Set-point Regulation of an Uncertain 6-DOF Magnetically Levitated Positioning Stage

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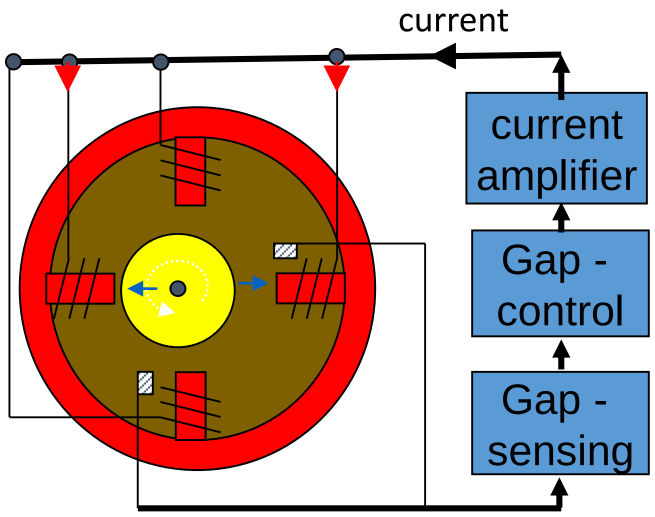
Outline

- Background – some of our activities in the field of AMB
- Six-DOF Precision Positioning Stage (mechanical structure and dynamical model)
- Iterative Output Control Law (theoretical)
- Iterative Output Control Law (practical)
- Experimental Results
- Summary and Conclusions

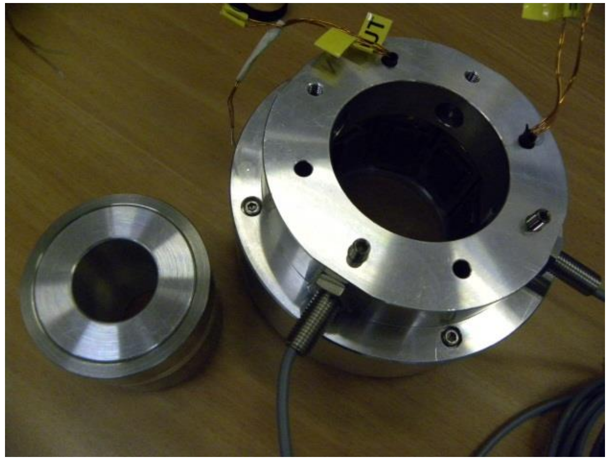
Background – some of our activities in the field of AMB

- Active Magnetic Bearings (AMB) allows rotation with no friction.
- It uses electromagnetic forces to prevent mechanical contact between the static (stator) and the moving (rotor) parts.
- Applications of AMBs include very high rotating systems, such as **turbo-molecular pumps** and **Flywheel Energy Storage Systems**.

The very basic principle of a radial AMB

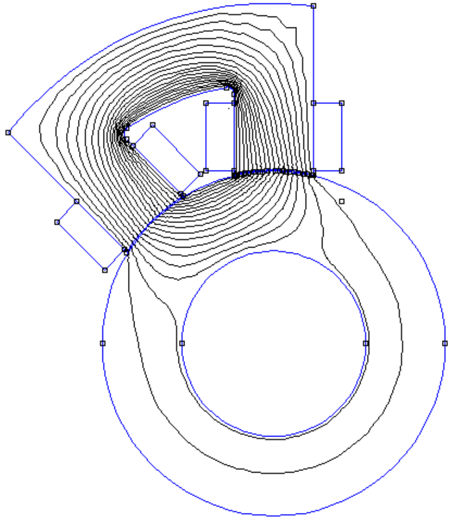
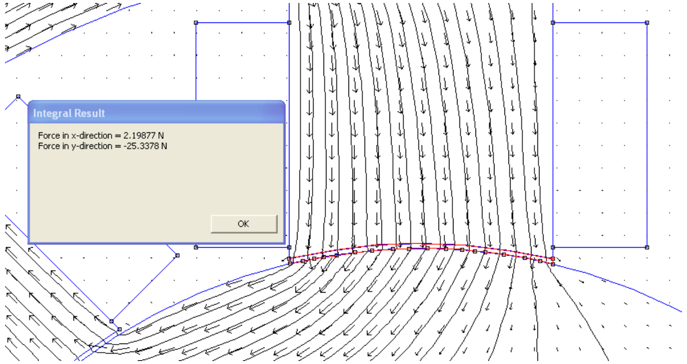


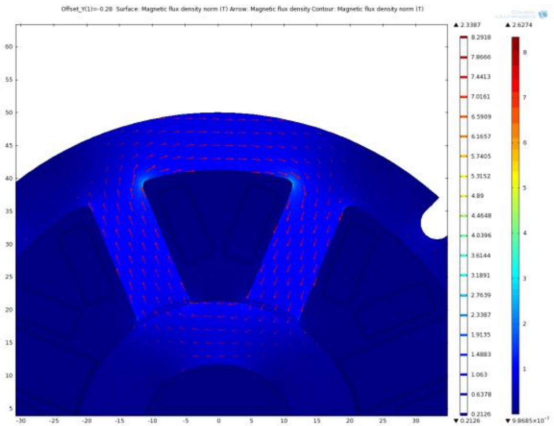
One of the AMBs we have developed at our Lab



Background – some of our activities in the field of AMB

The design includes magnetic and mechanical analysis (using finite elements software)

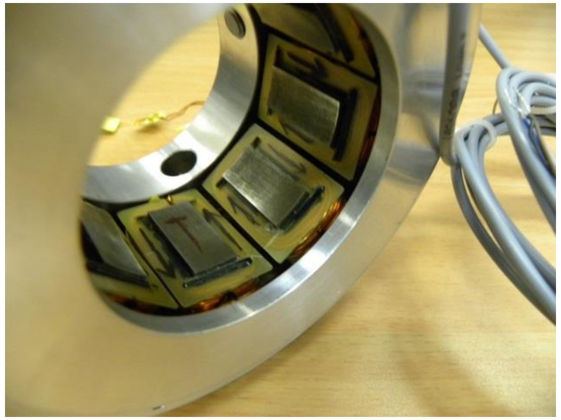
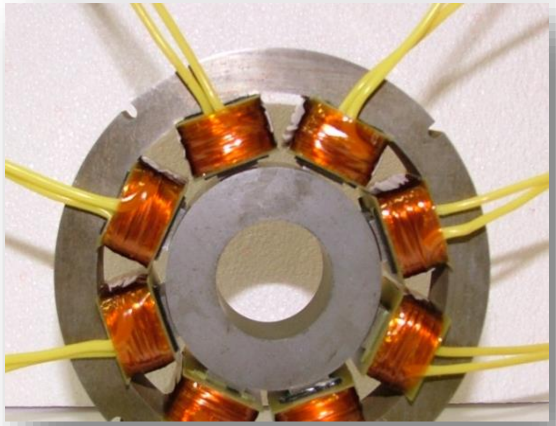






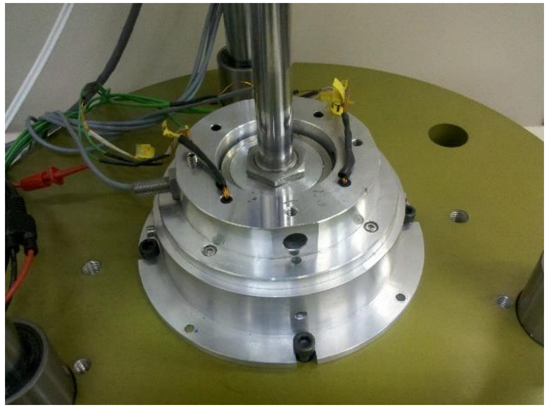
<u>Our interests includes</u>:

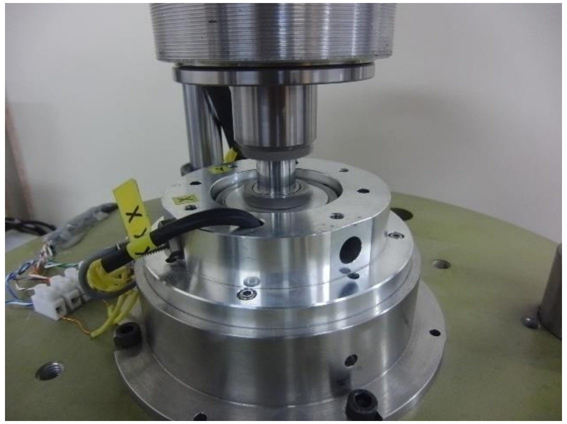
- Optimal design for minimum losses
- Adaptive control (unknown imbalance)
- AMB control, the case of elastic shaft.

Our AMBs are produced (**in-house**) from raw materials
(e.g., of electrical transformers)

Background – some of our activities in the field of AMB

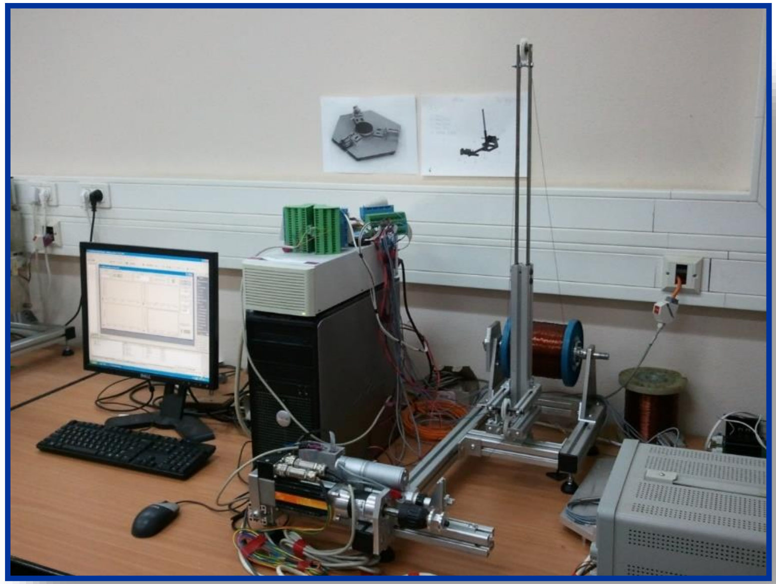




A 5 DOF AMB system at our Lab, it includes:

- Two radial AMBs
- One axial AMB (works against gravity)
- High speed brushless motor (up to 60,000 RPM)

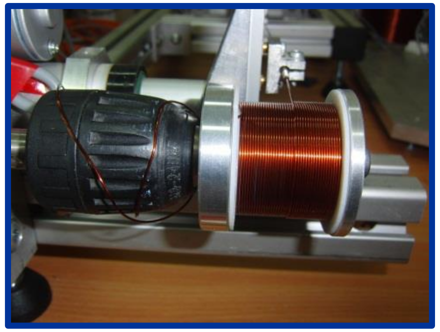
Background – some of our activities in the field of AMB

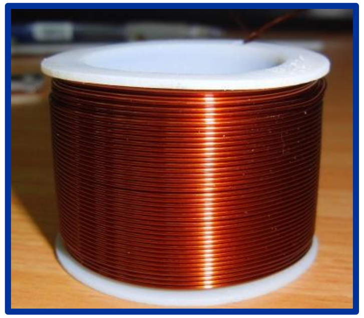


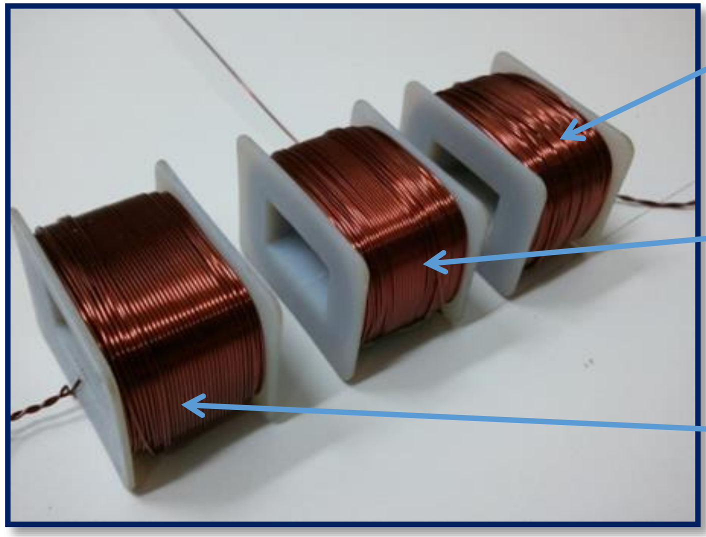
Many in-house skills have been acquired

Besides AMBs design and control we have developed a winding machine at our Lab.

- The wire tension is closed-loop controlled.
- Very slow winding allows maximum number of windings in a given volume.
- The bobbins (spools) are 3-D printed.

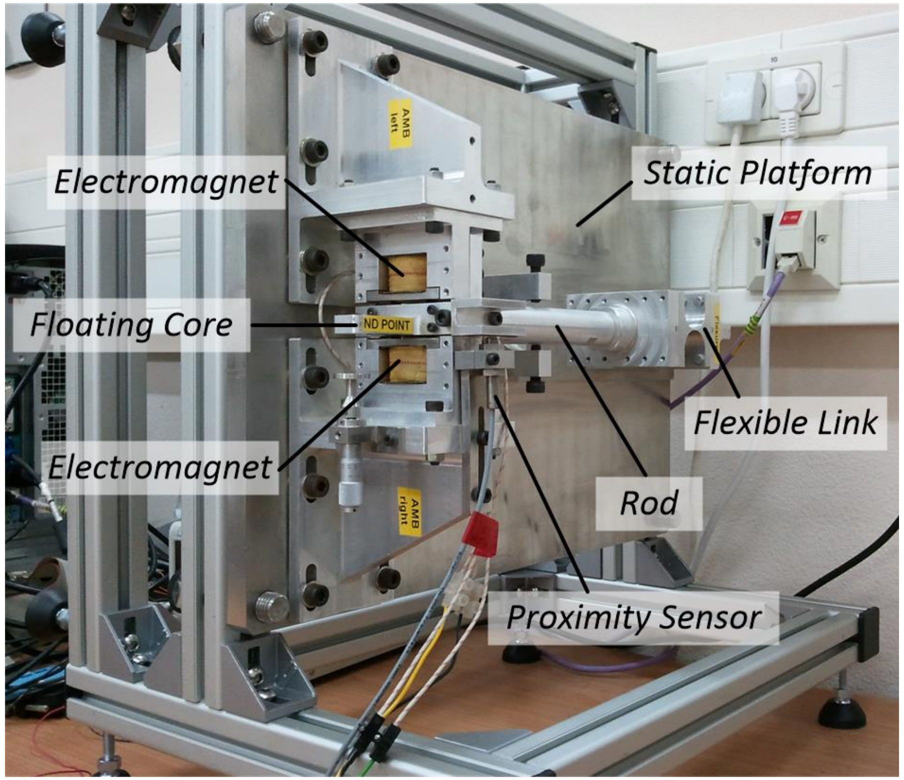






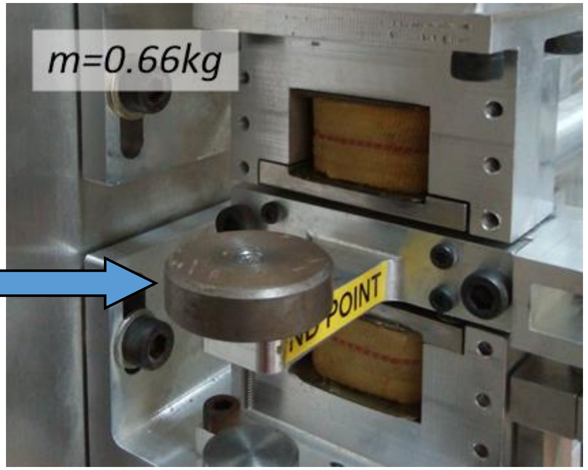
Background – some of our activities in the field of AMB

A single degree of freedom “AMB” (imbalance effects can be added by a small rotating eccentric mass)



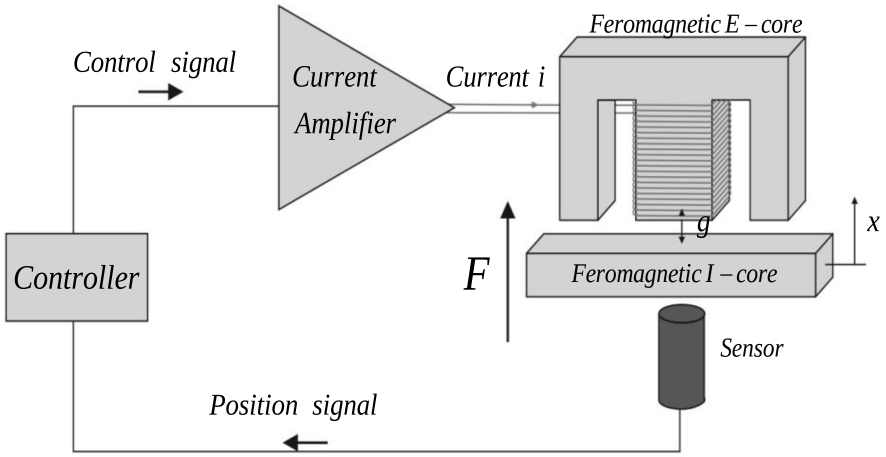
This system can be placed vertically or horizontally (depending on the gravity effect we want to achieve)

Uncertain load



Background – some of our activities in the field of AMB

The simplest model of the electromagnetic force (commonly utilized for control design)



$$f = c\frac{i^2}{g^2}$$

Systems consist of these actuators are **unstable**

A single DOF electro-magnetic actuator includes two E cores and a single I core.
Force can be applied in both directions (usually by linearizing the system around a bias current)

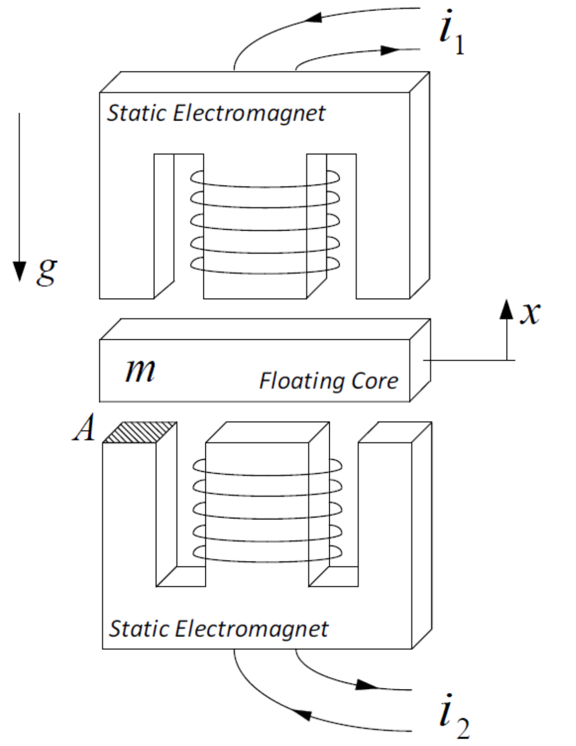
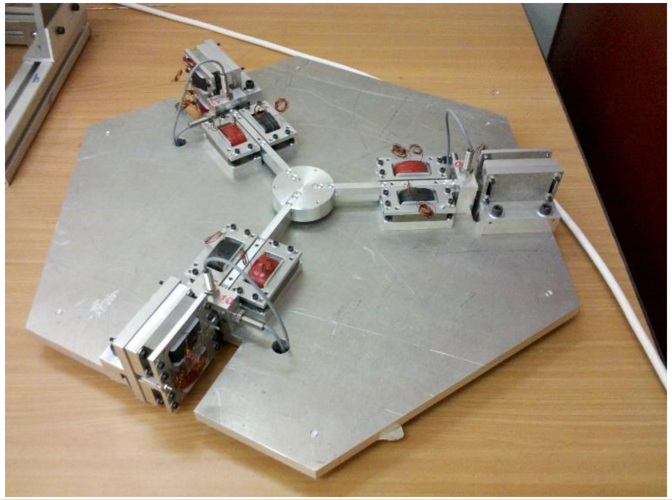


Fig. 1: SDOF AMB positioning system scheme

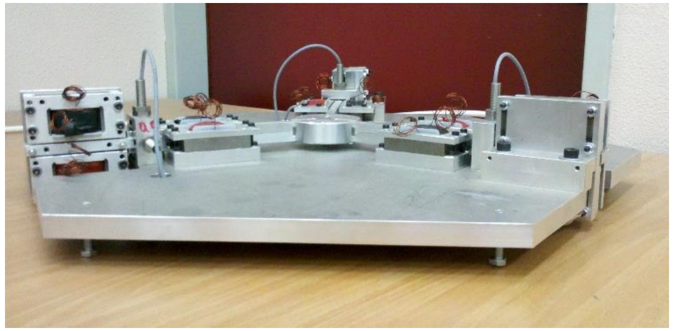
Background – some of our activities in the field of AMB

Two talented **final year students**:
Netanel Deri, Liron Edry

One very talented **MSc student**:
Sergei Basovich



6 DOF magnetically levitated positioning stage



All of this brings us to the subject of this talk:

Set-point Regulation of an Uncertain 6-DOF Magnetically Levitated Positioning Stage

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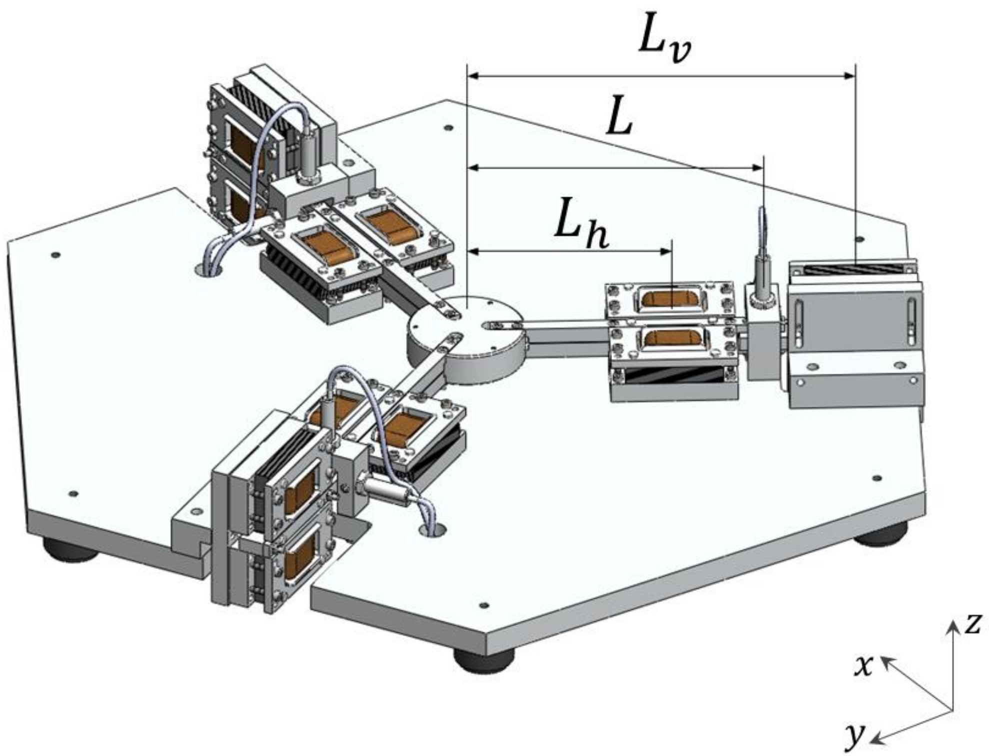
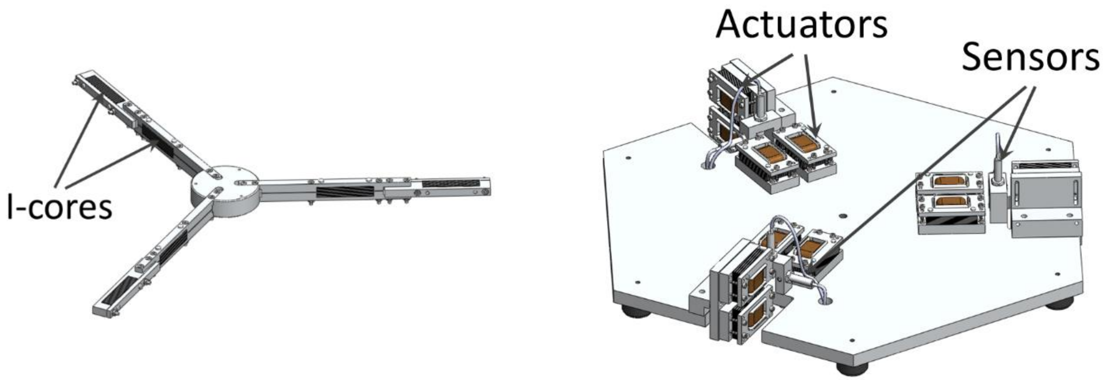
So, I could have started here, but . . .

Stage Structure

The stage consists of six electromagnetic actuators (Three are vertical and the other are horizontal)

The levitated-part consists of three arms connected in a joint, where each arm serves as a support for two I-cores.





The air gaps in all six actuators are measured by six proximity (eddy current) sensors to obtain information about the stage position and orientation.

The traveling range is $\pm 450\times 10^{-6}[m]$ $\pm 1500\times 10^{-6}[rad]$

Stage Structure

The mechanical data of the stage is . . .
(products of inertia are negligible)

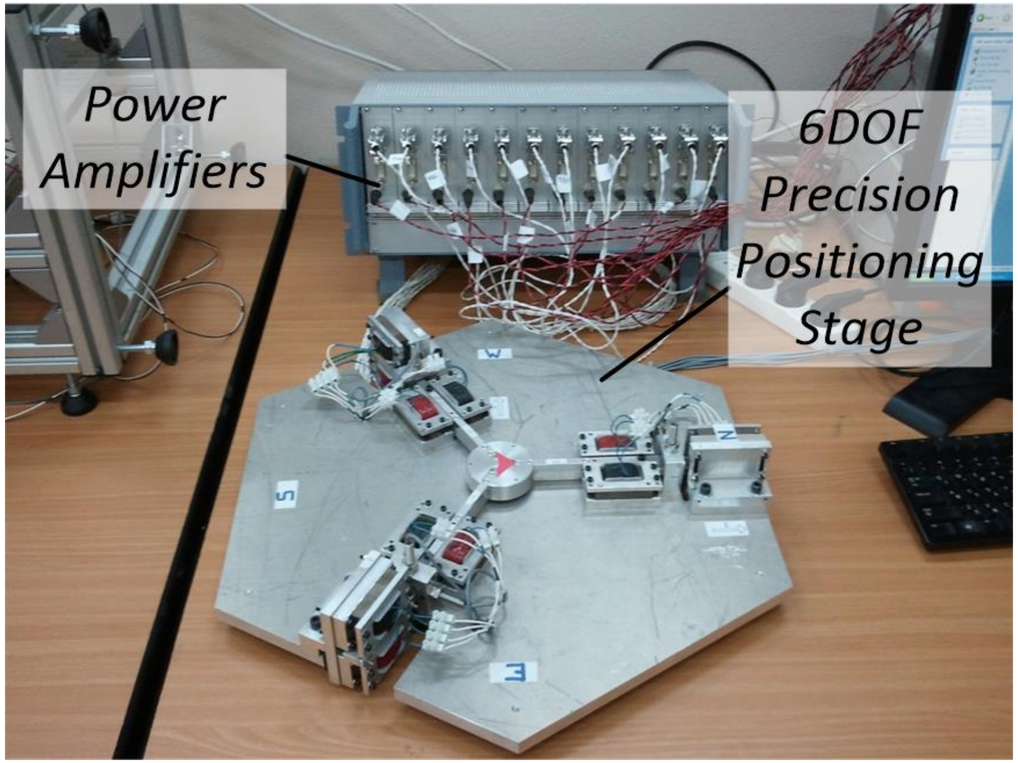


TABLE I: Stage Parameters

I[kgm^2] - inertia of the platen		
I_{xx}	I_{yy}	I_{zz}
5.4336e-3	5.4336e-3	1.0844e-2
I_{xy}	I_{yz}	I_{zx}
2.5717e-17	9.1320e-22	1.1316e-21
Platen nominal mass	*Electromagnetic coef.*	*Nominal air gap*
$m[kg]$	$c[Nm^2/A^2]$	$l_0[m]$
0.648	1.0809e-5	450e-6
Relevant dimensions [m]		
L_h	L	L_v
0.113	0.163	0.213

Stage Equations of Motion

The stage (levitated part) is modeled as a **rigid body with 6 DOF**

$$\boxed{M\ddot{q} + C(\dot{q})\dot{q} + w + \zeta = \Phi_{LF} \cdot f} \quad , \qquad q = [x, y, z, \varphi, \theta, \psi]^T$$

(**<u>small angles and small displacements are assumed</u>**)

The inertia matrix is $M = diag\left\{m_t, m_t, m_t, I_{xx}, I_{yy}, I_{zz}\right\}$

The actuator forces $f = [f_1, f_2, f_3, f_4, f_5, f_6]^T$

Assumed unknown

The gravity force $w = [0, 0, m_t g, 0, 0, 0]^T$

Torque due to a shifted c.g. (because of the payload) $\zeta = [0_{1\times3}, -\Delta\tau^T]^T$

Stage Equations of Motion

Transformation from actuator forces to body forces

$$\Phi_{LF} = \begin{bmatrix} Y & P \\ L_h P & -L_v Y \end{bmatrix}$$

where

$$Y = \begin{bmatrix} \sqrt{3}/2 & 0 & -\sqrt{3}/2 \\ -1/2 & 1 & -1/2 \\ 0 & 0 & 0 \end{bmatrix} \quad , \quad P = \begin{bmatrix} 0 & 0 & 0 \\ 0 & 0 & 0 \\ 1 & 1 & 1 \end{bmatrix}$$

Transformation from body coordinates to actuator coordinates

$$l = \Phi_{LQ}\, q$$

where

$$\Phi_{LQ} = \begin{bmatrix} Y^T & L_h P^T \\ P^T & -L_v Y^T \end{bmatrix} = \Phi_{LF}^T$$

Transformation from sensor coordinates to body coordinates

$$q = \Phi_{QS}\, s$$

where

$$\Phi_{QS}^{-1} = \begin{bmatrix} -Y^T & -LP^T \\ -P & LY^T \end{bmatrix}$$

Stage Equations of Motion

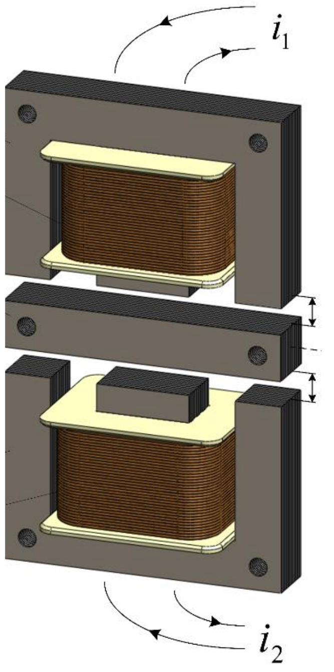
The actuator forces $$f_k = c\left(\frac{i_{k1}^2}{\left(l_0 + l_f - l_k(q)\right)^2} - \frac{i_{k2}^2}{\left(l_0 + l_f + l_k(q)\right)^2}\right) \quad , \; k = 1,2,3,4,5,6$$

l_f represents an additional length due to final permeability

Then, control currents (i_{k1} and i_{k2}) are applied based on:

$$i_{k1} = \begin{cases} \left(l_0 + l_f - l_k(q)\right)\sqrt{f_k / c}, & f_k > 0 \\ 0 & otherwise \end{cases}$$

$$i_{k2} = \begin{cases} \left(l_0 + l_f + l_k(q)\right)\sqrt{-f_k / c}, & f_k < 0 \\ 0 & otherwise \end{cases} \quad , \; k = 1,2,3,4,5,6$$



$l_0 = 450 \times 10^{-6} [m]$

$l_f = 1.8634 \times 10^{-6} [m]$

Stage Equations of Motion

The matrix $C(\dot{q})$ of the term $C(\dot{q})\dot{q}$ is given as,

$$C = \begin{bmatrix} 0_{3\times3} & 0_{3\times3} \\ 0_{3\times3} & C_0 \end{bmatrix}$$

Where,

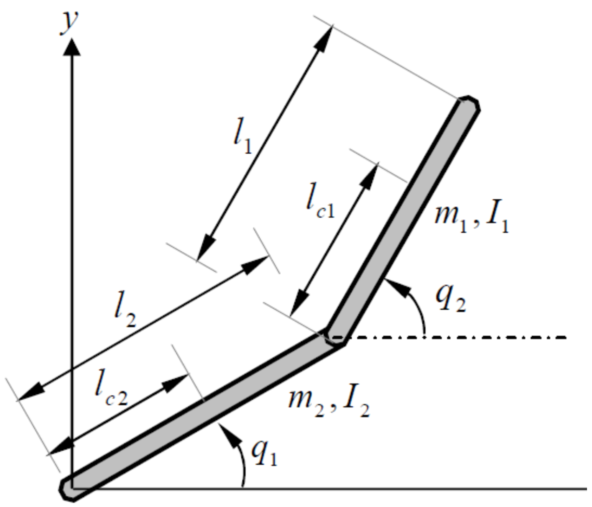
$$C_0 = \begin{bmatrix} 0 & I_{zz}\dot{\psi} & -I_{yy}\dot{\theta} \\ -I_{zz}\dot{\psi} & 0 & I_{xx}\dot{\varphi} \\ I_{yy}\dot{\theta} & -I_{xx}\dot{\varphi} & 0 \end{bmatrix}$$

It is important to note that the matrix $\dot{M} - 2C(\dot{q})$ is a skew symmetric matrix

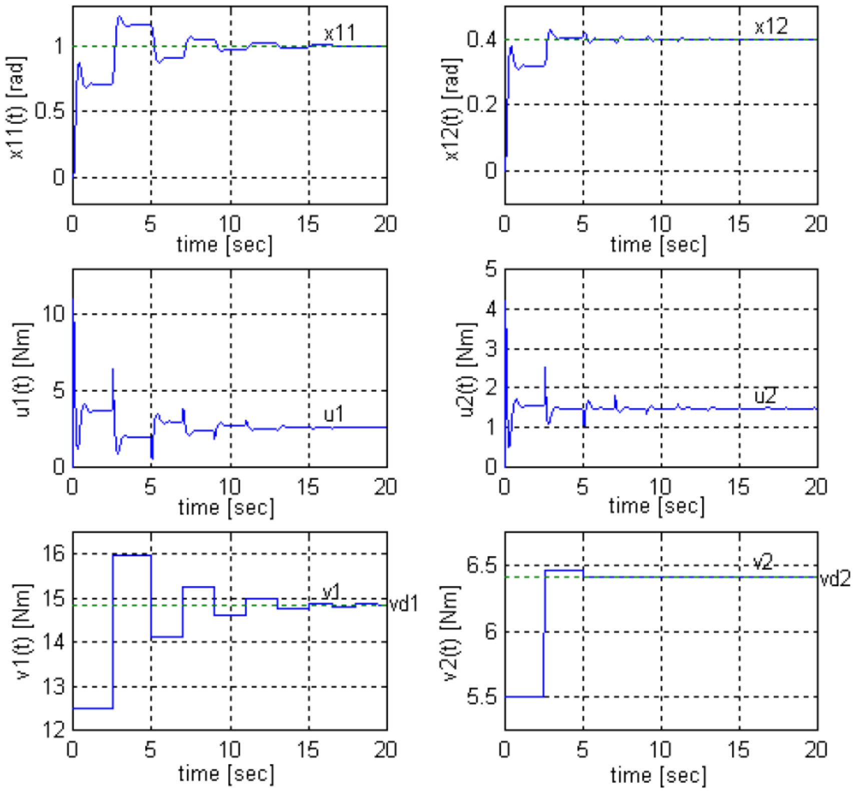
Iterative Output Control Law

Some **old** results from **robotics**

A. Ailon, “**Output controller based on iterative schemes for set-point regulation of uncertain flexible-joint robot models**,” *Automatica*, vol. 32, no. 10, pp. 1455-1461, 1996.



1998



$$D(q)\ddot{q} + C(q,\dot{q})\dot{q} + g(q) = u$$

The matrix $\dot{D}(q) - 2C(q,\dot{q})$ is *skew symmetric*

Iterative Output Control Law (Six-DOF Positioning Stage)

We define the uncertain term as $p \triangleq w + \zeta$

The state space representation,

$$\dot{x}_1 = x_2$$

$$\dot{x}_2 = M^{-1}(-C(x_2)x_2 - p + \Phi_{LF} \cdot f)$$

where, $x_1 = q, \quad x_2 = \dot{q}$

For the 6DOF stage,
we propose the following
iterative controller-observer

$$f = \Phi_{LF}^{-1}\left(-C_1(x_1 - x_1^d) - C_2\dot{z} + v\right)$$

$$\dot{z} = -K(z - x_1)$$

C_1, C_2, K positive definite & diagonal

Iterative Output Control Law (theoretical)

The compensating piecewise function v (constant at each iteration) is defined by the following update law

$$\boxed{v^{n+1} = v^n - S\left(x_1^n - x_1^d\right)}$$

Where, $S \triangleq C_1\left(I - \alpha I\right), \quad 0 < \alpha < \frac{1}{2}$

This can be schematically represented by the following process

$$\boxed{v^0 \overset{motion}{\Rightarrow} x_1^0 \overset{comp.}{\Rightarrow} v^1 \overset{motion}{\Rightarrow} x_1^1 \quad \ldots \quad v^n \overset{motion}{\Rightarrow} x_1^n \overset{comp.}{\Rightarrow} v^* \overset{motion}{\Rightarrow} x_1^d}$$

Iterative Output Control Law (theoretical)

Lemma 1 Let the system with the uncertain term $p \in \mathcal{R}^6$ be controlled by the controller-observer with an arbitrary $v \in \mathcal{R}^6$. Then, the equilibrium point $x_1 = \bar{x}_1$ of the closed loop system is asymptotically stable.

Proof Define a scalar function $H(x_1, x_2, z)$ as,

$$H(x_1, x_2, z) = \frac{1}{2}[x_2^T M x_2 + (x_1 - z)^T C_2 K (x_1 - z) + (x_1 - x_1^d - C_1^{-1} v)^T C_1 (x_1 - x_1^d - C_1^{-1} v)] + U_p(x_1)$$

where $U_p(x_1)$ satisfies $dU_p(x_1)/d x_1 = p$.

Evaluating $\dfrac{dH(x_1, 0, z)}{d x_r} = 0$ where $x_r = [x_1^T, z^T]^T$, yields the steady state equations of the closed loop.

$$\boxed{\begin{aligned} & p + C_1(x_1 - x_1^d) + C_2 K (x_1 - z) - v = 0 \\ & -C_2 K (x_1 - z) = 0 \end{aligned}}$$

Iterative Output Control Law (theoretical)

Proof (cont.)

Evaluating the Hessian ($d^2H(x_1,0,z)/d^2x_r$) we obtain, $\begin{bmatrix} C_1+C_2K & -C_2K \\ -C_2K & C_2K \end{bmatrix}$

which can be shown to be positive definite for $C_1, C_2, K > 0$.

Therefore the scalar function $H(x_1,x_2,z)$ is a convex function and it has a global minimum at $\bar{x}_r = [\bar{x}_1^T, \bar{z}^T]^T$ for a given constant vector v

Thus, a Lyapunov candidate function can be defined as $V = H(x_1,x_2,z) - H(\bar{x}_1,0,\bar{z})$

and its time derivative is $\dot{V} = -C_2K\dot{z}^2 \leq 0$

Hence, invoking the LaSalle's invariance principle, asymptotic stability of the equilibrium point $\bar{x}_r = [\bar{x}_1^T, \bar{z}^T]^T$ is concluded.

$\square$

Iterative Output Control Law (theoretical)

Lemma 2 Consider the stage model and define the map $T(v):\mathcal{R}^6 \to \mathcal{R}^6$ as,

$$T(v) = v - S\left(x_1 - x_1^d\right)$$

Then, the map $T(v)$ is a global contraction, i.e., there exists exactly one v^* such that.

$$T(v^*) = v^*$$

Proof For a given couple of vectors v^1 and v^2 (of the series $\{v^n\}$ generated by $T(v)$)

$$T(v^1) - T(v^2) = v^1 - v^2 - C_1(I - \alpha I)(x_1^1 - x_1^2)$$

From the equilibrium equations, it follows that $x_1^1 - x_1^2 = C_1^{-1}(v^1 - v^2)$

Since C_1 is diagonal, we have (from the last two equations) $\left\|T(v^1) - T(v^2)\right\| = \alpha\left\|v^1 - v^2\right\|$

Hence, the map $T(v)$ is contraction, with $T(v^*) = v^*$, and $\left\|v^* - v^n\right\| \leq \dfrac{\alpha^n}{1-\alpha}\left\|T(v^0) - v^0\right\|$ $\square$

Iterative Output Control Law (practical)

- Since the convergence of the closed-loop system to the desired set-point x_1^d involves an infinite time process, this algorithm is **impractical**.

- For a real control task, the controller should be used with **a decision module** that (within a single iteration) **concludes convergence** to a sufficiently close vicinity of the intermediate equilibrium point.

- As a result, due to the **differences between the theoretical and practical** intermediate equilibrium points, the last term of the practical process will slightly **deviate from the desired set point** x_1^d.

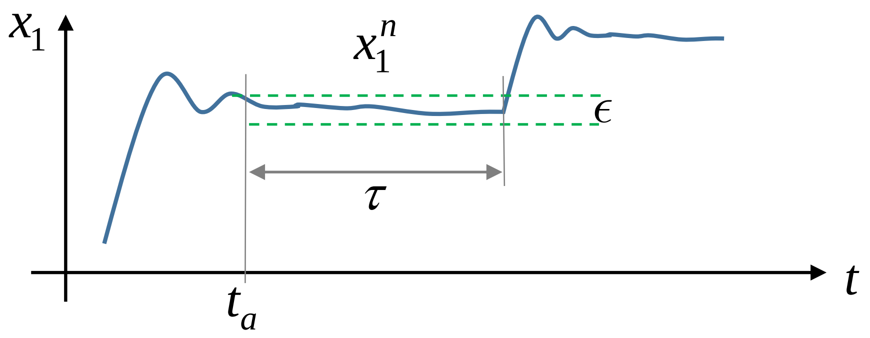
- How close to x_1^d can we get ? Lets put this in a mathematical framework

Iterative Output Control Law (practical)

Assumption

For a particular control task, positive constants ϵ and τ can be selected in such a way that from satisfaction of

$$\| x_1^n(t) - x_1^n(t_a) \| \le \epsilon, \quad \forall t \in [t_a, t_a + \tau], t_a \ge 0$$



during the n-th iteration, it can be concluded that

$$\| \varphi_v(t) - \bar{\varphi}_v(v) \| \le \zeta(\epsilon), \quad \forall t \in [t_a, \infty)$$

where $\varphi_v(t) = [x_1(t)^T, x_2(t)^T, z(t)^T]^T$ is the system trajectory,

$\bar{\varphi}_v(v) = [x_1^{nT}, 0, z_1^{nT}]^T$ is the n-th equilibrium point,

and $\zeta(\epsilon)$ is a constant (corresponding to a chosen ϵ)

Iterative Output Control Law (practical)

Practically, we are not using the map $T(v) = v - S\left(x_1 - x_1^d\right)$

hence, we define the practical map $E\left(v\right): \mathcal{R}^6 \to \mathcal{R}^6$

$$E(v) = v - S\left(x_1 + \Delta(v) - x_1^d\right)$$

where the error term $\Delta(v)$ satisfies

$$\left\|\Delta(v)\right\| \le \epsilon, \forall v \in \mathcal{R}^6$$

and we use following lemma.

Iterative Output Control Law (practical)

Lemma 3

Consider the map $E(v)$. Then, for any pair of vectors $\{v^1, v^2\}$ satisfying

$$\left\| v^1 - v^2 \right\| \geq \theta \triangleq \frac{2\lambda_{max}(S)\epsilon}{\alpha}$$

the following holds

$$\left\| E\left(v^1\right) - E\left(v^2\right) \right\| \leq 2\alpha \left\| v^1 - v^2 \right\|$$

Proof

Expanding $E\left(v^1\right) - E\left(v^2\right)$ we obtain

$$E\left(v^1\right) - E\left(v^2\right) = T\left(v^1\right) - T\left(v^2\right) + S\Delta\left(v^2\right) - S\Delta\left(v^1\right)$$

and $$\left\| E\left(v^1\right) - E\left(v^2\right) \right\| = \left\| T\left(v^1\right) - T\left(v^2\right) + S\Delta\left(v^2\right) - S\Delta\left(v^1\right) \right\|$$

Iterative Output Control Law (practical)

Proof (cont.)

For the right-hand side of the last equation, by the triangle inequality, we have

$$\left\| T\left(v^1\right) - T\left(v^2\right) + S\Delta\left(v^2\right) - S\Delta\left(v^1\right) \right\| \le \left\| T\left(v^1\right) - T\left(v^2\right) \right\| + \left\| S\Delta\left(v^2\right) - S\Delta\left(v^1\right) \right\|$$

For the right-hand side of the last equation,

$$\left\| T\left(v^1\right) - T\left(v^2\right) \right\| + \left\| S\Delta\left(v^2\right) - S\Delta\left(v^1\right) \right\| \le \alpha \left\| v^1 - v^2 \right\| + 2\lambda_{max}(S)\epsilon$$

For the right-hand side of the last equation, using the Lemma condition,

$$\alpha \left\| v^1 - v^2 \right\| + 2\lambda_{max}(S)\epsilon \le 2\alpha \left\| v^1 - v^2 \right\|$$

$\square$

Iterative Output Control Law (practical)

So, as long as $\left\| v^1 - v^2 \right\| \geq \theta \triangleq \dfrac{2\lambda_{max}(S)\epsilon}{\alpha}$

We have, $\left\| E\left(v^1\right) - E\left(v^2\right) \right\| \leq 2\alpha \left\| v^1 - v^2 \right\| \quad , \quad 0 < \alpha < \dfrac{1}{2}$

and the (practical) map $E(v)$ can be considered contraction.

Now suppose that for the sequence $\{v^n\}$ generated by $v^{n+1} = E(v^n), \quad n = 0,1\ldots$

there exists a minimal integer $m(v^0)$ for which $\left\| v^{m(v^0)-1} - v^{m(v^0)} \right\| < \theta$

The $m(v^0)$-th iteration is the final iteration.

Iterative Output Control Law (practical)

It is very important to be able to estimate the deviation from x_1^d after the final iteration

For that, we have **Lemma 4** and **Lemma 5** (in our paper), which are not presented here.

The final conclusion from these Lemmas is that,

$$\| x_1^d - x_1^{m(v^0)} \| \leq 3 \frac{\lambda_{max}(C_1)}{\lambda_{min}(C_1)} \epsilon$$

The upper bound of the steady state error norm (after the last iteration) **can be made as small as desired.**

Iterative Output Control Law (practical)

- The **traveling range** of the proposed positioning stage **is relatively** restricted.
- If not all the terms of the practical intermediate equilibrium point sequence are found inside the operational area, the **steady state equations are no longer valid**.
- Hence, another important practical aspect is the **boundedness** of the intermediate steady state response.
- To provide that, we introduce the **initialization phase** which augments the iterative process.

Iterative Output Control Law (practical)

The augmented iterative process is represented as,

$$\xrightarrow{initial.} \chi_1^i, v^0 \xrightarrow{motion} \chi_1^0 \xrightarrow{comp.} v^1 \xrightarrow{motion} \cdots$$

$$\cdots \chi_1^{m(v^0)-1} \xrightarrow{comp.} v^{m(v^0)} \xrightarrow{motion} \chi_1^{m(v^0)}$$

where, $\chi_1^n \triangleq x_1^n + \Delta(v^n)$ is the n-th term of the series of ***practical equilibrium points*** generated during the practical process.

Iterative Output Control Law (practical)

- The initialization phase represents the response of the system with $x_1^d = 0$ and $v = 0$.

- As a result of the initialization, the system will move to χ_1^i .

- The initialization phase assures that the update mechanism starts acting when χ_1^i is found inside the traveling range of the stage.

- The initial input v^0 is determined by $v^0 = -S\left(\chi_1^i - x_1^d\right)$

  and for the rest of the process we use $v^{n+1} = v^n - S\left(\chi_1^n - x_1^d\right)$

Iterative Output Control Law (practical)

Lemma 6 Consider the system with $x_1^d = 0$ and $v = 0$, and let δ^i be the air-gap vector corresponding to the practical equilibrium point χ_1^i . Then, for C_1 satisfying

$$\left\|\Phi_{LQ} C_1^{-1}\right\|_2 \cdot \beta < l_0 - \left\|\Phi_{LQ}\right\|_2 \cdot \epsilon \qquad (*)$$

where the scalar l_0 represents the nominal air gap value in each actuator and β is the upper bound of p, the following holds,

$$\left\|\delta^i\right\| < l_0$$

Lemma 7 Consider the system with the practical update law. Let δ^n and l^d be the air-gap vectors, corresponding to χ_1^n and x_1^d respectively. Then, for C_1 satisfying $(*)$ and for,

$$v^0 = -S(\chi_1^i + x_1^d)$$

the following holds,

$$|\delta_k^n| < l_0, \quad k = 1,2,3,4,5,6$$

Iterative Output Control Law (practical)

Assumptions required for the proofs of **Lemma 6** and **Lemma 7** (can be found in our paper),

- For the uncertain term p, a positive constant $\beta \triangleq \sup\|p\|$ exists and it is known.
- The constants ϵ and α can be selected such that $\epsilon < \frac{1-2\alpha}{2\cdot\|\Phi_{LQ}\|_2} l^0$
- All the component of the air gap vector l^d, corresponding to the desired set-point vector x_1^d, satisfies

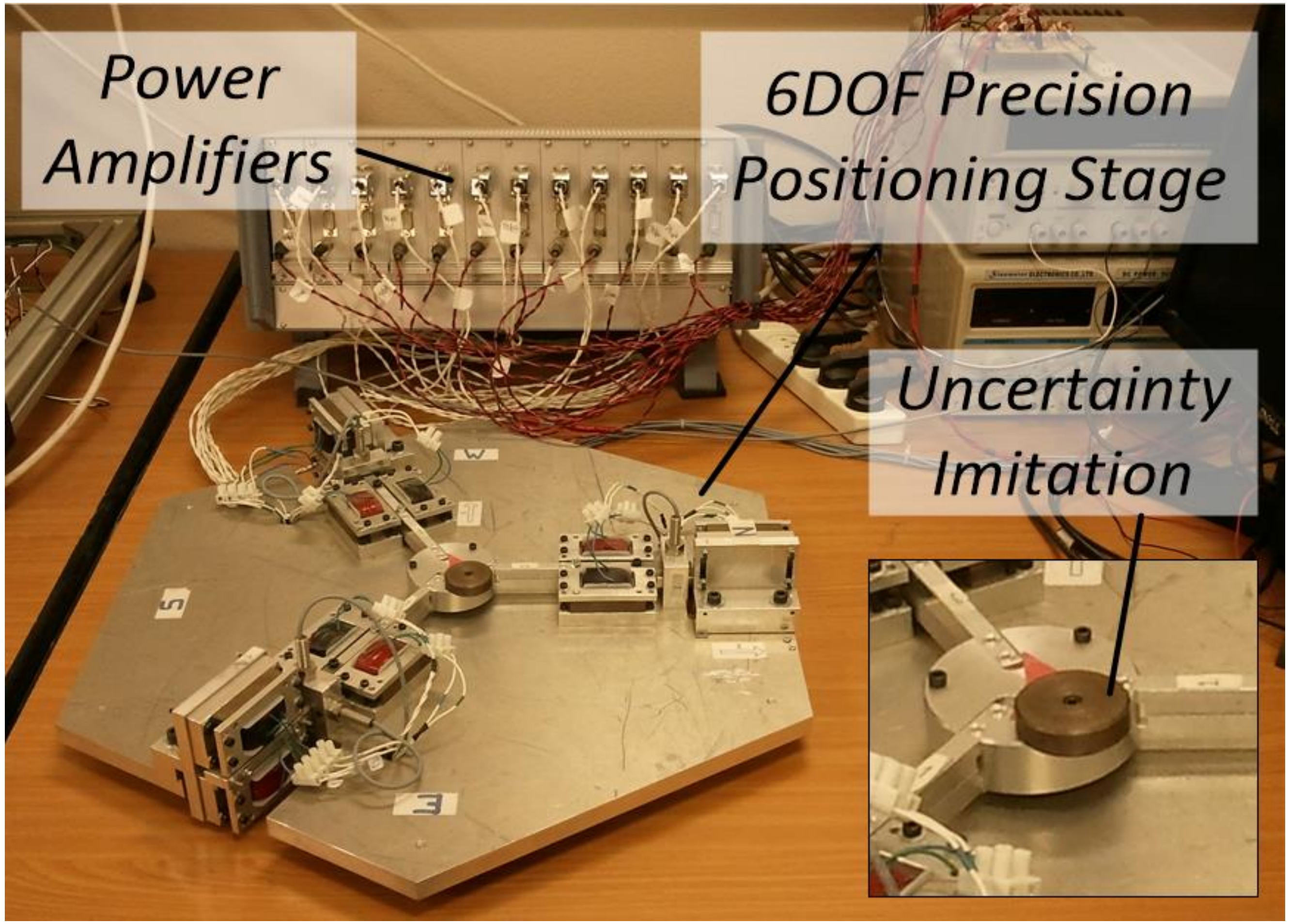
$$|l_k^d| < l_0 - \frac{2}{1-\alpha}\|\Phi_{LQ}\|_2 \cdot \epsilon, \quad k = 1,2,3,4,5,6$$

Experimental Results

The presented algorithm was verified experimentally.

To imitate the uncertainty we attached a steel payload of $m = 0.07\ \mathrm{kg}$ to the stage platen.

besides the additional negative force (w.r.t., z), it causes uncertain torques (w.r.t., x and y).



Experimental Results

To implement the controller we selected,

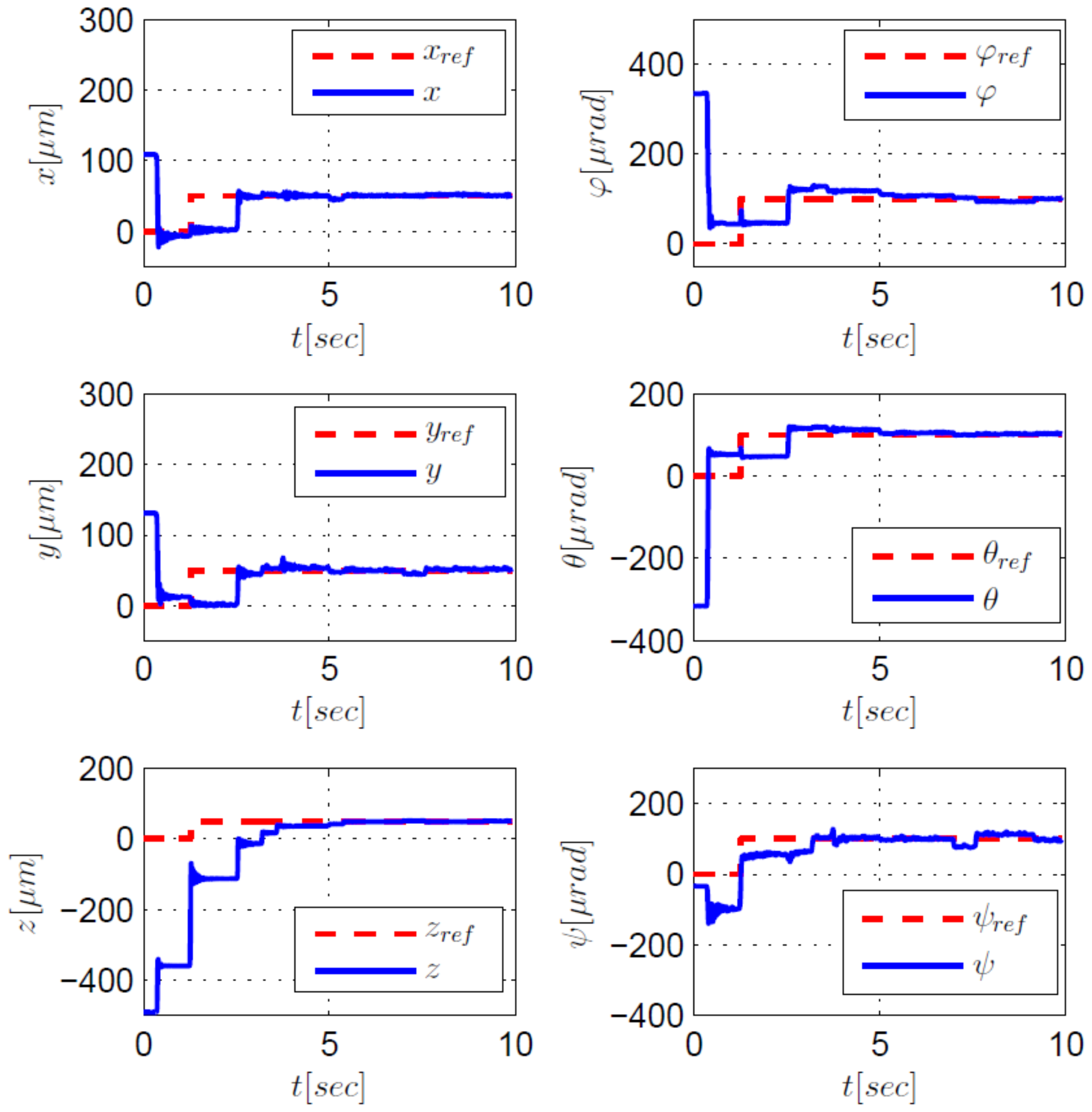
$$C_1 = 1000 \cdot diag\{32.5, 32.5, 32.5, 16.8, 16.8, 16.8\}$$

$$C_2 = diag\{25, 25, 31, 11.5, 11.5, 7.3\}$$

$$\alpha = 0.1, \quad \epsilon = 1\times 10^{-6}, \quad \tau = 0.2$$

while the required set point

$$x_1^d = 1\times 10^{-6}\{50, 50, 50, 100, 100, 100\}^T$$



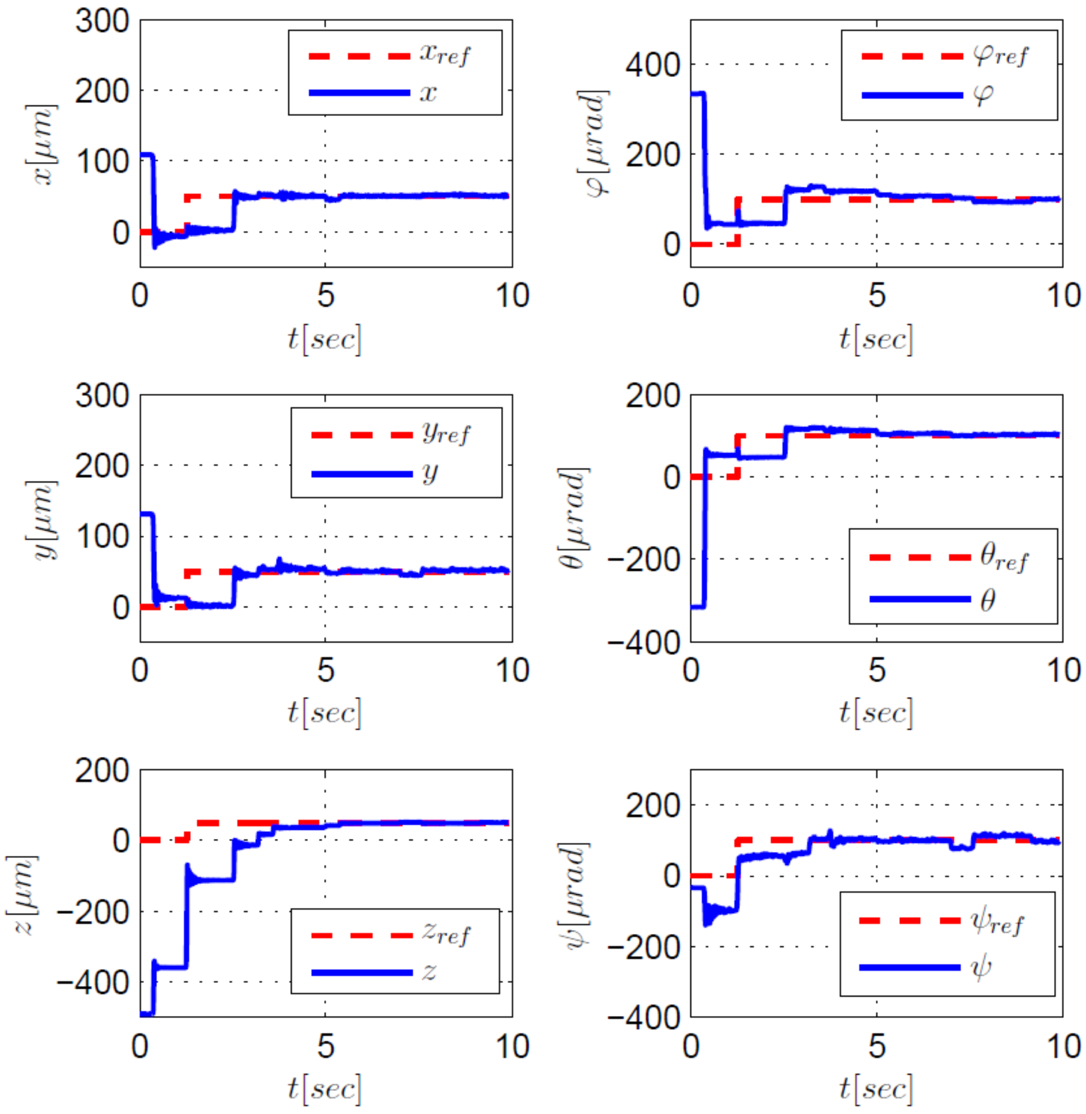
Experimental Results

All required assumptions for the initialization phase have verified.

At the time slot $0 < t < 0.35$ the system stays at initial conditions.

At the time slot $0.35 < t < 1.26$ it undergoes the initialization phase.

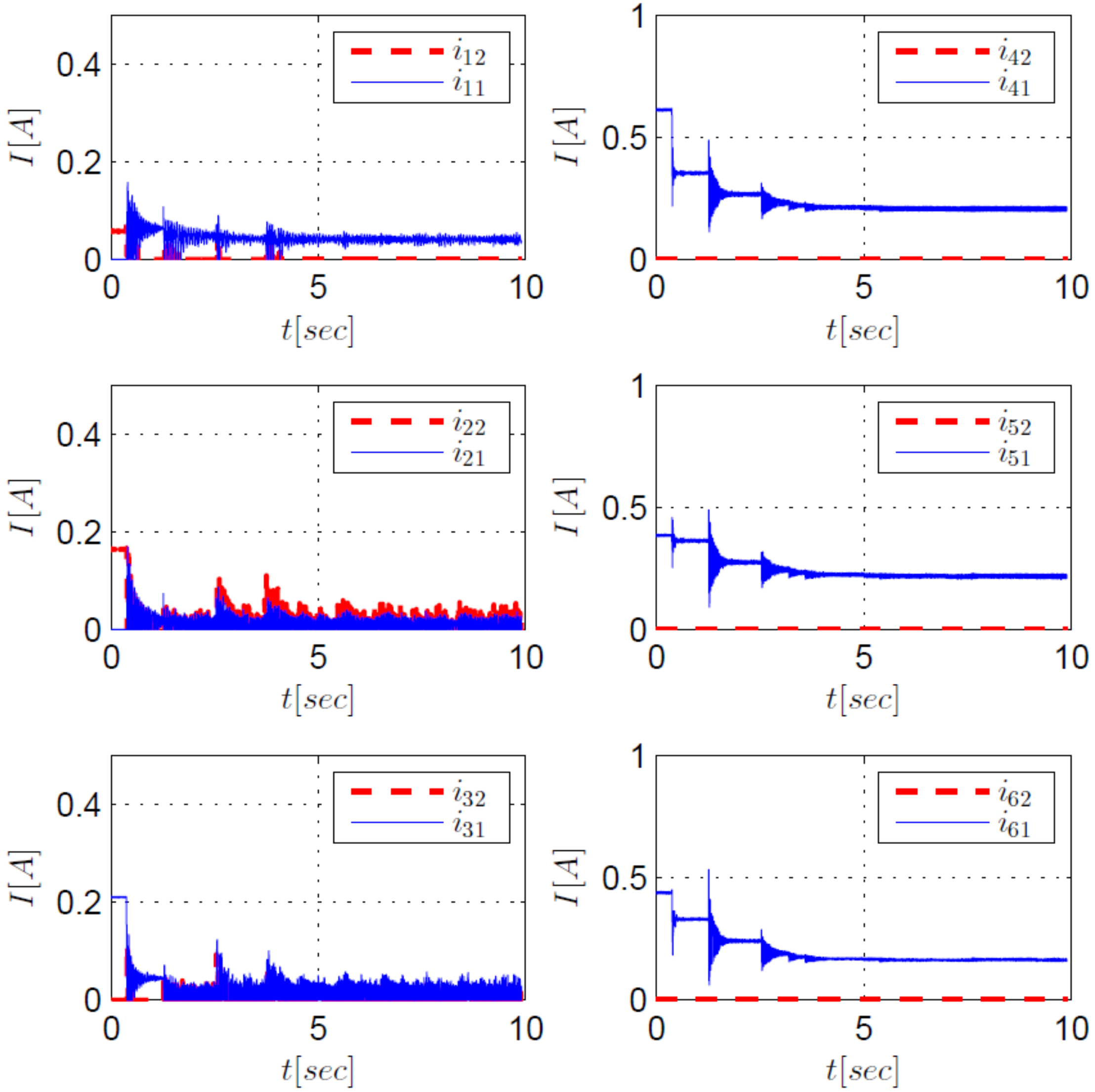
At the time section $1.26 < t < 2.53$ the system responses to v^0 and x_1^d .



Experimental Results

These are the control currents.

For the vertical actuators, only the upper coils were activated.



Experimental Results

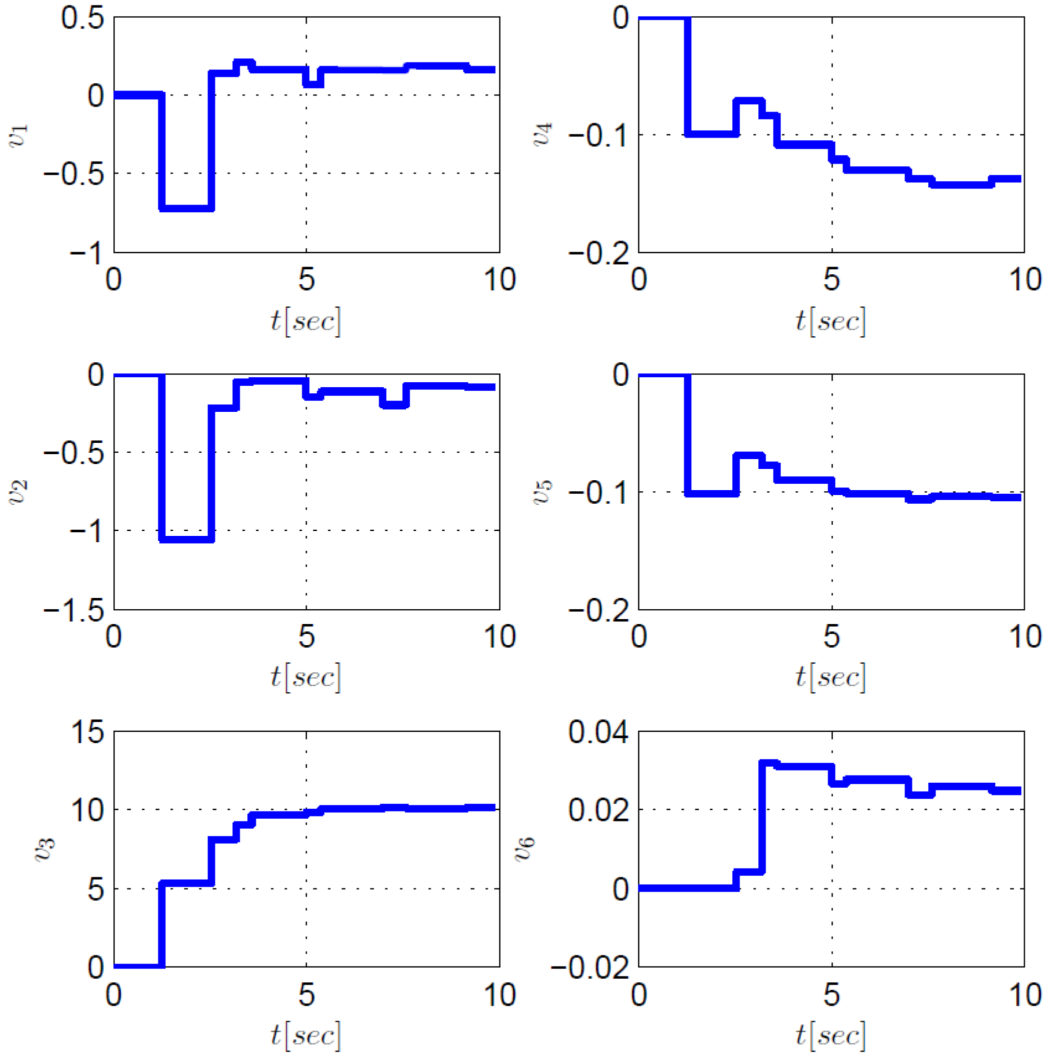
For the attached payload the uncertain vector p is estimated as,

$$p = \{0, 0, 7.044, -0.019, -0.011, 0\}^T$$

For this p, we have defined $\beta = 7.1$

In equilibrium condition, **for** $x_1 = x_1^d$, we suppose to have $\boxed{v = p}$

We understand that, v here is compensating for uncertainties that are not considered in the model



Summary and Conclusions

- A brief introduction to our activities in the field of AMB has been presented.
- A 6-DOF precision positioning stage, based purely on magnetic levitation principles (developed at our Lab) was presented
- Some old results from robotics have been modified to suit the case of magnetic levitation application (to the set point goal).
- An unknown payload was assumed (and hence, an unknown c.g.)
- The results were demonstrated experimentally.
- The proposed algorithm can be used as an identification routine, allowing realization of a simple controller (that is not based on iterations).