable to Estimate Receiver Downrange Based on Range Measurements From Two Satellites



Extended Kalman Filter is Unable to Estimate Receiver Altitude Based on Range Measurements From Two Satellites

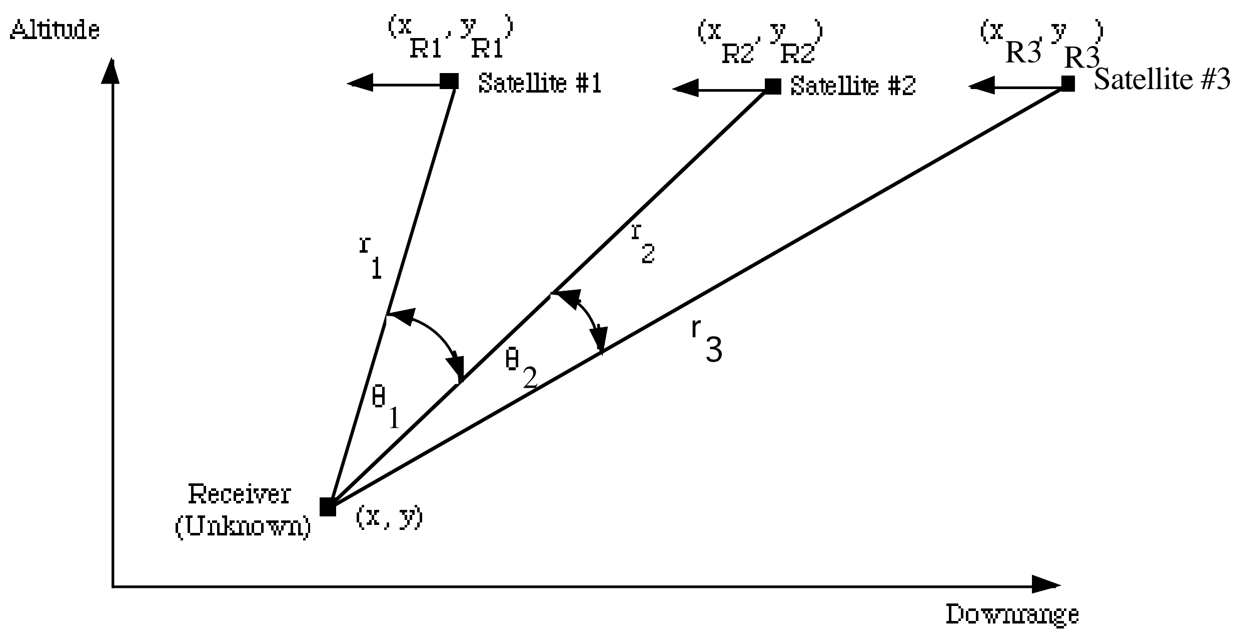


Process Noise Does Not Help Filter to Estimate Bias



Estimating Receiver Bias With Unknown Receiver Location and Three Satellites

Geometry for Estimating Receiver Location and Range Bias Based on Range Measurements From Three Satellites



Range measurements have bias in them

Designing Extended New Kalman Filter-1

Model of real world for stationary receiver

$$\dot{x} = 0$$

$$\dot{y} = 0$$

$$B\dot{I}AS = u_s$$

State space model

$$\begin{bmatrix} \dot{x} \\ \dot{y} \\ B\dot{I}AS \end{bmatrix} = \begin{bmatrix} 0 & 0 & 0 \\ 0 & 0 & 0 \\ 0 & 0 & 0 \end{bmatrix} \begin{bmatrix} x \\ y \\ BIAS \end{bmatrix} + \begin{bmatrix} 0 \\ 0 \\ u_s \end{bmatrix}$$

Systems dynamics matrix is zero

$$\mathbf{F} = \begin{bmatrix} 0 & 0 & 0 \\ 0 & 0 & 0 \\ 0 & 0 & 0 \end{bmatrix}$$

Therefore fundamental matrix is identity matrix

$$\mathbf{\Phi_k} = \begin{bmatrix} 1 & 0 & 0 \\ 0 & 1 & 0 \\ 0 & 0 & 1 \end{bmatrix}$$

Designing New Extended Kalman Filter-2

Continuous process noise matrix

$$\mathbf{Q} = \begin{bmatrix} 0 & 0 & 0 \\ 0 & 0 & 0 \\ 0 & 0 & \Phi_s \end{bmatrix}$$

Discrete process noise matrix

$$\mathbf{Q_k} = \int_0^{T_s} \mathbf{\Phi(\tau) Q \Phi^T(\tau) d\tau} \longrightarrow \mathbf{Q_k} = \begin{bmatrix} 0 & 0 & 0 \\ 0 & 0 & 0 \\ 0 & 0 & \Phi_s T_s \end{bmatrix}$$

True range from each satellite to receiver

$$r_1 = \sqrt{(x_{R1} - x)^2 + (y_{R1} - y)^2}$$

$$r_2 = \sqrt{(x_{R2} - x)^2 + (y_{R2} - y)^2}$$

$$r_3 = \sqrt{(x_{R3} - x)^2 + (y_{R3} - y)^2}$$

Measured range from each satellite

$$r_1^* = \sqrt{(x_{R1} - x)^2 + (y_{R1} - y)^2} + BIAS + v_{r1}$$

$$r_2^* = \sqrt{(x_{R2} - x)^2 + (y_{R2} - y)^2} + BIAS + v_{r2}$$

$$r_3^* = \sqrt{(x_{R3} - x)^2 + (y_{R3} - y)^2} + BIAS + v_{r3}$$

Designing New Extended Kalman Filter-3

Linearized measurement equation

$$\begin{bmatrix} \Delta r_1^* \\ \Delta r_2^* \\ \Delta r_3^* \end{bmatrix} = \begin{bmatrix} \frac{\partial r_1^*}{\partial x} & \frac{\partial r_1^*}{\partial y} & \frac{\partial r_1^*}{\partial BIAS} \\ \frac{\partial r_2^*}{\partial x} & \frac{\partial r_2^*}{\partial y} & \frac{\partial r_2^*}{\partial BIAS} \\ \frac{\partial r_3^*}{\partial x} & \frac{\partial r_3^*}{\partial y} & \frac{\partial r_3^*}{\partial BIAS} \end{bmatrix} \begin{bmatrix} \Delta x \\ \Delta y \\ \Delta BIAS \end{bmatrix} + \begin{bmatrix} v_{r1} \\ v_{r2} \\ v_{r3} \end{bmatrix}$$

Discrete measurement noise matrix

$$\mathbf{R_k} = \begin{bmatrix} \sigma_{r1}^2 & 0 & 0 \\ 0 & \sigma_{r2}^2 & 0 \\ 0 & 0 & \sigma_{r3}^2 \end{bmatrix}$$

Linearized measurement matrix

$$\mathbf{H_k} = \begin{bmatrix} \frac{\partial r_1^*}{\partial x} & \frac{\partial r_1^*}{\partial y} & \frac{\partial r_1^*}{\partial BIAS} \\ \frac{\partial r_2^*}{\partial x} & \frac{\partial r_2^*}{\partial y} & \frac{\partial r_2^*}{\partial BIAS} \\ \frac{\partial r_3^*}{\partial x} & \frac{\partial r_3^*}{\partial y} & \frac{\partial r_3^*}{\partial BIAS} \end{bmatrix}$$

Designing New Extended Kalman Filter-4

Evaluating partial derivatives

$$r_1^* = \sqrt{(x_{R1} - x)^2 + (y_{R1} - y)^2} + BIAS + v_{r1} \longrightarrow$$

$$\frac{\partial r_1^*}{\partial x} = .5\left[(x_{R1}-x)^2+(y_{R1}-y)^2\right]^{-.5} 2(x_{R1}-x)(-1) = \frac{-(x_{R1}-x)}{r_1}$$

$$\frac{\partial r_1^*}{\partial y} = .5\left[(x_{R1}-x)^2+(y_{R1}-y)^2\right]^{-.5} 2(y_{R1}-y)(-1) = \frac{-(y_{R1}-y)}{r_1}$$

$$\frac{\partial r_1^*}{\partial BIAS} = 1$$

$$r_2^* = \sqrt{(x_{R2} - x)^2 + (y_{R2} - y)^2} + BIAS + v_{r2} \longrightarrow$$

$$\frac{\partial r_2^*}{\partial x} = .5\left[(x_{R1}-x)^2+(y_{R1}-y)^2\right]^{-.5} 2(x_{R2}-x)(-1) = \frac{-(x_{R2}-x)}{r_2}$$

$$\frac{\partial r_2^*}{\partial y} = .5\left[(x_{R1}-x)^2+(y_{R1}-y)^2\right]^{-.5} 2(y_{R2}-y)(-1) = \frac{-(y_{R2}-y)}{r_2}$$

$$\frac{\partial r_2^*}{\partial BIAS} = 1$$

$$r_3^* = \sqrt{(x_{R3} - x)^2 + (y_{R3} - y)^2} + BIAS + v_{r3} \longrightarrow$$

$$\frac{\partial r_3^*}{\partial x} = .5\left[(x_{R3}-x)^2+(y_{R3}-y)^2\right]^{-.5} 2(x_{R3}-x)(-1) = \frac{-(x_{R3}-x)}{r_3}$$

$$\frac{\partial r_3^*}{\partial y} = .5\left[(x_{R3}-x)^2+(y_{R3}-y)^2\right]^{-.5} 2(y_{R3}-y)(-1) = \frac{-(y_{R3}-y)}{r_3}$$

$$\frac{\partial r_3^*}{\partial BIAS} = 1$$

Designing New Extended Kalman Filter-5

Linearized measurement matrix

$$\mathbf{H_k} = \begin{bmatrix} \frac{-(x_{R1}-x)}{r_1} & \frac{-(y_{R1}-y)}{r_1} & 1 \\ \frac{-(x_{R2}-x)}{r_2} & \frac{-(y_{R2}-y)}{r_2} & 1 \\ \frac{-(x_{R3}-x)}{r_3} & \frac{-(y_{R2}-y)}{r_2} & 1 \end{bmatrix}$$

Projected state estimates

$$\bar{x}_k = \hat{x}_{k-1}$$

$$\bar{y}_k = \hat{y}_{k-1}$$

$$\overline{BIAS}_k = \widehat{BIAS}_{k-1}$$

Projected ranges from each satellite

$$\bar{r}_{1_k} = \sqrt{(x_{R1_k} - \bar{x}_k)^2 + (y_{R1_k} - \bar{y}_k)^2} + \overline{BIAS}_k$$

$$\bar{r}_{2_k} = \sqrt{(x_{R2_k} - \bar{x}_k)^2 + (y_{R2_k} - \bar{y}_k)^2} + \overline{BIAS}_k$$

$$\bar{r}_{3_k} = \sqrt{(x_{R3_k} - \bar{x}_k)^2 + (y_{R3_k} - \bar{y}_k)^2} + \overline{BIAS}_k$$

Designing Extended Kalman Filter-6

Filtering equations

$$RES_{1_k} = r^*_{1_k} - \bar{r}_{1_k}$$

$$RES_{2_k} = r^*_{2_k} - \bar{r}_{2_k}$$

$$RES_{3_k} = r^*_{3_k} - \bar{r}_{3_k}$$

$$\hat{x}_k = \bar{x}_k + K_{11_k}RES_{1_k} + K_{12_k}RES_{2_k} + K_{13_k}RES_{3_k}$$

$$\hat{y}_k = \bar{y}_k + K_{21_k}RES_{1_k} + K_{22_k}RES_{2_k} + K_{23_k}RES_{3_k}$$

$$\widehat{BIAS}_k = \overline{BIAS}_k + K_{31_k}RES_{1_k} + K_{32_k}RES_{2_k} + K_{33_k}RES_{3_k}$$

True BASIC Version of New Extended Kalman Filter-1

```
OPTION NOLET
REM UNSAVE "DATFIL"
REM UNSAVE "COVFIL"
OPEN #1:NAME "DATFIL",ACCESS OUTPUT,CREATE NEW, ORGANIZATION TEXT
OPEN #2:NAME "COVFIL",ACCESS OUTPUT,CREATE NEW, ORGANIZATION TEXT
SET #1: MARGIN 1000
SET #2: MARGIN 1000
DIM PHI(3,3),P(3,3),M(3,3),PHIP(3,3),PHIPPHIT(3,3),GAIN(3,3)
DIM Q(3,3),HMAT(3,3),HM(3,3),MHT(3,3)
DIM PHIT(3,3),RMAT(3,3),HMHTRINV(3,3)
DIM HMHT(3,3),HT(3,3),KH(3,3),IDNP(3,3),IKH(3,3),HMHTR(3,3)
TF=100.
SIGNOISE=300.
X=0.
Y=0.
XH=1000.
YH=2000.
XR1=1000000.
YR1=20000.*3280.
XR2=50000000.
YR2=20000.*3280.
XR3=1000000000.
YR3=20000.*3280.
BIAS=10000.
BIASH=0.
PHIS=0.
ORDER=3
TS=1.
T=0.
S=0.
H=.01
MAT PHI=ZER(ORDER,ORDER)
MAT P=ZER(ORDER,ORDER)
MAT Q=ZER(ORDER,ORDER)
MAT IDNP=IDN(ORDER)
MAT RMAT=ZER(3,3)
Q(3,3)=PHIS*TS
```

Actual and estimated receiver location (X=0., Y=0., XH=1000., YH=2000.)

Actual and estimated range bias (BIAS=10000., BIASH=0.)

True BASIC Version of New Extended Kalman Filter-2

```
P(1,1)=1000.^2
P(2,2)=2000.^2
P(3,3)=(BIAS-BIASH)^2
RMAT(1,1)=SIGNOISE^2
RMAT(2,2)=SIGNOISE^2
RMAT(3,3)=SIGNOISE^2
DO WHILE T<=TF
        XR1OLD=XR1
        XR2OLD=XR2
        XR3OLD=XR3
        XR1D=-14600.
        XR2D=-14600.
        XR3D=-14600.
        XR1=XR1+H*XR1D
        XR2=XR2+H*XR2D
        XR3=XR3+H*XR3D
        T=T+H
        XR1D=-14600.
        XR2D=-14600.
        XR3D=-14600.
        XR1=.5*(XR1OLD+XR1+H*XR1D)
        XR2=.5*(XR2OLD+XR2+H*XR2D)
        XR3=.5*(XR3OLD+XR3+H*XR3D)
        S=S+H
        IF S>=(TS-.00001) THEN
                S=0.
                R1H=SQR((XR1-XH)^2+(YR1-YH)^2)+BIASH
                R2H=SQR((XR2-XH)^2+(YR2-YH)^2)+BIASH
                R3H=SQR((XR3-XH)^2+(YR3-YH)^2)+BIASH
                HMAT(1,1)=-(XR1-XH)/SQR((XR1-XH)^2+(YR1-YH)^2)
                HMAT(1,2)=-(YR1-YH)/SQR((XR1-XH)^2+(YR1-YH)^2)
                HMAT(1,3)=1.
                HMAT(2,1)=-(XR2-XH)/SQR((XR2-XH)^2+(YR2-YH)^2)
                HMAT(2,2)=-(YR2-YH)/SQR((XR2-XH)^2+(YR2-YH)^2)
                HMAT(2,3)=1.
                HMAT(3,1)=-(XR3-XH)/SQR((XR3-XH)^2+(YR3-YH)^2)
                HMAT(3,2)=-(YR3-YH)/SQR((XR3-XH)^2+(YR3-YH)^2)
                HMAT(3,3)=1.
                MAT HT=TRN(HMAT)
```

Initial covariance matrix (P(1,1) through P(3,3))

Measurement noise matrix (RMAT lines)

Numerical integration of satellite equations with second-order Runge-Kutta technique (XR1OLD=XR1 through XR3=.5*(...))

Linearized measurement matrix (R1H through HMAT(3,3)=1.)

True BASIC Version of New Extended Kalman Filter-3

```
PHI(1,1)=1.
PHI(2,2)=1.
PHI(3,3)=1.
MAT PHIT=TRN(PHI)
MAT PHIP=PHI*P
MAT PHIPPHIT=PHIP*PHIT
MAT M=PHIPPHIT+Q
MAT HM=HMAT*M
MAT HMHT=HM*HT
MAT HMHTR=HMHT+RMAT
MAT HMHTRINV=INV(HMHTR)
MAT MHT=M*HT
MAT GAIN=MHT*HMHTRINV
MAT KH=GAIN*HMAT
MAT IKH=IDNP-KH
MAT P=IKH*M
CALL GAUSS(R1NOISE,SIGNOISE)
CALL GAUSS(R2NOISE,SIGNOISE)
CALL GAUSS(R3NOISE,SIGNOISE)
R1=SQR((XR1-X)^2+(YR1-Y)^2)
R2=SQR((XR2-X)^2+(YR2-Y)^2)
R3=SQR((XR3-X)^2+(YR3-Y)^2)
RES1=R1+R1NOISE+BIAS-R1H
RES2=R2+R2NOISE+BIAS-R2H
RES3=R3+R3NOISE+BIAS-R3H
XH=XH+GAIN(1,1)*RES1+GAIN(1,2)*RES2+GAIN(1,3)*RES3
YH=YH+GAIN(2,1)*RES1+GAIN(2,2)*RES2+GAIN(2,3)*RES3
BIASH=BIASH+GAIN(3,1)*RES1+GAIN(3,2)*RES2+GAIN(3,3)*RES3
ERRX=X-XH
SP11=SQR(P(1,1))
ERRY=Y-YH
SP22=SQR(P(2,2))
ERRBIAS=BIAS-BIASH
SP33=SQR(P(3,3))
R1T=SQR((XR1-X)^2+(YR1-Y)^2)
R2T=SQR((XR2-X)^2+(YR2-Y)^2)
TEMP=(XR1-X)*(XR2-X)+(YR1-Y)*(YR2-Y)
CALL ATAN2(SQR((R1T*R2T)^2-TEMP^2), TEMP, TH1)
THET1=57.3*TH1
```

Fundamental matrix (PHI(1,1) through PHI(3,3))

Riccati equations (MAT PHIT through MAT P)

Filter (R3 through BIASH)

Actual and theoretical errors in estimates (ERRX through SP33)

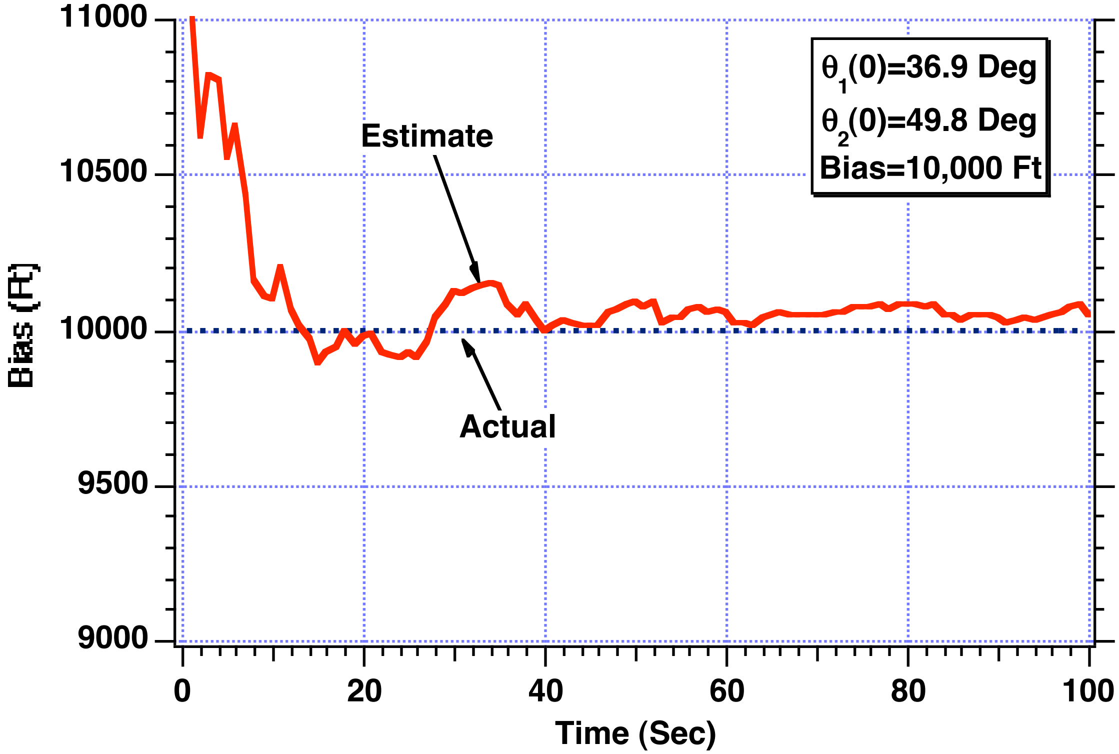
True BASIC Version of New Extended Kalman Filter-4

```
                    R3T=SQR((XR3-X)^2+(YR3-Y)^2)
                    TEMP3=(XR2-X)*(XR3-X)+(YR2-Y)*(YR3-Y)
                    CALL ATAN2(SQR((R2T*R3T)^2-TEMP3^2), TEMP3, TH2)
                    THET2=57.3*TH2
                    PRINT T,X,XH,Y,YH,BIAS,BIASH,THET1,THET2
                    PRINT #1:T,X,XH,Y,YH,BIAS,BIASH,THET1,THET2
                    PRINT #2:T,ERRX,SP11,-SP11,ERRY,SP22,-SP22,ERRBIAS,SP33,-SP33
          END IF
LOOP
CLOSE #1
CLOSE #2
END

SUB GAUSS(X,SIG)
LET X=RND+RND+RND+RND+RND+RND-3
LET X=1.414*X*SIG
END SUB

SUB ATAN2 (Y, X, Z)
IF X < 0 THEN
          IF Y < 0 THEN
                    LET Z = ATN(Y / X) - 3.14159
          ELSE
                    LET Z = ATN(Y / X) + 3.14159
          END IF
ELSE
          LET Z = ATN(Y / X)
END IF
END SUB
```

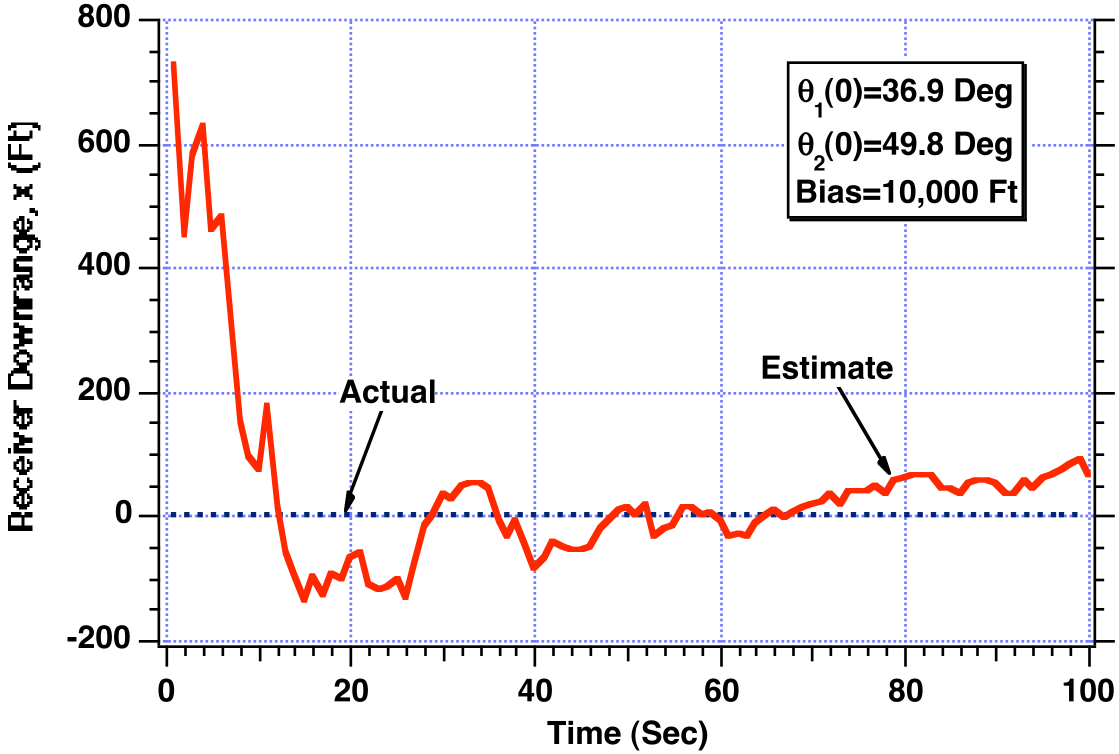
Subroutine for zero mean Gaussian noise

Arctangent subroutine which is good in all 4 quadrants

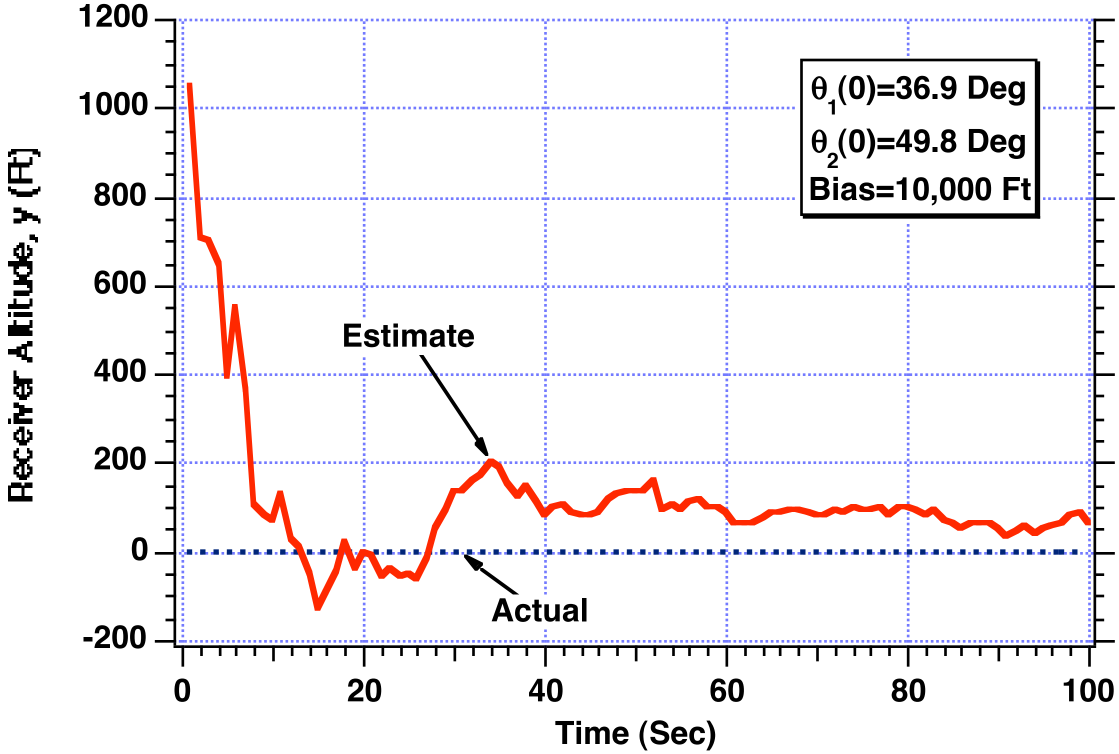
Three Satellite Range Measurements Enable New Extended Kalman Filter to Estimate Bias



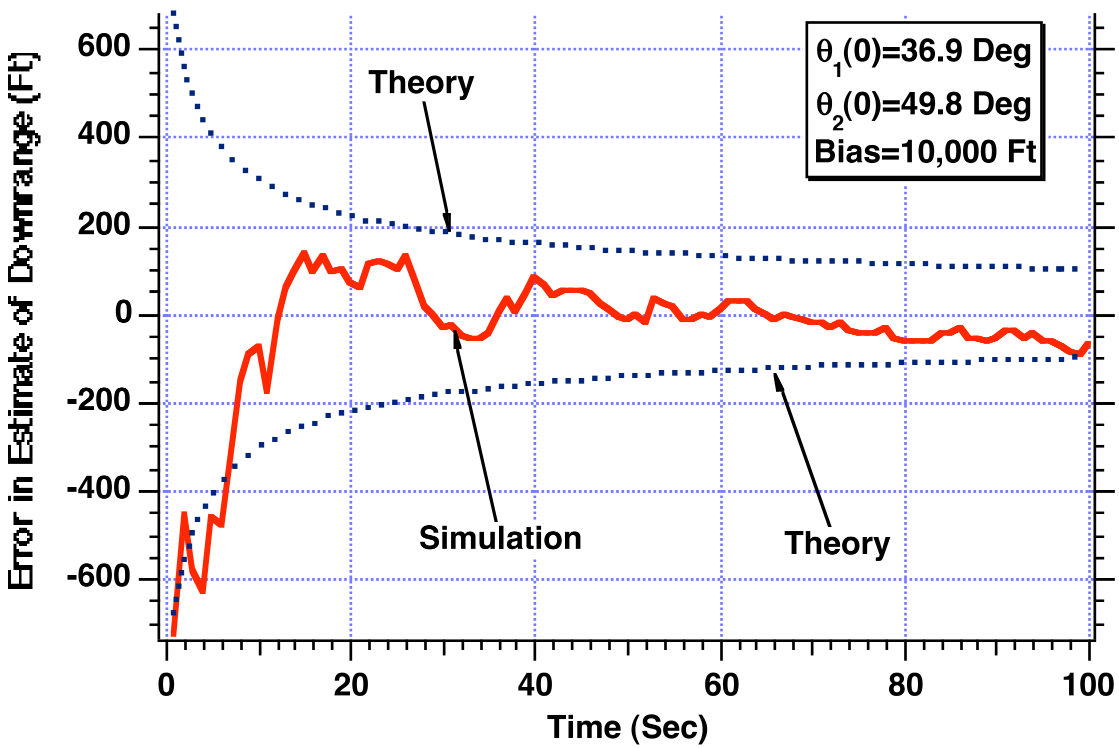
Three Satellite Range Measurements Enable New Extended Kalman Filter to Estimate Receiver Downrange



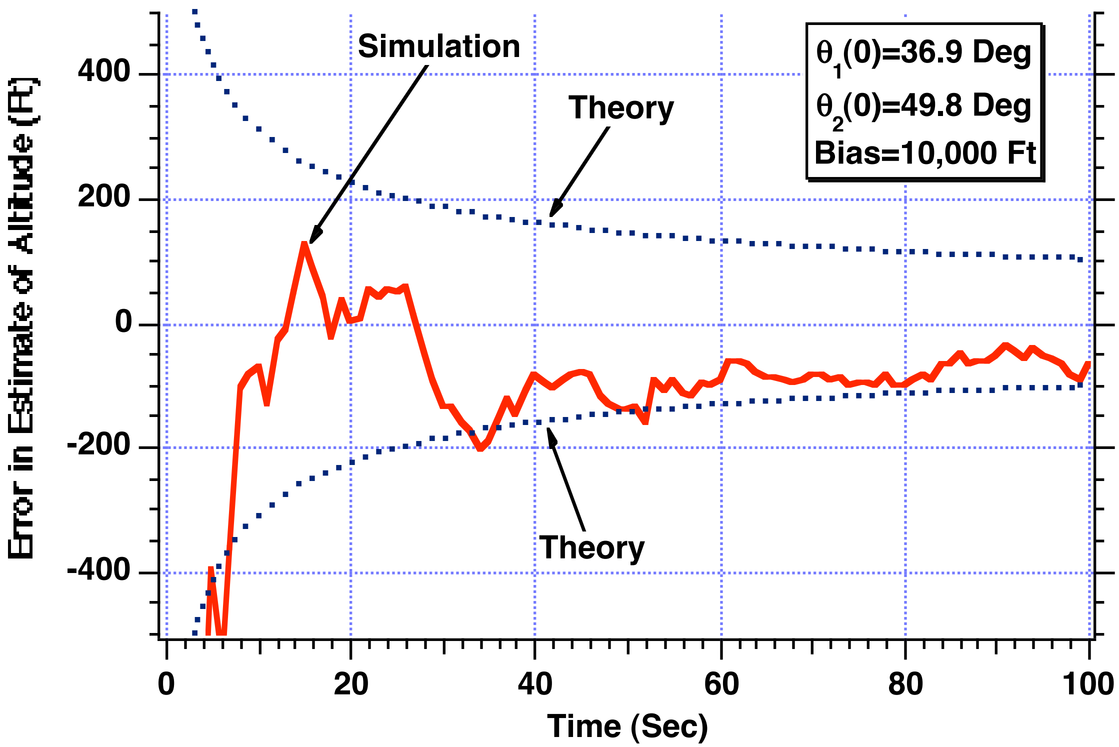
Three Satellite Range Measurements Enable New Extended Kalman Filter to Estimate Receiver Altitude



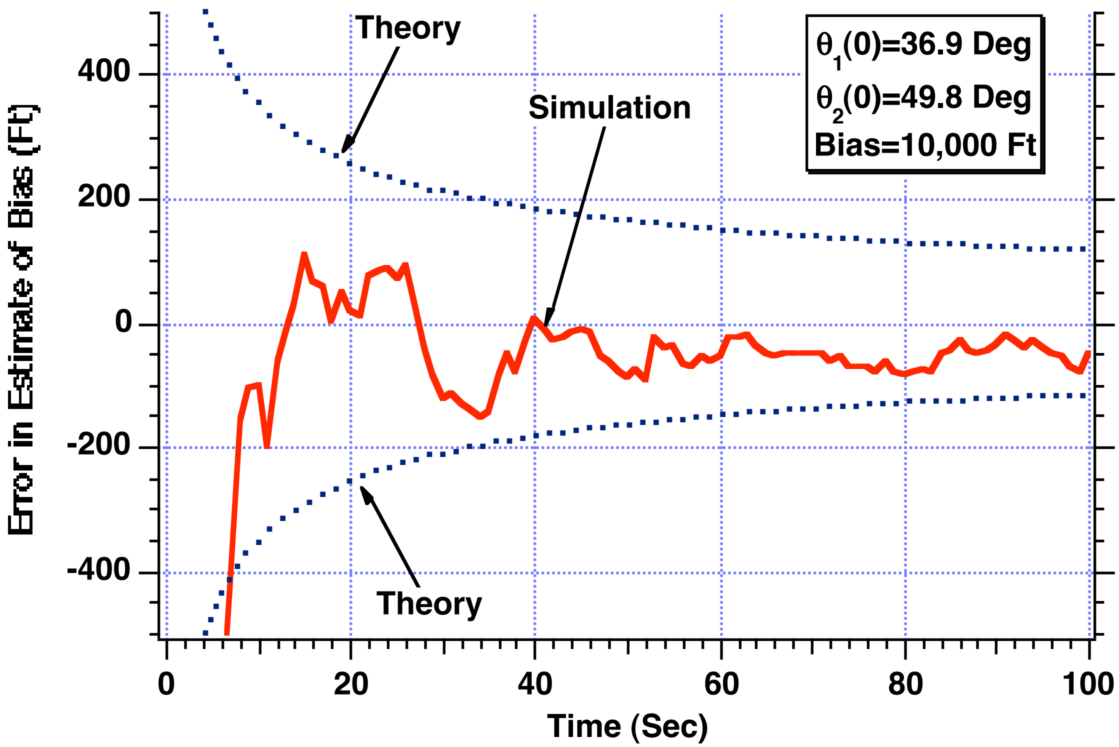
New Extended Kalman Filter Appears to be Yielding Correct Estimates of Receiver Downrange



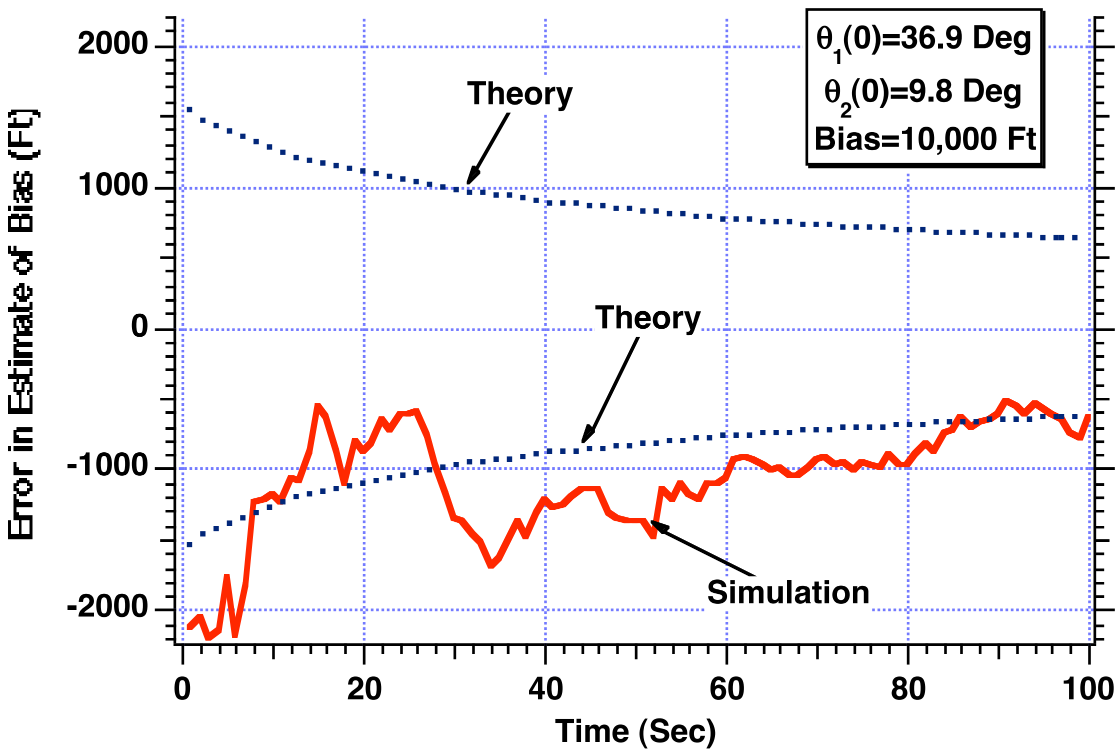
New Extended Kalman Filter Appears to be Yielding Correct Estimates of Receiver Altitude



New Extended Kalman Filter Appears to be Yielding Correct Estimates of Bias



Errors in the Estimate of Bias Get Larger as the Angle Between Second and Third Satellite Decreases



Biases
Summary

- If the satellite was causing the bias in the range measurement we could estimate the bias by taking range measurements from one satellite to a receiver whose location was known precisely

- If the receiver was causing the bias and the receiver location was unknown, three satellites were required in the two dimensional problem for an extended Kalman filter to estimate the bias and receiver location