Control Design for a MEMS Accelerometer with Tunneling Sensing

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Thank you for introducing me to
a fascinating field for control applications

Outline

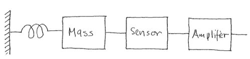
1. Introduction
2. Control Architecture for Force Feedback
3. A Tunneling Accelerometer
4. Experiments
5. Summary

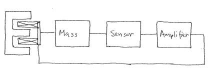
Introduction

- Interesting and useful devices in dynamic development
  AFM, Accelerometers, Gyroscopes, Hard disks, Optical memories
  ...
- Small scale
  Scaling of surface l^2 vs volume l^3: surface effects important
- Oscillatory (nonlinear) dynamics with low damping
- Noise: Brownian motion, Johnson-Nyquist, tunneling, pink, ...
- Parameter uncertainty and parameter variations
- Fast sampling MHz, challenging implementation
- Control is often mission critical, noise, robustness, dynamics, nonlinearities all have to be balanced
- Rich area for applying control BUT not standard control problems

Force Feedback

- Classic idea with tremendous impact
- Game changer in instrument design





Open loop, all components matter

Bandwidth $\omega_b = \sqrt{k/m}$

Sensitivity = k_a/k

Invariant $\omega_b^2 S = k_a/m$

Closed loop, actuator only critical element

Bandwidth and sensitivity given by feedback

Error signal also useful!

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Design a Sensor not a Controller

Key idea: Exploit error signal and not just the feedback signal

Model of *sensor system*

$$\frac{dx}{dt} = Ax + B_w w + Bu \qquad y = Cx,$$

x sensor state, w signal to be measured u actuation signal. Design instrument to have w and u *co-located* ($B_w = kB$).

Model for *signal to be measured*

$$\frac{dw}{dt} = 0, \text{ (constant but unknown)}$$
$$\frac{dz}{dt} = A_w z, \qquad w = C_w z \text{ (general)}$$

Characterized by A_w. Tune sensor to spectrum of acceleration to be measured, for example in automotive applications.

Control Structure

System model

$$\frac{dx}{dt} = Ax + B_w w + Bu, \qquad y = Cx$$
$$\frac{dz}{dt} = A_w z, \qquad w = C_w z$$

Standard controller structure based on Kalman filter and state feedback

$$\frac{d\hat{x}}{dt} = A\hat{x} + B_w C_w \hat{z} + Bu + L_x(y - C\hat{x})$$
$$\frac{d\hat{z}}{dt} = A_w \hat{z} + L_w(y - C\hat{x}) = A_w \hat{z} + L_w(y - \hat{y})$$
$$u = -K_x \hat{x} - K_z \hat{z}.$$

- Design instrument to make $B_w C_w$ close to B
- Determine filter gains L and L_w to give good estimates
- Determine feedback gains K and K_w to give small errors

Instrument Transfer Function

Transfer function from w to $\hat{w}$!

$$G_{\hat{w}w} = \frac{F(s)}{I + F(s)}$$
$$F(s) = C_w(sI - A_w)^{-1}L_w(sI - A - L_xC)^{-1}B_w$$

For $A_w = 0$ (constant but unknown or slowly varying acceleration) the expression simplifies to

$$G_{\hat{w}w} = \frac{L_zC(sI - A + L_xC)^{-1}B_w}{s + L_zC(sI - A + L_xC)^{-1}B_w}, \qquad G_{\hat{w}w}(0) = 1$$

- $G_{\hat{w}w}$ does not depend on feedback gains K_x and K_z!
- $G_{\hat{w}w}$ does not depend on B
- $G_{\hat{w}w}$ depends on filter gains L_x, L_z

Many design options:

- Optimize with respect to disturbances and uncertainty
- Shape the frequency response $G_{\hat{w}w}$

Choosing Feedback Gains

Closed loop dynamics

$$\begin{aligned}\frac{dx}{dt} &= Ax + B_w w - BK_x\hat{x} - BK_zC_w\hat{w} \\ &= (A - BK_x)x + (B_wC_w - BK_z)z + BK_x\tilde{x} + B_wK_z\tilde{z}\end{aligned}$$

- Physical interpretation
- Make effect of external signal w small by matching BK_z to B_wC_w (instrument design). The term $(B_wC_w - BK_z)z$ vanishes if $BK_z = B_wC_w$
- Make terms proportional to $\tilde{x}$ and $\tilde{z}$ small by good estimator design
- Choose K_x to balance decay rate (eigenvalues of $A - BK_x$) to disturbance amplification (BK_x)
- Design gains for robustness

Sensor Resolution

$$\frac{dx}{dt} = Ax + B_w w + Bu, \qquad y = Cx, \qquad \frac{dz}{dt} = A_w z, \qquad w = C_w z$$

Augmented state $x = (x; z)$ small abuse of notation

$$A_a = \begin{bmatrix} A & B_wC_w \\ 0 & A_w \end{bmatrix}, \; B_a = \begin{bmatrix} B \\ 0 \end{bmatrix}, \; C_a = \begin{bmatrix} C & 0 \end{bmatrix}, \; C_{wa} = \begin{bmatrix} 0 & C_w \end{bmatrix}$$

$$\begin{aligned} dx &= A_a x dt + B_a u dt + dv \\ dy &= C_a x dt + de \\ R_x &= E dv\, dv^T, \qquad R_e = E de\, de^T \end{aligned}$$

Kalman filter

$$A_aP + PA_a + R_x - PC_a^TR_y^{-1}C_aP = 0, \qquad L = \begin{bmatrix} L_x \\ L_w \end{bmatrix} = PC_a^TR_y^{-1}$$

Variance of estimate $\sigma^2_{\hat{w}} = C_{wa}PC_{wa}^T$

Constant Acceleration, Fixed Estimator Gains

$$\frac{dx}{dt} = Ax + B_w w + Bu, \qquad y = Cx, \qquad \frac{dz}{dt} = A_w z, \qquad w = C_w z$$

Augmented state with small abuse of notation $x = (x; z)$

$$A_a = \begin{bmatrix} A & B_wC_w \\ 0 & A_w \end{bmatrix}, \; B_a = \begin{bmatrix} B \\ 0 \end{bmatrix}, \; C_a = \begin{bmatrix} C & 0 \end{bmatrix}, \; C_{wa} = \begin{bmatrix} 0 & C_w \end{bmatrix}$$

$$\begin{aligned} dx &= (A_a - LC_a)x dt + B_a u dt + dv \\ dy &= C_a x dt + de \\ R_x &= E dv\, dv^T \\ R_e &= E de\, de^T \end{aligned}$$

Variances of estimation error given by the Lyapunov equation

$$A_aP + PA_a + R_x = 0$$

Variance of estimate $\sigma^2_{\hat{w}} = C_{wa}PC_{wa}^T$

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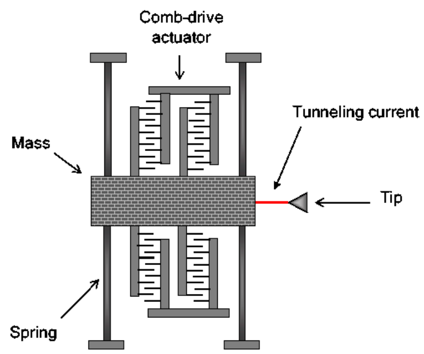
The Tunneling Accelerometer



Courtesy of Laura Oropeza-Ramon

Two Control Problems

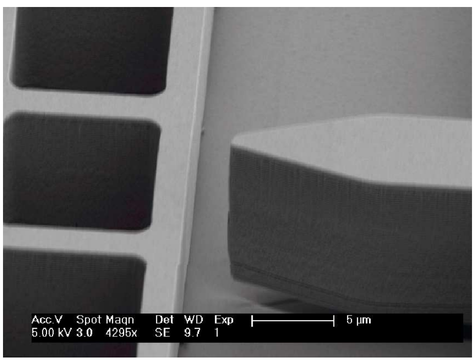
Initialization - Move tip to tunneling position

- Tip starts at initial postion about 1 µm from the surface.
- No signal is available until tunneling starts about 1 nm= 10 Å from the surface.
- Controller is blind until tunneling occurs!
- Tip destroyed if it hits the mass!

Steady state operation - Maintain tunneling

- Measure acceleration
- Tradeoff between bandwidth and noise

Tunneling Tip



Courtesy of Laura Oropeza-Ramon

Tunneling Model

The tunneling current can be modeled by

$$I = k_t^0 V_v e^{-\alpha x\sqrt{\phi}}$$

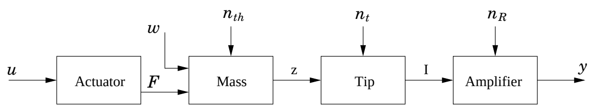
Notice exponential dependence on x! Linearize

$$\frac{dI}{dx} = -\alpha\sqrt{\phi}k_t^0 V_v e^{-\alpha x\sqrt{\phi}} = -\alpha\sqrt{\phi}I = k_t I, \qquad \boxed{\delta I = -k_t I_e \delta x}$$

Notice that gain is proportional to current and that it is constant if we regulate the current well. Typical parameters

- Tunneling current: $I_e = 2$ nA
- *Effective* distance between tip and sample, $x_e = 10$ Å
- Tunneling gain: $k_t I_e = 0.5$ nA/Å $= 5$ A/m for $I_e = 2$ nA
- Amplifier gain: $k_{va} = 2 \times 10^7$ V/A, $V_e = 0.04$ V
- Sensor gain: $k_{vx} = 10^8$ V/m $=$ 10 mV/Å

Block Diagram



Actuator: $F = \dfrac{N\epsilon_0 h}{d}(V_0 + u)^2$, $\quad \boxed{\delta F = k_a \delta u, \quad k_a = 2\dfrac{N\epsilon_0 h V_0}{d}}$

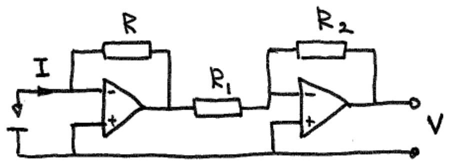
Mass: $m\dfrac{d^2 z}{dt^2} + c\dfrac{dz}{dt} + kz = F + mw + n_{th}$

Tunneling tip:

$I = k_t^0 V_v e^{-\alpha x\sqrt{\phi}}$, $\quad \boxed{\delta I = k_t I_e \delta x + n_t, \quad k_t = \alpha\sqrt{\phi}}$

Amplifier: $V = k_v(RI + n_R)$ (simplified)

Preamplifier



Notice tunneling current of the order of nA! Additional capacitors needed to stabilize the circuit. Opamps also have dynamics.

Noise Sources

- Thermal noise white noise gives a force on the mass with spectral density $4ck_BT$ (dissipation fluctuation theorem), c damping coefficient, $k_B = 1.38 \times 10^{-23}$ [J/Kelvin] Boltzmann's constant and T temperature
- Tunneling noise modeled as shot noise which is white noise with spectral density $q_0 2I$, where $q_0 = 1.6 \times 10^{-19}$ C is the charge of the electron and I is the current.
- Model resistors by an ideal resistor with a voltage source in series representing the Johnson-Nyquist noise which is white noise with spectral density $4k_BTR$
- Amplifier noise
- Pink noise ($1/f$ noise)

Sensor Model

Constant but unknown acceleration, simplified preamp model

$$dx = A_a x dt + B_a u dt + dv, \qquad dy = C_a x dt + de,$$

$$A_a = \begin{pmatrix} 0 & 1 & 0 \\ -k/m & -c/m & -1 \\ 0 & 0 & 0 \end{pmatrix}, \qquad B_a = \begin{pmatrix} 0 \\ k_a/m \\ 0 \end{pmatrix}$$

$$C_y = \begin{pmatrix} k_s & 0 & 0 \end{pmatrix}, \qquad C_w = \begin{pmatrix} 0 & 0 & 1 \end{pmatrix}$$

$$R_x = Edv\,dv^T = \mathrm{diag}(0\,, 2ckBT/m^2,\ r_w)$$

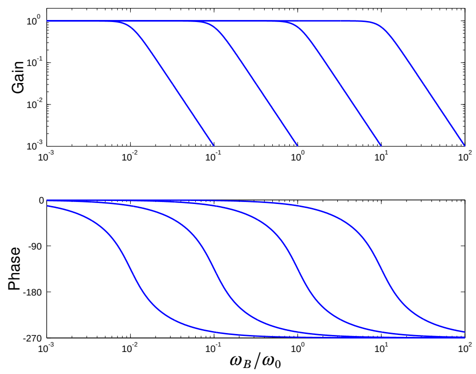
$$R_y = E(de)^2 = k_v^2(2k_BTR + R^2 q_0 I_0).$$

Sensor transfer function

$$G_{\hat{w}w}(s) = \frac{l_3 k_s}{s^3 + (k_s l_1 + c/m)s^2 + (k_s(l_1 c/m + l_2) + k/m) + l_3 k_s}$$

Pick l_1, l_2 and l_3 to shape the transfer function $G_{\hat{w}w}(s)$

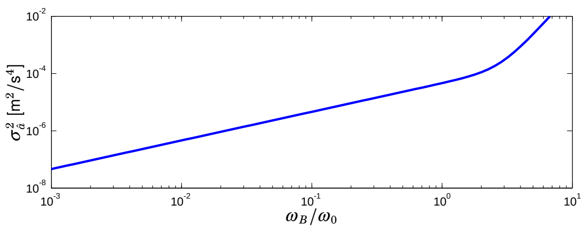
Sensor Transfer Function



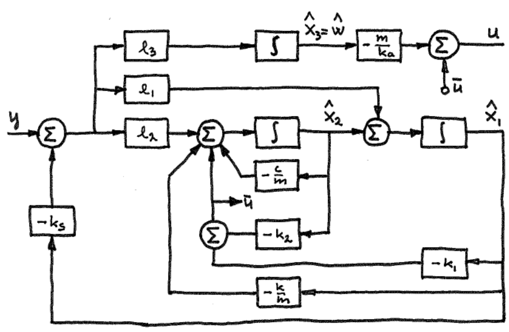
$\alpha_c = 1$, $\zeta_c = 0.5$, $\dfrac{\omega_B}{\omega_0} =$ 0.01,0.01, 0.1, 1.0, and 10

Trade-off between Bandwidth and Variance

- Choose filter gains to shape sensor transfer function
- Bandwidth-variance compromise
- Design issues



Controller Architecture

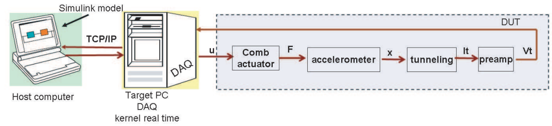


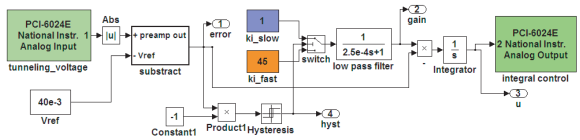
Physical interpretations!

Outline

First Attempt

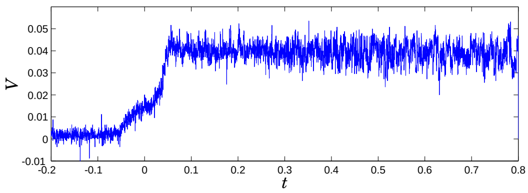
- Initialize - Initiate tunneling, get safely from 1 µm to 1 nm
- Switched integrating controller
- Regulate - maintain tunneling





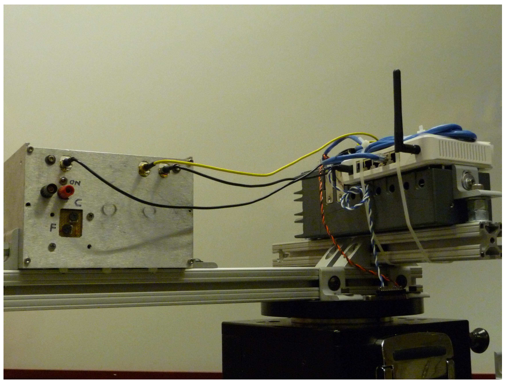
Hunt for Noise Sources

- Originally very high noise levels
- Guide-lines from physical modeling very useful



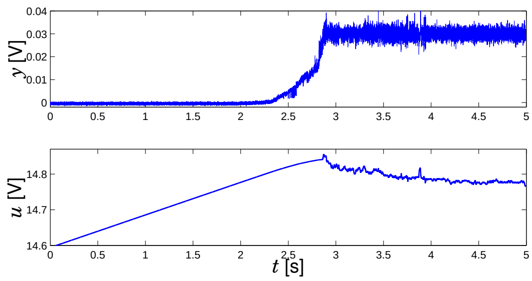
- Redesign electronics: preamplifier, DAC with better resolution
- Replace PC by National Instruments Compact Rio

Improved Experimental Set-up



Courtesy of Chris Burgner

Improved Electronics

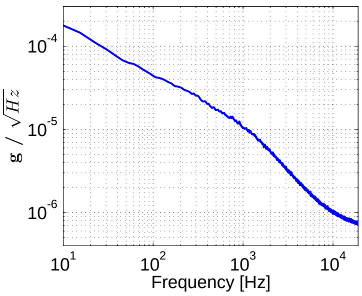


- Tunneling starts at t=2.4
- Standard devviation $\sigma = 3$ mV $\approx$ 0.3 Å

Control Signal has Long Term Drift $1/f$

Model material irregularities as small RC systems in thermal equilibrium with energy logarithmethically distributed

$$\varphi(\omega) = \int_0^\infty \frac{a}{\omega^2 + a^2} d \log a = \frac{\pi}{2\omega}$$



Outline

Summary

- Interesting application area for control
- Systems with low damping

  Truxal 1961: The design of feedback systems to effect satisfactorily the control of *very lightly damped* physical systems is perhaps the most basic of the difficult control problems.
- Noise

  Thermal, Johnson-Nyquist, tunneling, $1/f$
- Noise model very important for improving electronics, tunneling current very small a few nA
- Integrated systems and control design
- A design framework

  Insight and understanding
  Controller structure
  Design trade-offs
  State models are attractive numerically

References

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Parameters

Boltzmann's constant	k_B	1.3807×10^{-23} J/K
Charge of electron	q_0	1.602×10^{-19} C
Tunneling constant	α	1.025 $1/\text{Å}\sqrt{\text{eV}}$
Tunneling barrier	ϕ	0.05 eV
Temperature	T	293 K
Mass	m	4.917 μg
Resonant frequency	f_0	4.2 kHz
Q-value	Q	10
Actuator gain	k_a	9.2×10^{-7} N/V
Tunneling gain	k_t	4 A/m
Preamp resistance	R	10.2 MΩ
Voltage gain	k_v	2
Sensor gain	$k_s = k_t k_v R$	21.6 MV/m