Auxiliary Command Input Shaping Technique to Reduce Disturbance Induced Vibration

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May, 2017
Motivation

- Flexible structures would be easier to control when vibrational modes are attenuated. Thus, there is motivation to combine a pre-filter and feedback.

- **Input Shaping**: shape the input in such a way that the vibration modes are not excited

- Traditional **input shaping** posses a weak point: Inability to suppress vibrations caused by external disturbances. Only command-incurred vibrations are attenuated

- Thus, we will focus on:

**Applying input shaping approaches to flexible systems to reduce disturbance-incurred residual vibrations**
Problem Definition

Consider a crane conveying a load through a rest-to-rest manoeuvre. Once the manoeuvre is accomplished, the load is dropped to its final location. Dropping the load incurs a sudden change in the external force induced by the load on the crane. The sudden change in the external force may be seen as an external, impulse-like, disturbance.
The term **Input Shaping**\(^\circledast\) (introduced by Singer et al, 1990):

The operation of convolving a desired input command with an impulse sequence

1. Applying an impulse, \(A_1\), will cause system to vibrate
2. Applying second impulse, \(A_2\), at a later time cancels vibration
3. The second impulse must be applied at the correct time and must have the appropriate magnitude for complete cancelation
The amplitudes and time instances of the impulses in an input shaper are determined by solving a set of constraint equations. A variety of constraints are used:

- Robustness constraints (*Singer et al.* 1990)
- Impulse amplitude constraints (*Singhose et al.* 1994)
- Optimal time requirement (*Pao & Singhose*, 1995)
For example, the ZV shaper \((\textit{Singer et al, 1990})\), for 2\(^{nd}\) Ord. linear system, with damping ratio \(\zeta\), and damped natural frequency, \(\omega_d = \omega_n \sqrt{1 - \zeta^2}\):

\[
\begin{align*}
1 + K & \quad \text{at } t_1 = 0 \\
\frac{K}{1 + K} & \quad \text{at } t_2 = 0.5 T_d
\end{align*}
\]

\[
K = e^{\frac{-\zeta \pi}{\sqrt{1 - \zeta^2}}}
\]

\[
T_d = \frac{2\pi}{\omega_d} \quad \text{- Damped period, in [sec]}
\]
In the ZV-shaper, $t_2$ depends on $T_d$, and cannot be shortened arbitrarily. This is a major weakness of the ZV-shaper.

- A limited attempt to overcome this drawback was presented by Singhose et al. (1994), where negative impulses are allowed in the filter. Yet, impulse instances still depends on $T_d$.
- A different approach is taken by Magee & Book (1998) where the Optimal Arbitrary Time-delay Filter (OATF) is introduced:

\[
IS_{OATF}(t) = \sum_{j=1}^{3} A_j \delta(t - (j - 1) \cdot \Delta)
\]  

$\delta(t)$ - Dirac delta function  
$\Delta$ - Shaper time delay, arbitrarily chosen
Chang et al (2004) introduce the CIST - Commandless Input Shaping Technique:

- When a disturbance is induced, a set of two impulse is directly applied to the system via an actuator.
- An analytic solution for the shaper parameters for different actuator dynamics and 2 types of disturbances was proposed.
- Yet, dynamic systems can not be controlled by direct impulses, thus, CIST seems impractical!!.
Consider:

\[ \text{ACIST} \]

\[ \text{ACIST}_\text{cmd}(t) + IS \]

\[ d(t) \]

\[ y(t) \]

\[ y(t) - \text{Output} \]

\[ r_A(t) - \text{Auxiliary reference command} \]

\[ IS - \text{Impulse set which convolves } r_A(t) \]

The disturbance:

\[ d(t) = A_d \delta(t - t_d) \]  

(2)

We seek to bring the total response, \( y(t) \), to zero after some finite time \( t \geq t_n \) by a well designed ACIST.
Simple ACIST

Assume:

\[ IS(t) = A_1 \delta(t - t_1) + A_2 \delta(t - t_2) \]  
\[ r_A(t) = R_A \cdot 1(t_r) \]  

The output, \( y(t) \), is

\[ y(t) = R_A(A_1 + A_2) + R_A e^{-\zeta \omega_n t} \sqrt{C^2 + S^2} \sin(\omega_d t + \psi) \]  

where \( \psi \) is a phase shift and (after setting \( A_2 = -A_1 \))

\[ C = \frac{A_d \omega_n}{R} \frac{1}{\sqrt{1 - \zeta^2}} + \frac{A_1 \zeta}{2\zeta^2 - 1} - \frac{A_1 e^{\omega_n \zeta t_2}}{2\zeta^2 - 1} \left[ \cos\left(\frac{\pi}{2} - \omega_d t_2\right) + \frac{\zeta \cos(\omega_d t_2)}{\sqrt{1 - \zeta^2}} \right] \]

\[ S = \frac{A_1}{2\zeta^2 - 1} - \frac{A_1 e^{\omega_n \zeta t_2}}{2\zeta^2 - 1} \left[ \sin\left(\frac{\pi}{2} - \omega_d t_2\right) + \frac{\zeta \sin(\omega_d t_2)}{\sqrt{1 - \zeta^2}} \right] \]  

(5)
For zero vibration we set $C = 0$ and $S = 0$, which yields:

\[
\frac{A_d \omega_n (2\zeta^2 - 1)}{R A A_1 \sqrt{1 - \zeta^2}} + \frac{\zeta}{\sqrt{1 - \zeta^2}} = e^{\omega_n \zeta t} \left[ \sin(\omega_d t) + \frac{\zeta \cos(\omega_d t)}{\sqrt{1 - \zeta^2}} \right] \tag{6}
\]

\[
1 = e^{\omega_n \zeta t} \left[ \cos(\omega_d t) - \frac{\zeta \sin(\omega_d t)}{\sqrt{1 - \zeta^2}} \right] \tag{7}
\]

For $\zeta = 0$ we get

\[
t_2 = \frac{n\pi}{\omega_d}, \quad n = 0, 2, 4, \ldots, \infty \tag{8}
\]

\[
A_1 \to \infty \tag{9}
\]

For $\zeta \neq 0$ we solve numerically to get $t_2$, and

\[
A_1 = \frac{A_d \omega_n (3\zeta^2 - 2\zeta^4 - 1)e^{-\omega_n \zeta t}}{R \sqrt{1 - \zeta^2} \sin(\omega_d t)} \tag{10}
\]
The simple **ACIST** reduces residual vibration drastically for \( t > t_2 \). However, the magnitude of the transient response is much higher than the response without **ACIST** !!
The ACIST with OATF

Enhance the **ACIST** by using the 3-impulse OATF

\[ IS_{OATF}(t) = \sum_{j=1}^{3} A_j \delta(t-(j-1)\cdot\Delta) \]  

(11)

The total response is

\[ y(t) = R_A(A_1 + A_2 + A_3) + R_A e^{-\zeta \omega_n t} \sqrt{C^2 + S^2} \sin(\omega_d t + \psi) \]  

(12)

where

\[ C = \frac{A_d \omega_n}{R_A \sqrt{1-\zeta^2}} + \frac{A_1 \zeta}{\sqrt{1-\zeta^2}} + \frac{A_2 e^{\omega n \zeta \Delta}}{2 \zeta^2 - 1} \left[ \sin(\omega_d \Delta) + \frac{\zeta \cos(\omega_d \Delta)}{\sqrt{1-\zeta^2}} \right] + \]

\[ + \frac{A_3 e^{2 \omega n \zeta \Delta}}{2 \zeta^2 - 1} \left[ \sin(2\omega_d \Delta) + \frac{\zeta \cos(2\omega_d \Delta)}{\sqrt{1-\zeta^2}} \right] \]

\[ S = \frac{A_1}{2 \zeta^2 - 1} + \frac{A_2 e^{\omega n \zeta \Delta}}{2 \zeta^2 - 1} \left[ \cos(\omega_d \Delta) - \frac{\zeta \sin(\omega_d \Delta)}{\sqrt{1-\zeta^2}} \right] + \]

\[ + \frac{A_3 e^{2 \omega n \zeta \Delta}}{2 \zeta^2 - 1} \left[ \cos(2\omega_d \Delta) - \frac{\zeta \sin(2\omega_d \Delta)}{\sqrt{1-\zeta^2}} \right] \]  

(13)
For zero vibration we set

\[
0 = A_1 + A_2 + A_3
\]

\[
C = 0
\]

\[
S = 0
\]  

(14)

which yields:

\[
A_3 = -KK\zeta \frac{e^{-\omega n\zeta\Delta} - \cos(\omega_d \Delta) + \zeta \sin(\omega_d \Delta)}{2 \cos(\omega_d \Delta) - (e^{\omega n\zeta\Delta} + e^{-\omega n\zeta\Delta})}
\]  

(15)

\[
A_2 = -A_3 \frac{1 - e^{2\omega n\zeta\Delta}}{1 - e^{\omega n\zeta\Delta}} \frac{(\cos(2\omega_d \Delta) - \zeta \sin(2\omega_d \Delta))}{(\cos(\omega_d \Delta) - \zeta \sin(\omega_d \Delta))}
\]  

(16)

\[
A_1 = -A_2 - A_3
\]  

(17)

where: \(K = \frac{A_d \omega n}{2 R_A \sin(\omega n\Delta)}\), \(K_{\zeta} = \frac{2\zeta^2 - 1}{\sqrt{1-\zeta^2}}\), \(\zeta = \frac{\zeta}{\sqrt{1-\zeta^2}}\)
ACIST via Optimization

Same results can be obtained by the following: Consider the dynamic state-space equations:

\[
\dot{x}(t) = \begin{bmatrix} 0 & 1 \\ -\omega_n^2 & -2\zeta\omega_n \end{bmatrix} x(t) + \begin{bmatrix} 0 \\ \omega_n^2 \end{bmatrix} u(t) + \begin{bmatrix} 1 & 0 \end{bmatrix} x(t)
\]

\[y(t) = \begin{bmatrix} 1 & 0 \end{bmatrix} x(t)\]

(18)

The solution \(x(t)\) is

\[
x(t) = \phi(t, 0)x(0) + R_A \int_0^t \phi(t, \tau) B \sum_{j=1}^{3} A_j \delta(t - (j - 1)\Delta) d\tau
\]

\[\phi(t, \tau) = e^{\Gamma(t - \tau)} - \text{state transition matrix from } \tau \text{ to } t.\]
Solving the integral in (19) for $t \geq 2\Delta$ gives

$$x(t) = \phi(t, 0)x(0) + \Psi(t)f_A \quad \forall t \geq 2\Delta \quad (20)$$

where

$$\Psi(t) = \left[ \Phi[2\Delta](t) - \Phi[0](t), \Phi[2\Delta](t) - \Phi[\Delta](t) \right] \quad (21)$$

$$f_A = R_A[A_1, A_2]^T \quad (22)$$

and

$$\Phi[t_k](t) = e^{-\zeta \omega_d (t-t_k)} \begin{bmatrix} \dot{\zeta} \sin(\omega_d (t-t_k)) + \cos(\omega_d (t-t_k)) \\ -\frac{\omega_d}{1-\zeta^2} \sin(\omega_d (t-t_k)) \end{bmatrix}, \quad t_k = 0, \Delta, 2\Delta \quad (23)$$
For zero residual vibration at $t \geq 2\Delta$ we set $x(2\Delta) = [0, 0]^T$. Minimize

$$J = \frac{1}{2}x(2\Delta)^T W x(2\Delta)$$

with $W$ some weighting matrix. Substituting (20) into (24) and solving for $t = 2\Delta$

$$J = \frac{1}{2} (\phi(2\Delta,0)x(0))^T W \phi(2\Delta,0)x(0) +$$

$$+ \frac{1}{2} (\Psi(2\Delta)f_A)^T W \Psi(2\Delta)f_A + (\phi(2\Delta,0)x(0))^T W \Psi(2\Delta)f_A$$

Differentiating (25) with respect to $f_A$ and equating to zero:

$$f_A = - [\Psi(2\Delta)^T W \Psi(2\Delta)]^{-1} [(\phi(2\Delta,0)x(0))^T W \Psi(2\Delta)]$$

A direct solution for (26) with $W = I$ will give the same results as in (15) and (16). Furthermore, it can be shown that the second optimality condition $\frac{\partial^2 J}{\partial f_A^2}$ is fulfilled and $\Psi(2\Delta)^T W \Psi(2\Delta)$ is invertible when $\sin(\omega_n\Delta) \neq 0$. 
Test Results

To validate the results we used a simple test-bed
Test Results - Cont.

Test results of the **ACIST** with **OATF** shaper

![ACIST with OATF Test Results](image)

**Tip Pos [Pixel]**

**ACIST** Cmd [V]

**Time [s]**

18 / 19
Conclusions

- A technique to reduce the residual vibration of a flexible system was presented.
- The shaper parameters are obtained via algebraic solution based on the disturbance and system parameters.
- It was shown that the shaper is optimal in terms of residual vibration.
- Hardware experiments verified the technique performance.