Optimal Control in
Delayed Bilateral Teleoperation

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Joint work with Jang Ho Cho and Liran Malachi

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Outline

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Teleoperation

Remote operation of robotic/dynamical systems is required

- to perform tasks in unreachable/hazardous environments

  (space/ocean exploration, nuclear power, mining)

- in robotic systems used for medical surgery

  (minimally invasive surgery, scaled environment, surgeon may be miles away)

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Master device is used to control slave device in the task environment

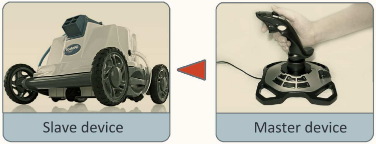
Slave device

Master device

Unilateral vs. bilateral teleoperation

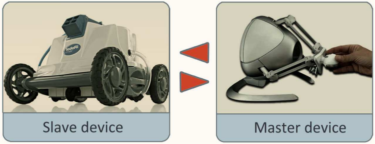
Unilateral teleoperation

- signals are sent in one direction (master → slave)
- no force feedback



Bilateral teleoperation

- signals are sent in both directions
- force/haptic feedback is allowed



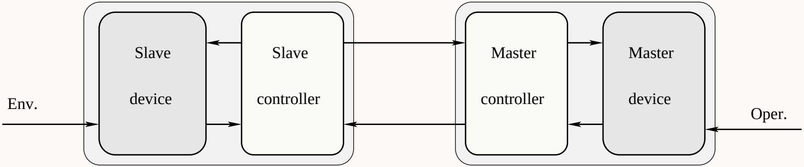
The goal is to achieve transparency of the teleoperation system.

(Two-directional force/position tracking to make the operator “fill” the task environment)

Control of bilateral teleoperation systems

Both master and slave are dynamical systems and need to be controlled

Control goal is to couple the master and slave dynamics



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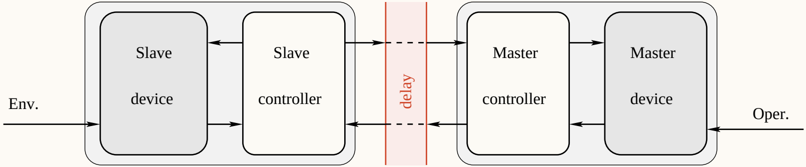

Major challenges:

- Communications delays $\Rightarrow$ distributed control problem

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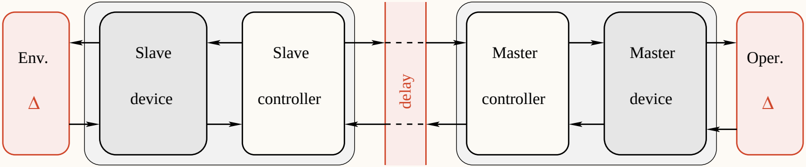

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- Communications delays ⇒ distributed control problem
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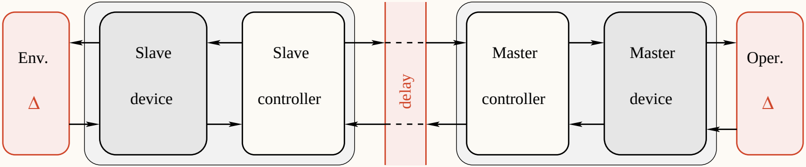

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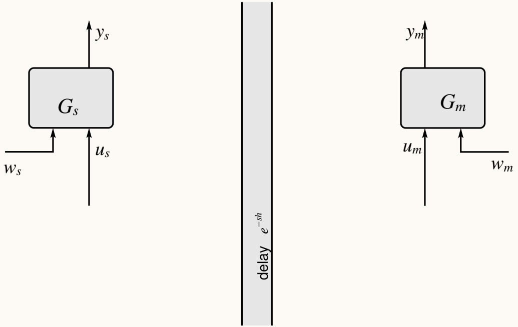
Control setting



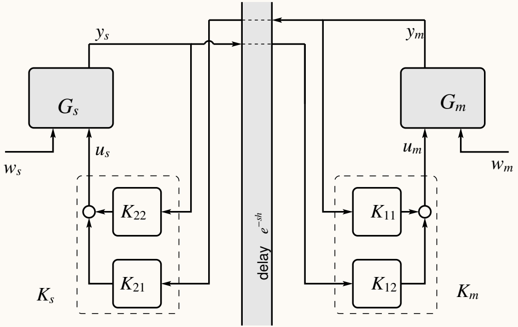
G_m - dynamics of the master device and operator arm

w_m - operator command, y_m - master measurements

G_s - dynamics of the slave device and known part of environment

w_s - unknown part of environment, y_s - slave measurements

Control setting



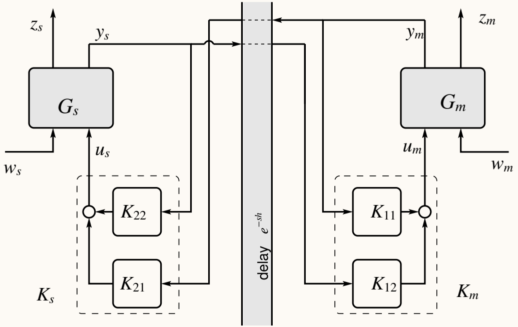
K_{11}, K_{22} - master and slave local controllers (based on local information)

K_{12}, K_{21} - master and slave bilateral controllers

(based on information from the “other side” - delayed)

u_m, u_s - master and slave control signals

Control setting



z_m, z_s - position, force, velocity . . . other signals to be coupled

performance index is defined as $$z := \begin{bmatrix} (z_m - z_s)' & u_m' & u_s' \end{bmatrix}'$$

small z = transparency achieved with reasonable control effort

Being more specific . . .

Master and slave dynamics can be defined as

$$G_m : \begin{cases} M_m\ddot{\xi}_m + b_m\dot{\xi}_m = f_m + u_m \\ w_m = f_m \\ y_m = z_m = \begin{bmatrix} f_m \\ \xi_m \end{bmatrix} \end{cases} \qquad G_s : \begin{cases} M_s\ddot{\xi}_s + b_s\dot{\xi}_s = f_s + u_s \\ w_s = f_s \\ y_s = z_z = \begin{bmatrix} f_s \\ \xi_s \end{bmatrix} \end{cases}$$

ξ_* - position vector, f_* - external force, u_* - control signal,

M_m, b_m - inertia and damping matrices

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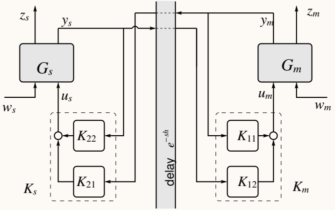
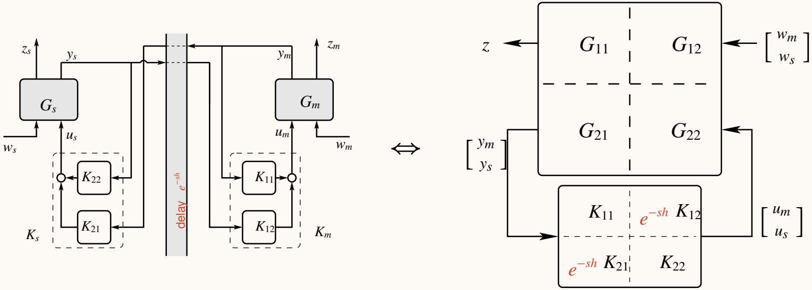
Our setting is an abstraction

of the above formulation



Casting as a generalized control setup



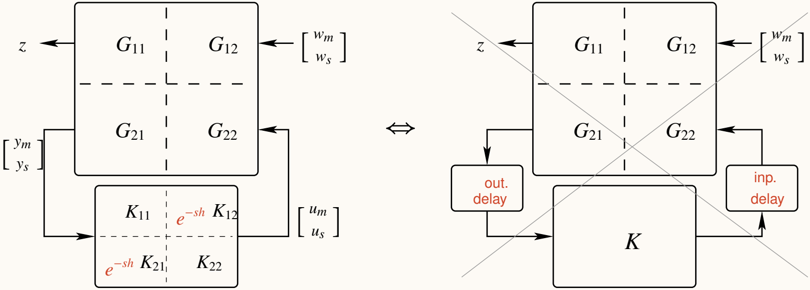
Information constraints ⇒ constraints on controller structure (distributed control)

$$\begin{bmatrix} u_m \\ u_s \end{bmatrix} = \begin{bmatrix} K_{11} & e^{-sh}K_{12} \\ e^{-sh}K_{21} & K_{22} \end{bmatrix} \begin{bmatrix} y_m \\ y_s \end{bmatrix}$$

Off-diagonal blocks of K have to be delayed

Denote the set of admissible controllers by $\mathcal{S}$

Casting as a generalized control setup



Delays can not be extracted $\Rightarrow$ Distributed control problem

Is not equivalent to problem with input/output delays

No ready to use methods for handling this problem are available

Distributed control as a theoretical challenge

No analytical solutions for the general case

Optimal controller might be nonlinear (H.S.Witsenhausen,1968)

Attempts to find tractable solutions for special cases:

- positive systems (Tanaka and Langbord, 2010; Rantzer, 2011)
- convex optimization (Boyd, 2004; Guo et al., 2010)
- quadratic invariance (Rotkowitz and Lall, 2006)

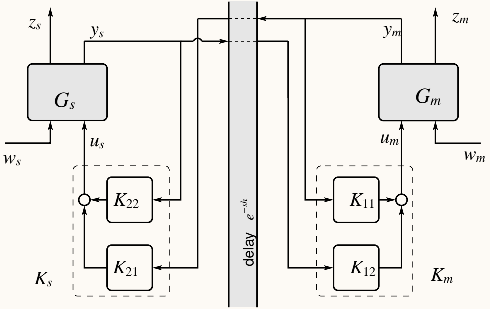
Theoretical research motivated by practical applications:

- formation control (Boyd, 2004; Gattami et al., 2011)
- wind farm control (Rantzer and Madjidian, 2010)
- power networks (Scherpen, 2011)
- traffic control (van Schuppen, 2011)
- bilateral teleoperation

Outline

Independent design of master/slave controllers

1. Design tracking systems for master and slave independently
2. Plug them to work together via communication channel



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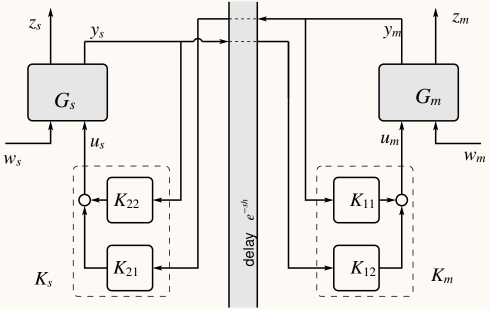

- Overall system contains feedback interconnection with time delays
- ☹ No guarantees on the joint behavior

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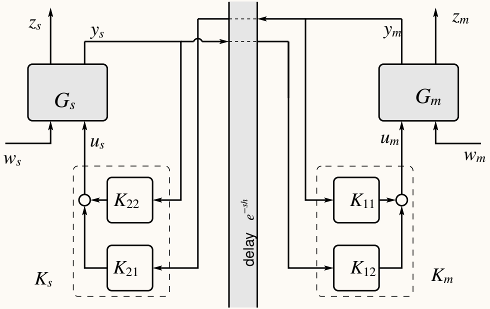


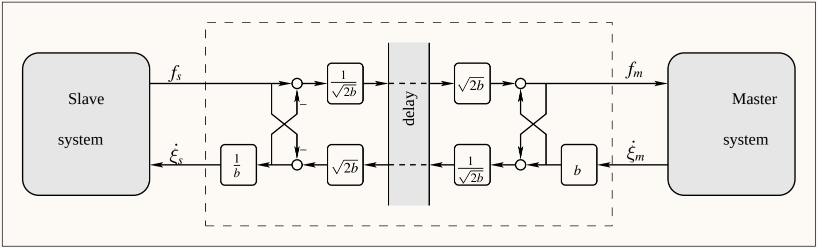
- Overall system contains feedback interconnection with time delays
- ☹ No guarantees on the joint behavior
- ☹ Stability is an issue

Passivity based methods

1. Transformation is applied

   to guarantee passivity of communication channel

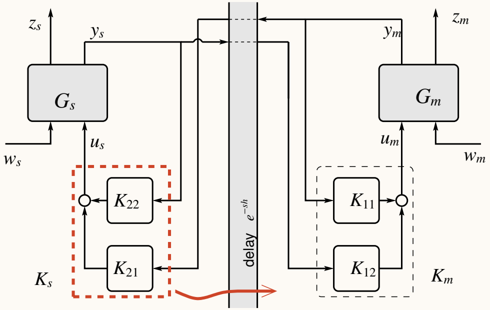
2. Master and slave systems are designed to be passive as well



- ☺ Stability is guaranteed regardless the delay length
- ☹ Restrictive design
- ☹ Synthesis procedure is not intuitive

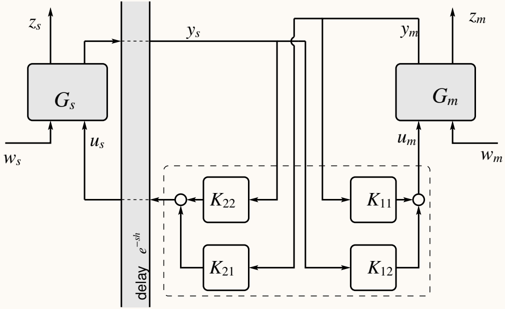
$H^2/H^\infty/\mu$ controller synthesis

No ready to use optimization methods for decentralized problem . . .



$H^2/H^\infty/\mu$ controller synthesis - centralized

Implement all parts of the controller as a single block from one of the sides



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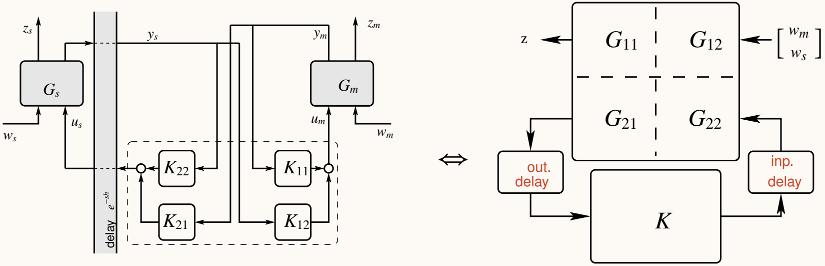


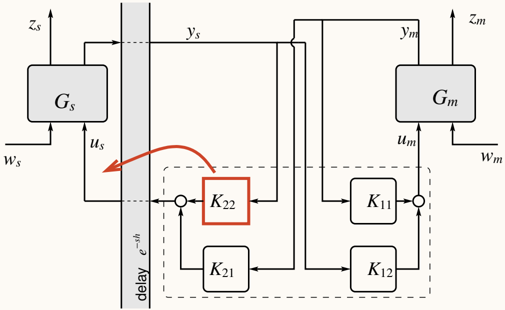
- Reduces the problem to centralized setting with input/output delays

☺ Standard techniques can be applied
  - time discretization / Pade approximation / loop shifting

☹ Introduces unnecessary delays

$H^2/H^\infty/\mu$ controller synthesis - centralized (contd.)

To make the control scheme less restrictive



$H^2/H^\infty/\mu$ controller synthesis - centralized (contd.)

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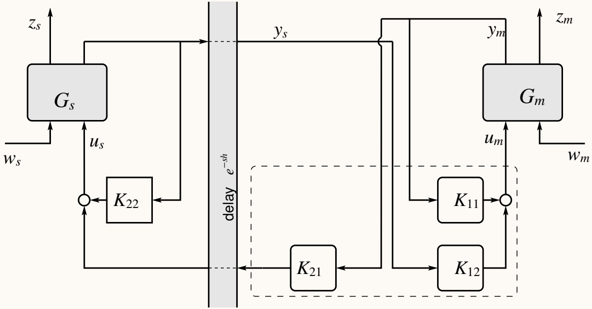


Controller for the remote site can be designed separately

- ☺ Does not introduce unnecessary delays
- ☹ No guarantee of global optimality
- ☹ Not intuitive iterative synthesis procedure

Intermediate summary

Existing methods for control of delayed bilateral teleoperation systems

- restrict controller's structure
- do not result in holistic optimization-based synthesis procedure

Common belief:

- Global optimization is not feasible

  (Because the problem falls into a category of distributed control problems)

Intermediate summary

Existing methods for control of delayed bilateral teleoperation systems

- restrict controller's structure
- do not result in holistic optimization-based synthesis procedure

- Is global optimization feasible?

  (Because the problem falls into a category of distributed control problems)

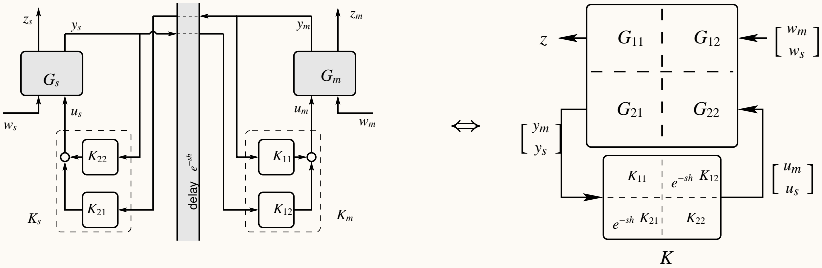
We are going to show that intrinsic properties of teleoperation setup

facilitate analytical solution of the problem . . .

Outline

Problem formulation

Going back to the original control setting



Consider H^2 optimization

$$\min_{\text{stab. } K \in \mathcal{S}} \|G_{11} + G_{12}K(I - G_{22}K)^{-1}G_{21}\|_2$$

Problem formulation

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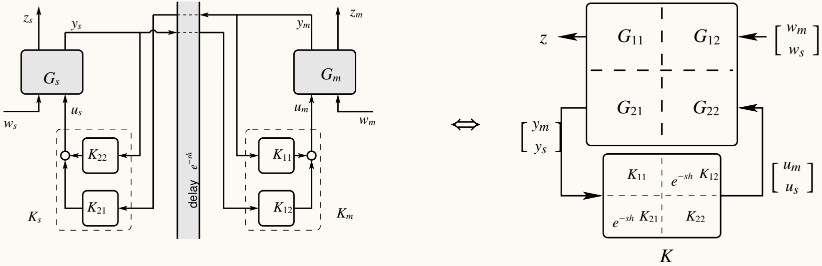

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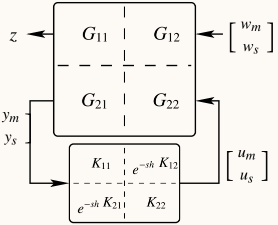
Why/How does structural constraint $K \in \mathcal{S}$ complicates the problem?

One possible explanation: It impedes the use of Youla parameterization.

Youla parameterization

$$T = G_{11} + G_{12}K(I - G_{22}K)^{-1}G_{21}$$



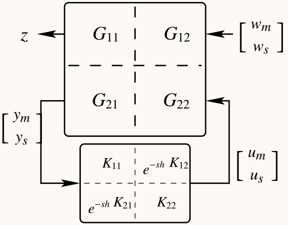
Assuming stable plant

- all stabilizing controllers: $K = Q(I + G_{22}Q)^{-1}, \quad \forall Q \in H^{\infty}$
- all stabilized systems: $T = G_{11} + G_{12}QG_{21}, \quad \forall Q \in H^{\infty}$

The problem is affine in terms of Q

Youla parameterization

$$\underline{T = G_{11} + G_{12}K(I - G_{22}K)^{-1}G_{21}}$$



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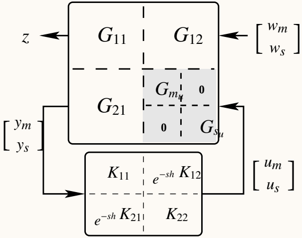
The problem is affine in terms of Q

How to account for structural constraint, i.e., $Q \in \text{?} \quad \Leftrightarrow \quad K \in \mathcal{S}$

Generally, ? is difficult to find . . . yet, in the case of teleoperation . . .

The structure of G_{22}

Master and slave dynamics are originally decoupled $\Rightarrow$ G_{22} is block-diagonal



It can be shown that

$$\underbrace{\begin{bmatrix} K_{11} & e^{-sh}K_{12} \\ e^{-sh}K_{21} & K_{22} \end{bmatrix}}_{K\in\mathcal{S}} \begin{bmatrix} G_{m_u} & 0 \\ 0 & G_{s_u} \end{bmatrix} \underbrace{\begin{bmatrix} K_{11} & e^{-sh}K_{12} \\ e^{-sh}K_{21} & K_{22} \end{bmatrix}}_{K\in\mathcal{S}} = \underbrace{\begin{bmatrix} * & e^{-sh}* \\ e^{-sh}* & * \end{bmatrix}}_{\in\mathcal{S}}$$

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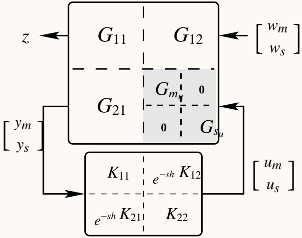

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and as a result $K \in \mathcal{S} \iff Q \in \mathcal{S}$

Quadratic invariance

Problems for which $K \in \mathcal{S} \Leftrightarrow Q \in \mathcal{S}$ are called quadratically invariant

- necessary and sufficient condition $KG_{22}K \in \mathcal{S}$
- the notion is proposed in (Rotkowitz and Lall, 2006)
- earlier research on related topics (Desoer, '80s; Voulgaris, 2000)

Active study during the last years

- characterization and interpretation
  - partially ordered sets (Shah and Parrilo, 2006)
  - fully connected networks with time delays (Rotkowitz etc., 2010)
- quest for a complete analytical solution
  - two players with one-directional communication (Swigart and Lall, 2010)
  - state-feedback with decoupled disturbances (Shah and Parrilo, 2011)

Quadratic invariance - consequences

Parameterization of all stabilizing controllers $K = Q(I + G_{22}Q)^{-1}, \ \forall Q \in \mathcal{S} \cap H^{\infty}$

Parameterization of all stabilized systems $T = G_{11} + G_{12}QG_{21}, \ \forall Q \in \mathcal{S} \cap H^{\infty}$

The problem reduces to

model matching with structural constraints on the design parameter

$$\min_{\text{stab. } K \in \mathcal{S}} ||\mathcal{F}_l(G, K)||_2 = \min_{Q \in \mathcal{S} \cap H^{\infty}} ||G_{11} + G_{12}QG_{21}||_2$$

The questions are:

- How to find optimal Q?
- Given optimal Q,

  what is the structure of optimal K and how to implement it?

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Controller structure

Detailed form of $K = Q(I + G_{22}Q)^{-1}$ is

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Deriving explicit formulae for the components of K leads to bulky expressions

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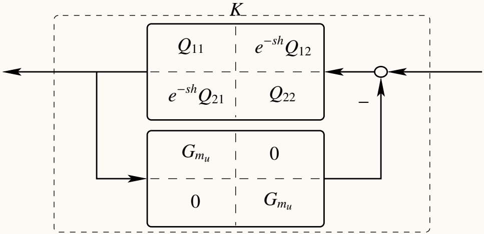
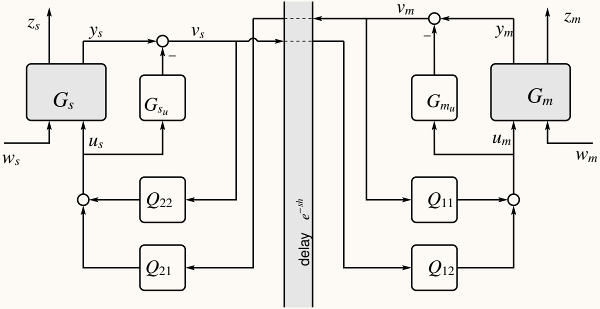
Deriving explicit formulae for the components of K leads to bulky expressions

Instead, consider graphical interpretation

$K = Q(I + G_{22}Q)^{-1} \quad \Leftrightarrow$



Controller structure (contd.)



Communication is based on v_m and v_s signals

- parts of the outputs driven by external disturbances

Alternative for passivity-based schemes

- stability is guaranteed regardless the delay length
- unlike passivity-based schemes, structure not restrictive

Back to optimization

To find optimal Q we need to solve

$$\min_{Q \in \mathcal{S} \cap H^\infty} ||G_{11} + G_{12} Q G_{21}||_2$$

- Model matching optimization
  with structural constraints on the design parameter
- Fundamental open problem
  in the context of quadratically invariant distributed control
- Only some special cases have been solved so far
  (Swigart and Lall, 2010; Lessard and Lall, 2011; Kristalny and Shah, 2012; Lampersi and Doyle, 2012)

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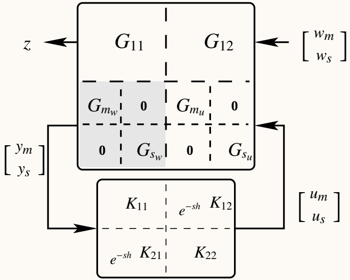
Again natural properties of teleoperation setup facilitate the solution . . .

The structure of G_{21}

Master and slave do not have common disturbances $\Rightarrow$ G_{21} is block diagonal



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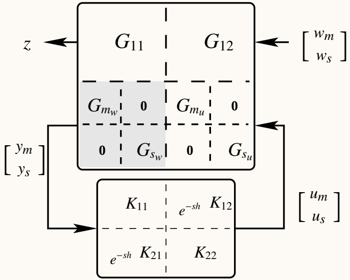
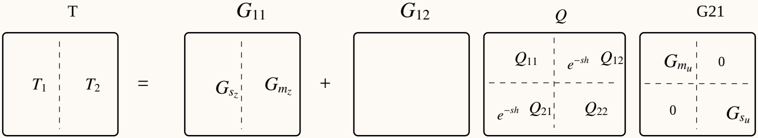
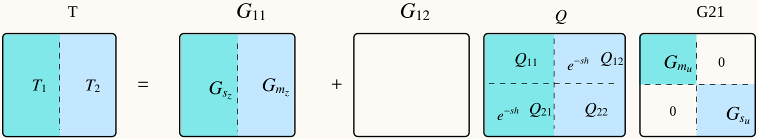


As a result,



Splitting the problem

We can consider each column apart



$$T_1 = G_{m_z} + G_{12}\begin{bmatrix} Q_{11} \\ e^{-sh}Q_{21} \end{bmatrix} G_{m_w}$$

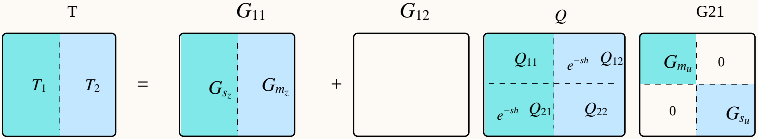
(depends on Q_{11}, Q_{21} only)

$$T_2 = G_{s_z} + G_{12}\begin{bmatrix} e^{-sh}Q_{12} \\ Q_{22} \end{bmatrix} G_{s_w}$$

(depends on Q_{12}, Q_{22} only)

Splitting the problem

We can consider each column apart



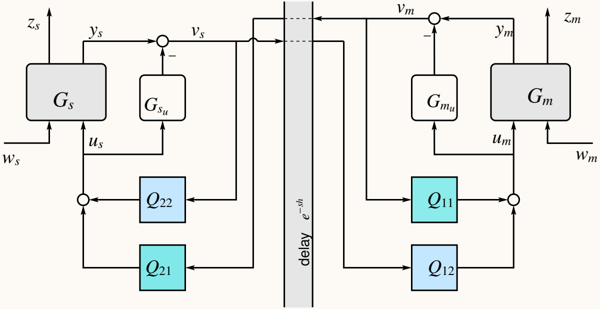
$$T_1 = G_{m_z} + G_{12} \begin{bmatrix} Q_{11} \\ e^{-sh} Q_{21} \end{bmatrix} G_{m_w}$$ (depends on Q_{11}, Q_{21} only)

$$T_2 = G_{s_z} + G_{12} \begin{bmatrix} e^{-sh} Q_{12} \\ Q_{22} \end{bmatrix} G_{s_w}$$ (depends on Q_{12}, Q_{22} only)

H^2 norm satisfies $$||T||_2^2 = ||T_1||_2^2 + ||T_2||_2^2$$

The problems splits into $$\min_{Q_{11,21} \in \mathcal{H}^\infty} ||T_1||_2 \ , \qquad \min_{Q_{12,22} \in \mathcal{H}^\infty} ||T_2||_2$$

Splitting the problem - interpretation



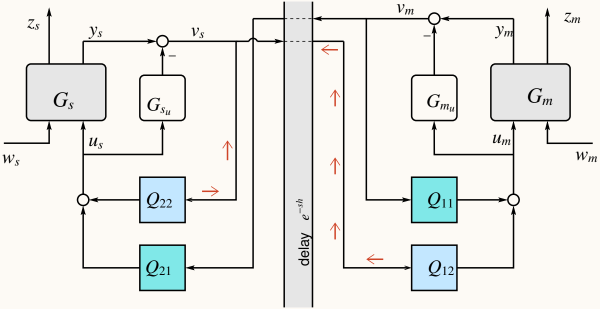
Two independent problems:

1. Q_{11}, Q_{21} - control of overall system based on the master measurements
2. Q_{12}, Q_{22} - control of overall system based on the slave measurements

(The problem splits with respect to the measurements

and not the controlled objects)

Splitting the problem - interpretation



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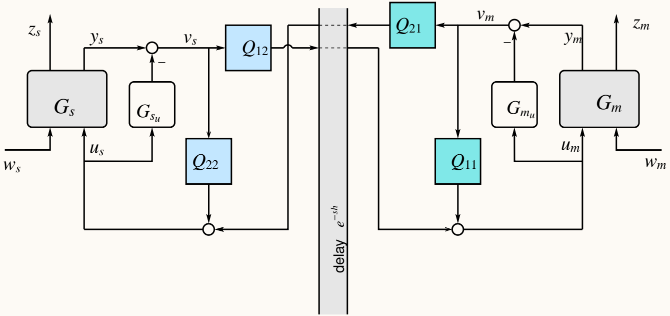


Two independent problems:

1. Q_{11}, Q_{21} - control of overall system based on the master measurements
2. Q_{12}, Q_{22} - control of overall system based on the slave measurements

(The problem splits with respect to the measurements

and not the controlled objects)

Splitting the problem - why does it help?

Consider the first column

(the second can be handled in a similar manner)

$$T_1 = G_{m_z} + G_{12} \begin{bmatrix} Q_{11} \\ e^{-sh} Q_{21} \end{bmatrix} G_{m_w}$$

Delay can be extracted out of the design parameter

$$T_1 = G_{m_z} + G_{12} \underbrace{\begin{bmatrix} I & 0 \\ 0 & e^{-sh} \end{bmatrix}}_{\Lambda} \begin{bmatrix} Q_{11} \\ Q_{21} \end{bmatrix} G_{m_w}$$

This shifts the problem into another category (problem with input delay)

Using Pade or time discretization delay can be absorbed into the state

- blurs problem structure, increases dimension of AREs
- leads to high-order black-box controllers

Is there an elegant way to deal with delays?

Exploiting recent results (contd.)

Recent results in the area of delayed systems control:

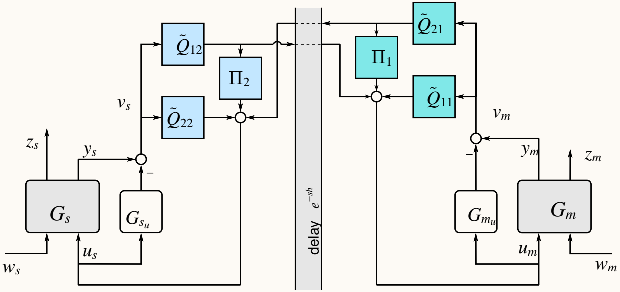
"Dead-Time Compensation for Systems with Multiple I/O Delays:
A Loop Shifting Approach," IEEE TAC, 2011

by L. Mirkin and Z. J. Palmor and D. Shneiderman

The resulting solution

Solution in terms of ARE of the same dimension as in delay-free case

Optimal controller has easy to implement structure:

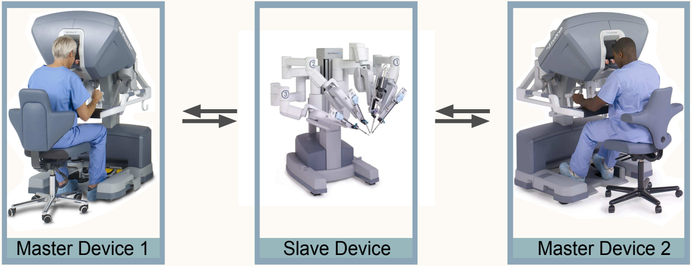


Explicit state-space formulae for $\tilde{Q}_*$ are derived

FIR blocks $\Pi_{1/2}$ are the only infinite dimensional components

Outline

Cooperative teleoperation



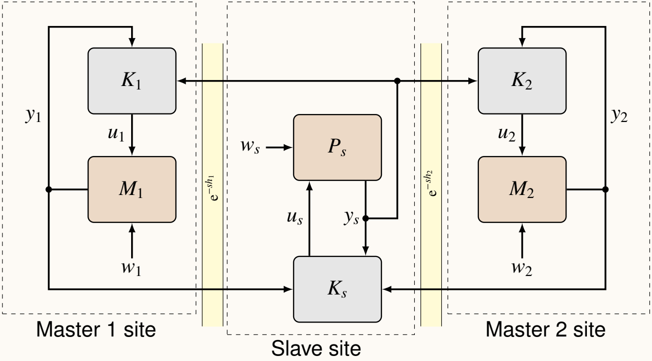
Cooperative bilateral teleoperation:

cooperative operation of multiple master/slave pairs.

Needed to perform complex tasks,

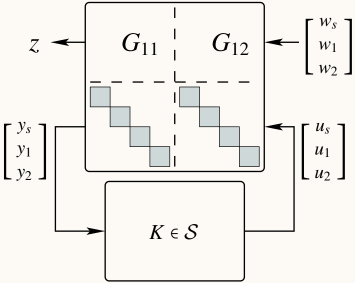
which cannot be conducted by a single operator.

Cooperative teleoperation - natural control architecture



- Slave communicates with two masters
- Each site has a local controller

Cooperative teleoperation - generalized plant



$$\mathcal{S} : \begin{bmatrix} * & e^{-sh_1} * & e^{-sh_2} * \\ e^{-sh_1} * & * & 0 \\ e^{-sh_2} * & 0 & * \end{bmatrix}$$

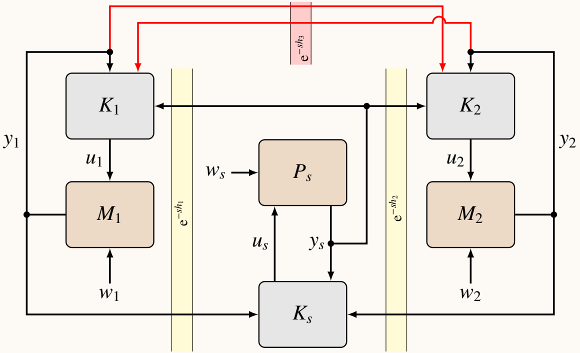
Unlike the single mater/slave case, this setup is not QI

$$KG_{22}K \notin \mathcal{S}$$

But there is a way to circumvent this difficulty ...

Cooperative teleoperation - modified QI control architecture

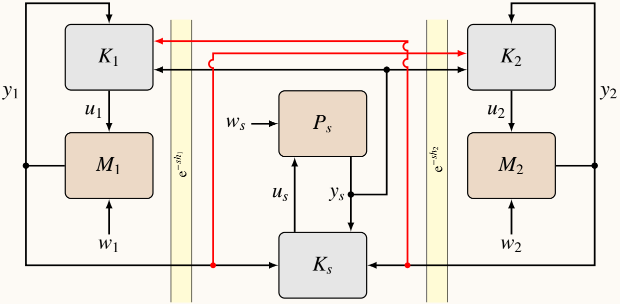
Allowing communication between masters makes the problem QI



$$K = \begin{bmatrix} K_{ss} & e^{-sh_1} K_{s1} & e^{-sh_2} K_{s2} \\ e^{-sh_1} K_{1s} & K_{11} & e^{-sh_3} K_{12} \\ e^{-sh_2} K_{2s} & e^{-sh_3} K_{21} & K_{22} \end{bmatrix}$$

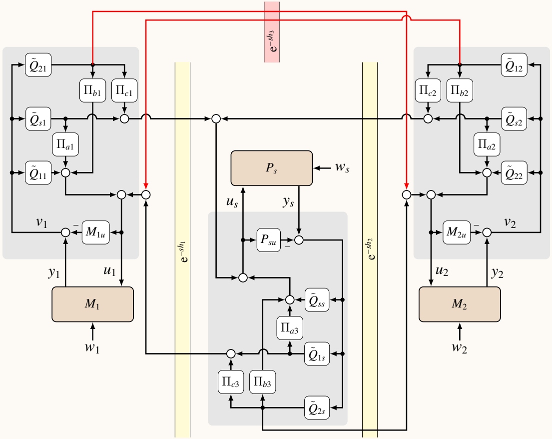
Cooperative teleoperation - modified QI control architecture

Communication can be implemented through the slave site



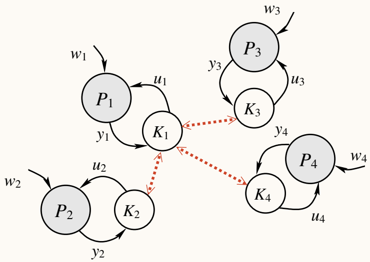
$$K = \begin{bmatrix} K_{ss} & e^{-sh_1} K_{s1} & e^{-sh_2} K_{s2} \\ e^{-sh_1} K_{1s} & K_{11} & e^{-(sh_1+sh_2)} K_{12} \\ e^{-sh_2} K_{2s} & e^{-(sh_1+sh_2)} K_{21} & K_{22} \end{bmatrix}$$

Resulting solution - controller structure



Possible generalization

Coordination of arbitrary number of agents over delayed communication



P_i - subsystems

w_i, y_i, u_i - local signals

K_i - local controllers

Required properties:

- uncoupled agent dynamics
- independent external disturbances

The requirement on joint behaviour is the only coupling term.

Open problems

- How to construct optimization criteria
- Experimental validation
- Robustness issues

Thank you for attention!

Riccati equations

AREs