AUV
Modeling, Control, Obstacle avoidance

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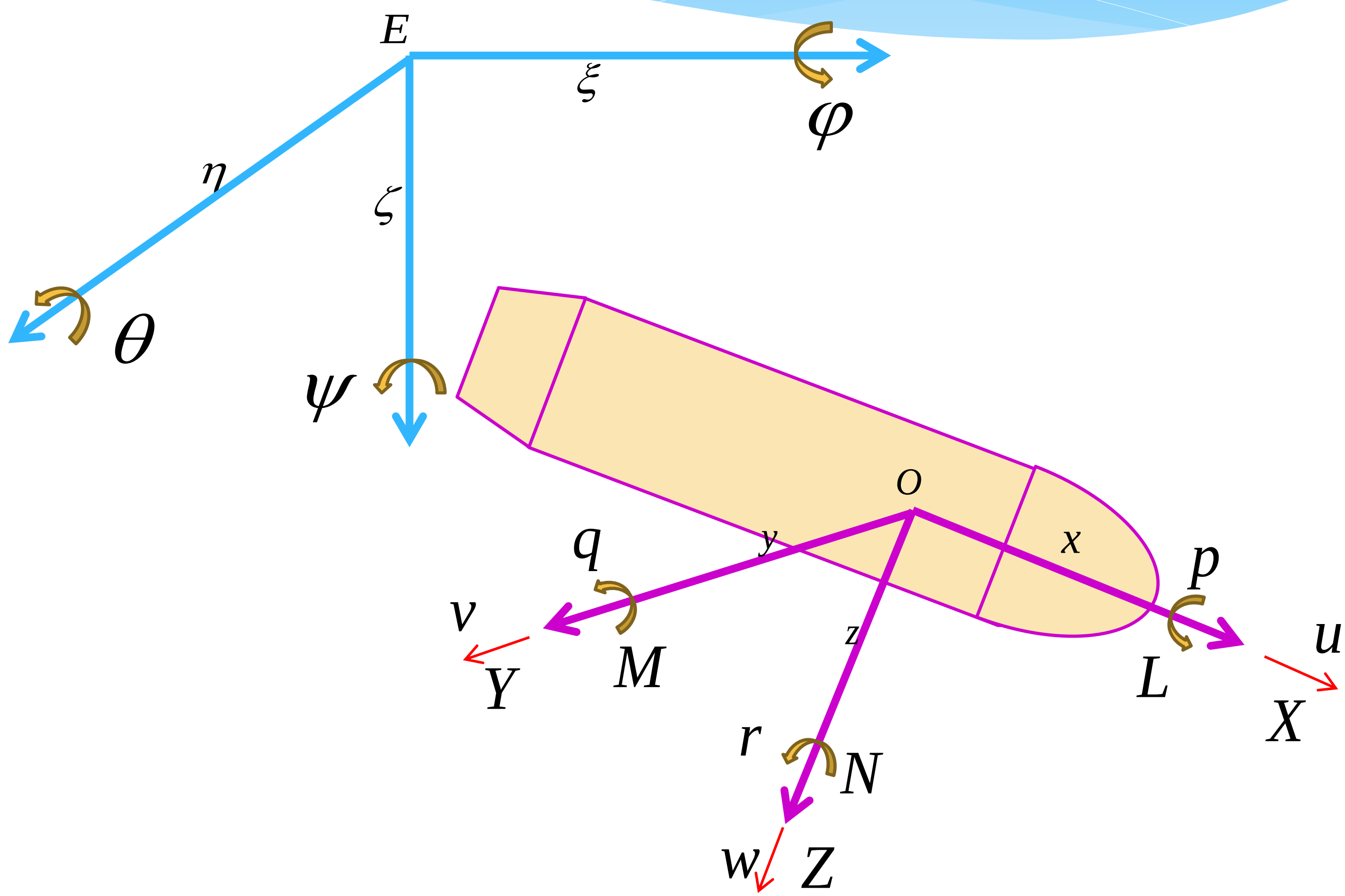
Outlines

- Coordinate system, general 6Dof inertial and relative
- Modeling MIMO sysem using Fourier Integral method
- MIMO Control design
  - Specifications
  - Loop shaping
- Optimal path planner using the DIDO solver
- Experiments

Coordinate System

Technical terms (terminology)

- **x**
  - Surge: u , X
  - Roll: p , L
- **y**
  - Sway: v , Y
  - Pitch: q , M
- **z**
  - Heave: w , Z
  - Yaw: r , N



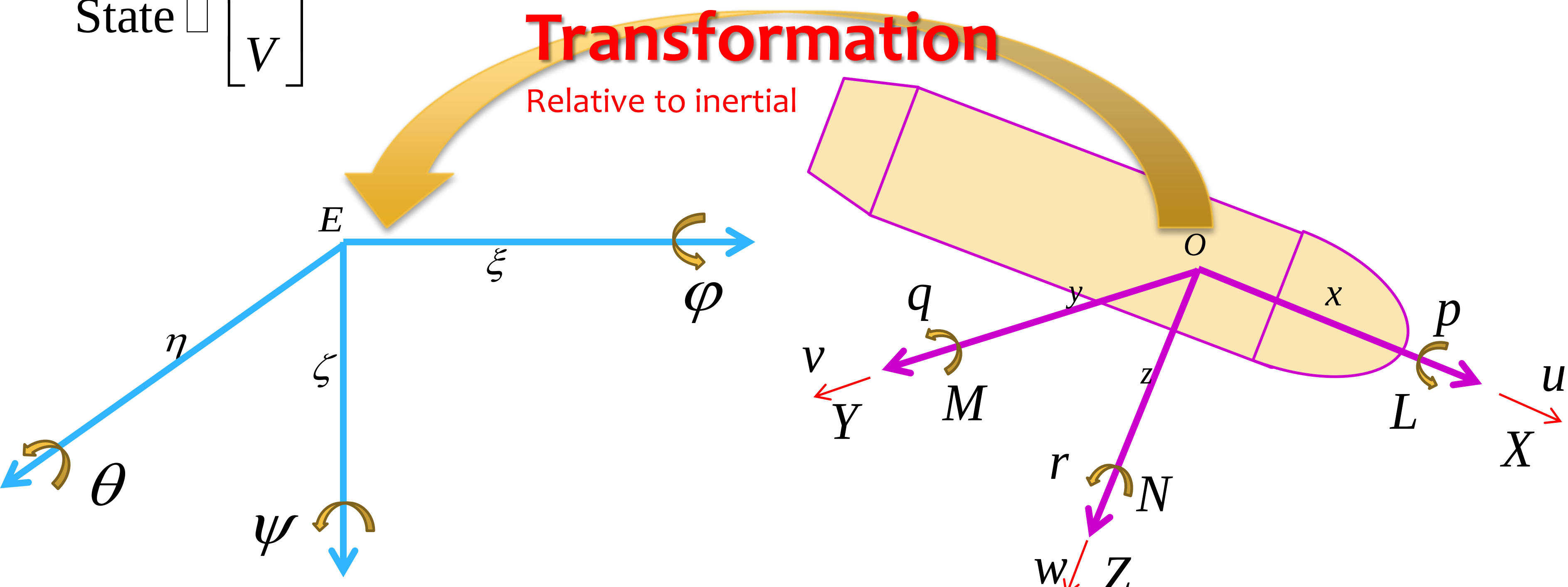
State Control

AUV State

$$R = [r' \,\vdots\, \Lambda']' = [\xi \quad \eta \quad \zeta \,\vdots\, \varphi \quad \theta \quad \psi]'$$

$$V = [U' \,\vdots\, \Omega']' = [u \quad v \quad w \,\vdots\, p \quad q \quad r]'$$

$$\text{State} \triangleq \begin{bmatrix} R \\ V \end{bmatrix}$$



Transformation

- Transformation from fixed-body frame to inertial coordinate system

Inertial velocity

$$\begin{bmatrix} \dot{\xi}_G \\ \dot{\eta}_G \\ \dot{\zeta}_G \\ \dot{\varphi} \\ \dot{\theta} \\ \dot{\psi} \end{bmatrix} = \begin{bmatrix} T_1 & 0_3 \\ 0_3 & T_2 \end{bmatrix} \begin{bmatrix} u \\ v \\ w \\ p \\ q \\ r \end{bmatrix}$$

Relative velocity

- Transform matrixes

$$T_1 = \begin{bmatrix} \cos\psi\cos\theta & \cos\psi\sin\theta\sin\varphi - \sin\psi\cos\varphi & \cos\psi\sin\theta\cos\varphi + \sin\psi\sin\varphi \\ \sin\psi\cos\theta & \sin\psi\sin\theta\sin\varphi + \cos\psi\cos\varphi & \sin\psi\sin\theta\cos\varphi - \cos\psi\sin\varphi \\ -\sin\theta & \cos\theta\sin\varphi & \cos\theta\cos\varphi \end{bmatrix}$$

$$T_2 = \begin{bmatrix} 1 & \tan\theta\sin\varphi & \tan\theta\cos\varphi \\ 0 & \cos\varphi & -\sin\varphi \\ 0 & \sin\varphi\sec\theta & \cos\varphi\sec\theta \end{bmatrix}$$

Momentum Theorem

$$\boldsymbol{momentum} \triangleq \boldsymbol{Mass \cdot Velocity} \quad (\boldsymbol{scalar\ product})$$

$$\boldsymbol{external\ forces\ and\ moments} = \frac{\boldsymbol{d}}{\boldsymbol{dt}}[\boldsymbol{momentum}]$$

Origin is located at the center of buoyancy and the center of gravity lies at the point $\boldsymbol{r_G}$.

- ***Newton's equation of motion*** $\sum \boldsymbol{F} = \boldsymbol{m a_G}$

$\boldsymbol{a_G} \triangleq$ center mass acceleration, m $\triangleq$ body mass , $\sum \boldsymbol{F} \triangleq$ external forces

$$\boldsymbol{a_G} = \frac{\partial \boldsymbol{U}}{\partial \boldsymbol{t}} + \boldsymbol{\Omega} \mathrm{x} \boldsymbol{U} + \dot{\boldsymbol{\Omega}} \mathrm{x} \boldsymbol{r_G} + \boldsymbol{\Omega} \mathrm{x} \boldsymbol{\Omega} \mathrm{x} \boldsymbol{r_G}$$

Ω=[p,q,r]’ , U=[u,v,w]’

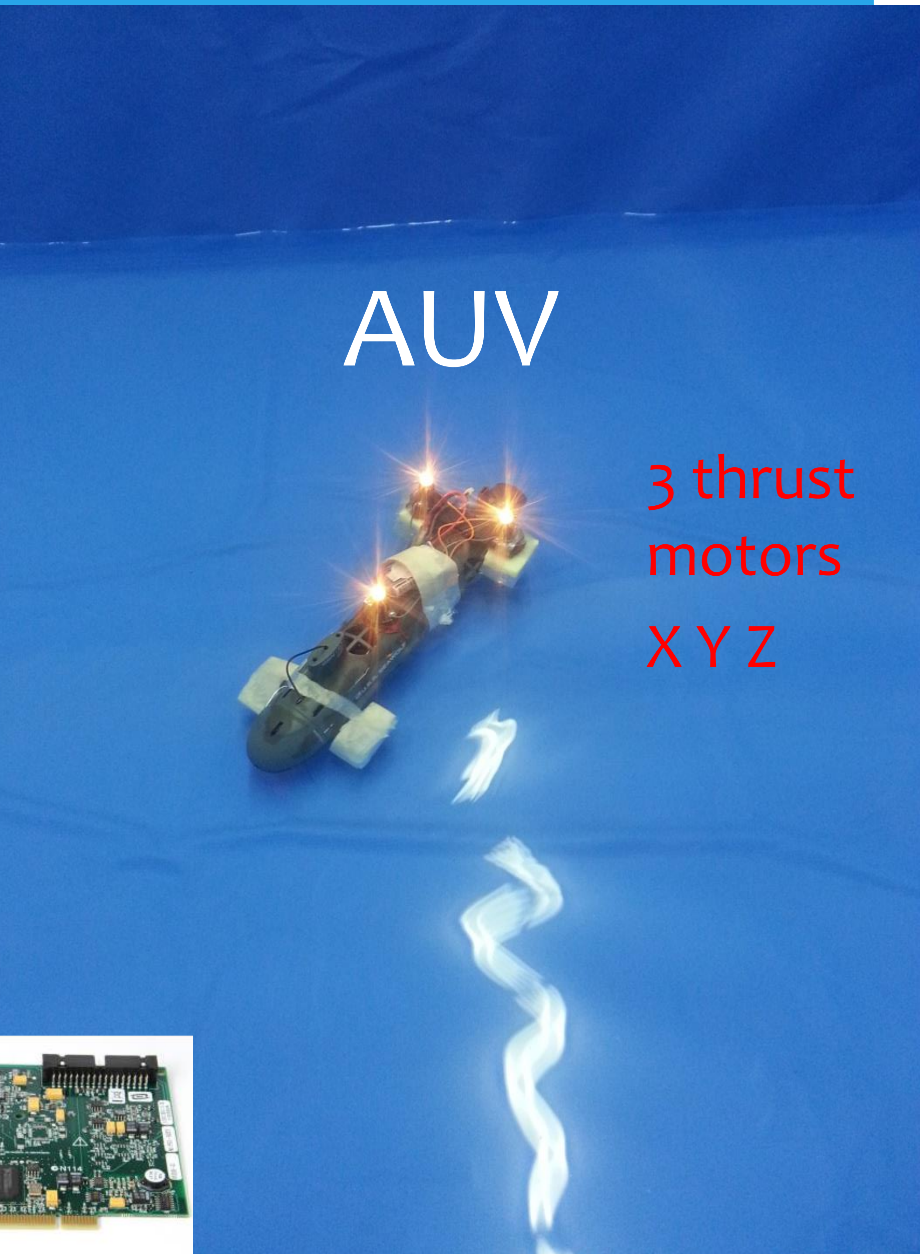
- ***Euler's equation of motion*** $\sum \boldsymbol{M_B} = \dot{\boldsymbol{H}}_{\boldsymbol{G}} + \boldsymbol{r_G} \mathrm{x} \boldsymbol{m a_G}$

$\dot{\boldsymbol{H}}_{\boldsymbol{G}} \triangleq$ rate of change of angular momentum about the center of gravity , $\sum \boldsymbol{M_B} \triangleq$ external moments

$$\dot{\boldsymbol{H}}_{\boldsymbol{G}} = [\boldsymbol{I}]\dot{\boldsymbol{\Omega}} + \boldsymbol{\Omega} \mathrm{x} [\boldsymbol{I}] \boldsymbol{\Omega}$$

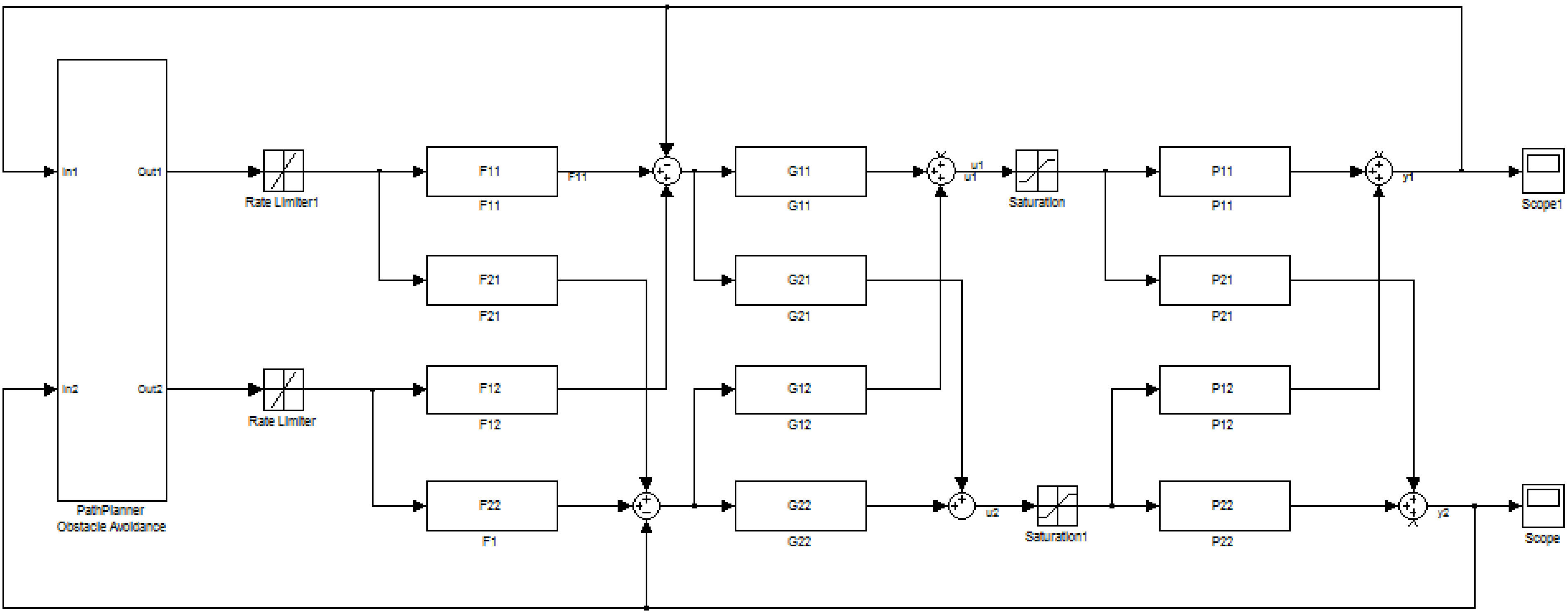
$[\boldsymbol{I}] \triangleq$ diagonal inertia matrix ($\mathrm{diag}\left[I_x \ I_y \ I_z\right]$)

Laboratory Setup



Control Architecture

- 2 Dof MIMO:
- Path planner (MPC)
- F is the prefilter, G is the controller, P is the plant.
- Additions:
  - rate limit on the position reference command
  - Saturation on the control signal



Modeling

- Modeling using Fourier integral method
- Execute on all variations of u and y:
  - From u1 to y1 and y2
  - From u2 to y2 and y1

$$\angle Plant(j\omega) = \delta$$

$$\left|Plant(j\omega)\right| = \frac{B}{A}$$

$$y_s = \frac{1}{nT}\int_0^{nT} y(t)\sin(\omega t)\,dt = \frac{1}{nT}\int_0^{nT}\left(B\sin(\omega t+\delta)+e(t)\right)\sin(\omega t)\,dt =$$

$$= \frac{1}{nT}\int_0^{nT} B\sin(\omega t+\delta)\sin(\omega t)\,dt + \frac{1}{nT}\int_0^{nT} e(t)\sin(\omega t)\,dt =$$

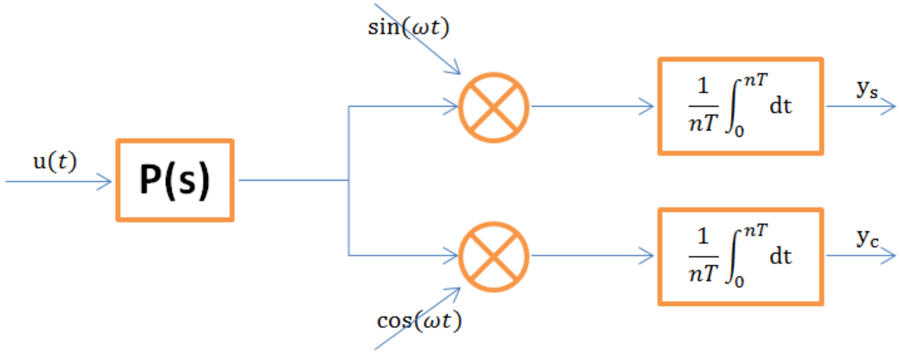
$$= \frac{B}{2}\cos(\delta) - \underbrace{\frac{B}{2nT}\int_0^{nT}\cos(\omega t+\delta)\,dt}_{=0} + \underbrace{\frac{1}{nT}\int_0^{nT} e(t)\sin(\omega t)\,dt}_{=0,\,uncorrelated\ signals}$$

$$y_s = \frac{B}{2}\cos(\delta)$$

$$y_c = \frac{B}{2}\sin(\delta)$$

$$\delta = \arctan 2(y_s, y_c)$$

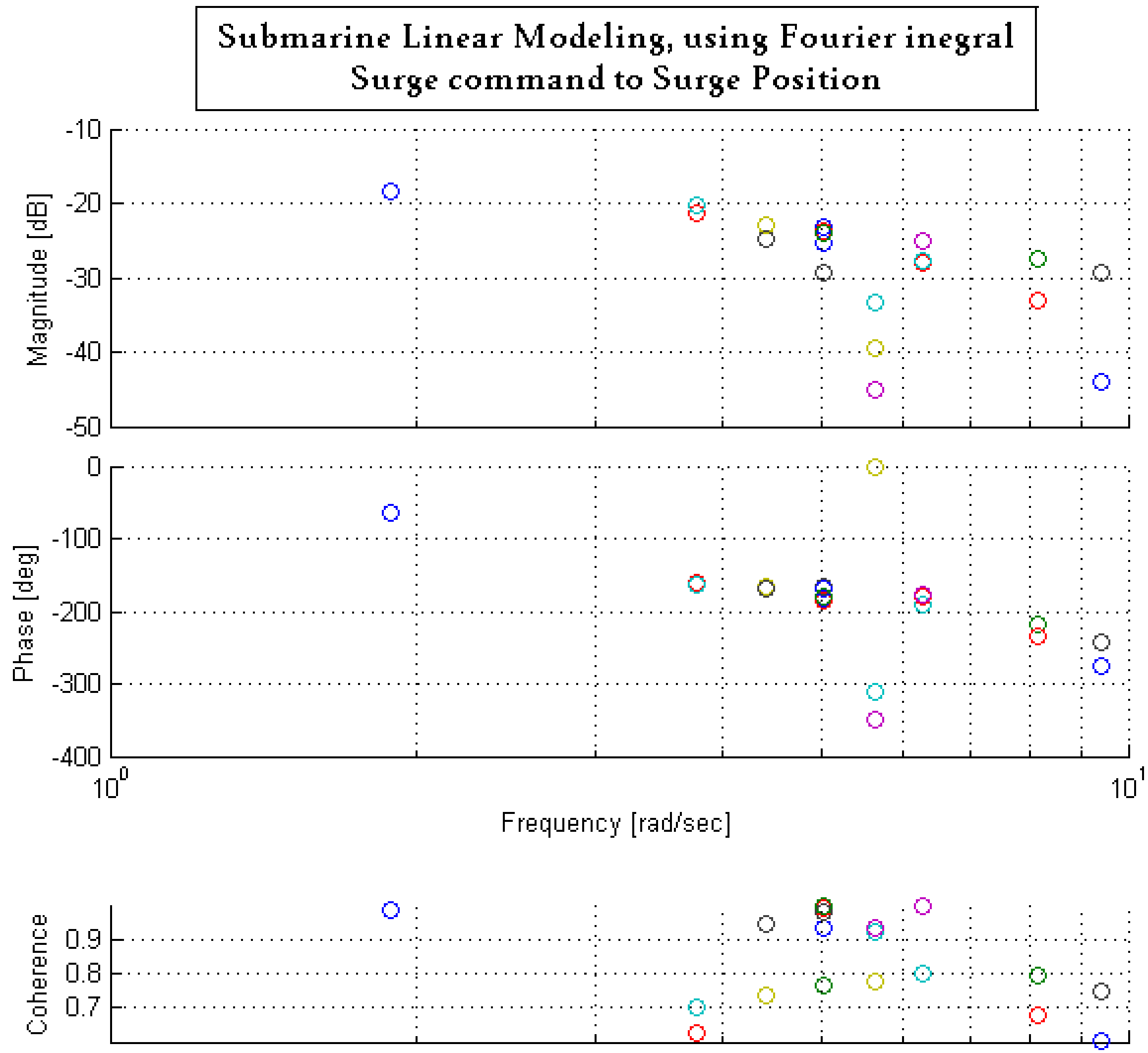
$$B = 2\sqrt{y_s^{\,2} + y_c^{\,2}}$$



Modeling

- Modeling results (1/4 of the plants)

- From Thrust motor input to Longitude position



Control Design

- Specifications:
  - stabilize the close loop
  - Step response
    - Settling time 10 [s]
    - Overshoot < 10%
  - Disturbance rejection
    - 6 [dB] for all frequencies
  - Cross coupling sensitivity
    - 3 [dB] for all frequencies

All time domain specifications translated into frequency domain using 2nd order transfer function formulations.

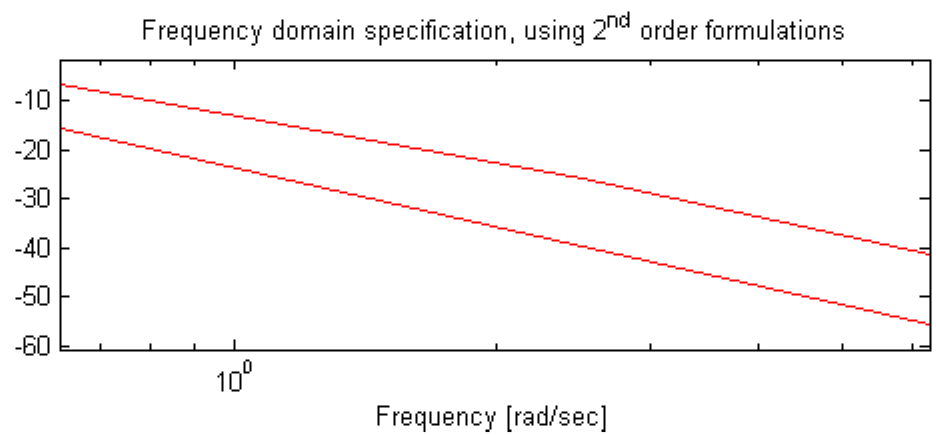
Control Design specifications

- Time domain specifications translated into frequency domain specifications using 2nd order formulations

$$G(s)=\frac{\omega_n}{s^2+2\zeta\omega_n s+\omega_n^2}$$

$$T_{setteling}=\frac{4}{\zeta\omega_n},\ for\ 2\%\ error$$

$$OS=1+e^{-\frac{\pi\zeta}{\sqrt{1-\zeta^2}}}$$

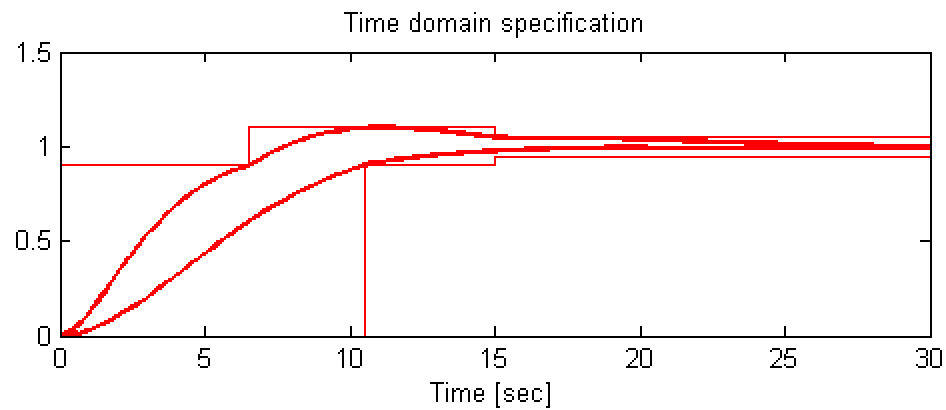


- Servo specification

$$a(\omega)\le\left|ClosedLoop\right|\le b(\omega)$$

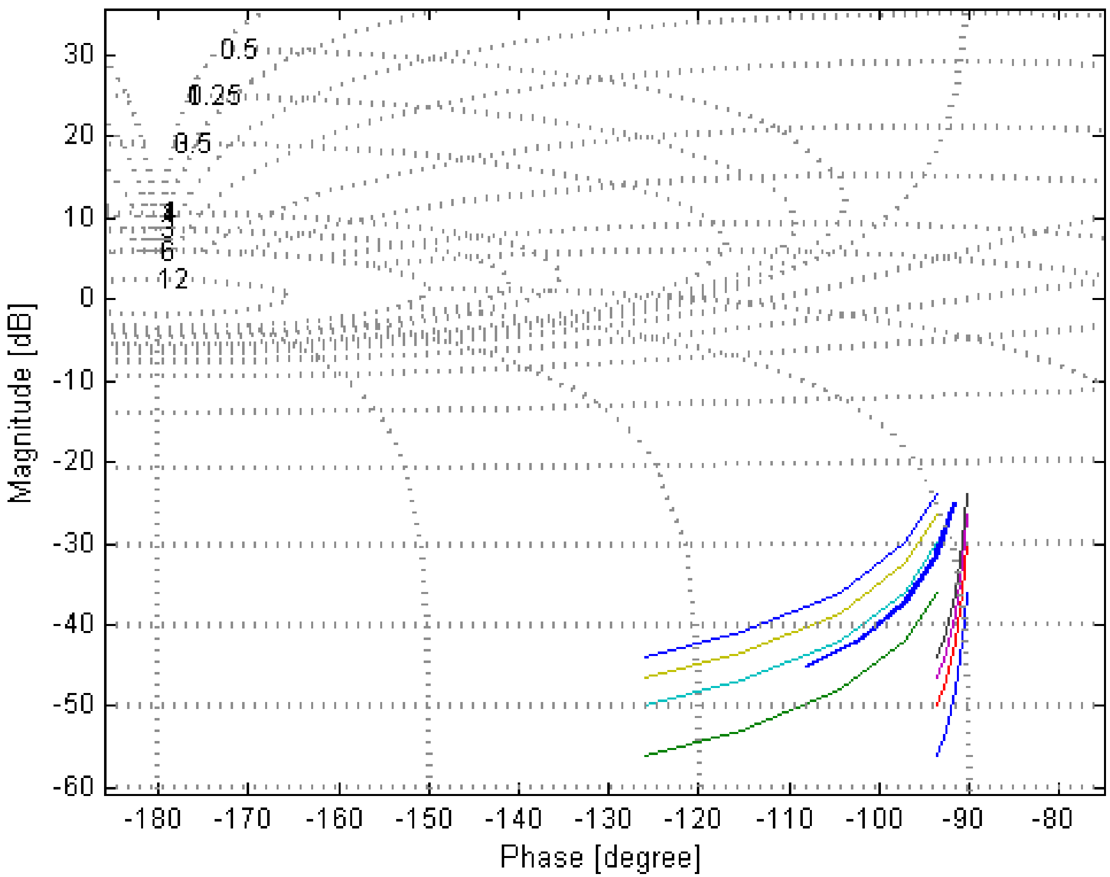
- Sensitivity specification

$$\left|sensitivity\right|\le c(\omega)$$



Control Design Cases

- Specifications must be satisfied for all plants (for all cases)
- The identified points from the modeling on Nichols chart
- All cases can be bounded



Control Design Template

- The chosen linear plant structure:

$$\frac{k_{11}}{s}e^{-\tau_{11}s}$$

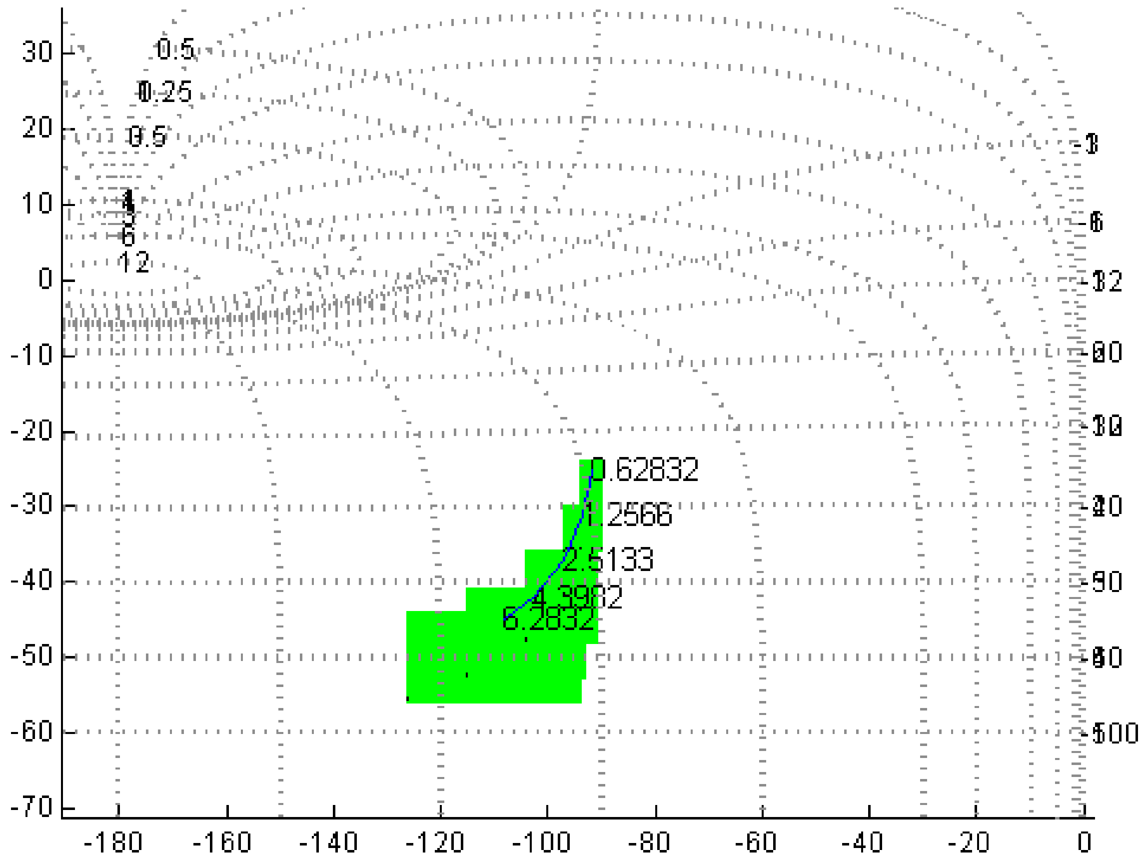
- The nominal plant is

$$\frac{0.035}{s}e^{-0.05s}$$

- The uncertainties are:

$$k_{11} \in [0.01, 0.04]$$

$$\tau_{11} \in [0.01, 0.1]$$



Control Design
Horowitz-Sidi bouds

- Tolerance bounds

$$\left|\frac{\bar{S}_{\max}}{\bar{S}_{\min}}\right| = \frac{\max_i \left|\frac{P_i G}{1+P_i G}\right|}{\min_i \left|\frac{P_i G}{1+P_i G}\right|} \le \frac{b}{a}$$

- Sensitivity bounds

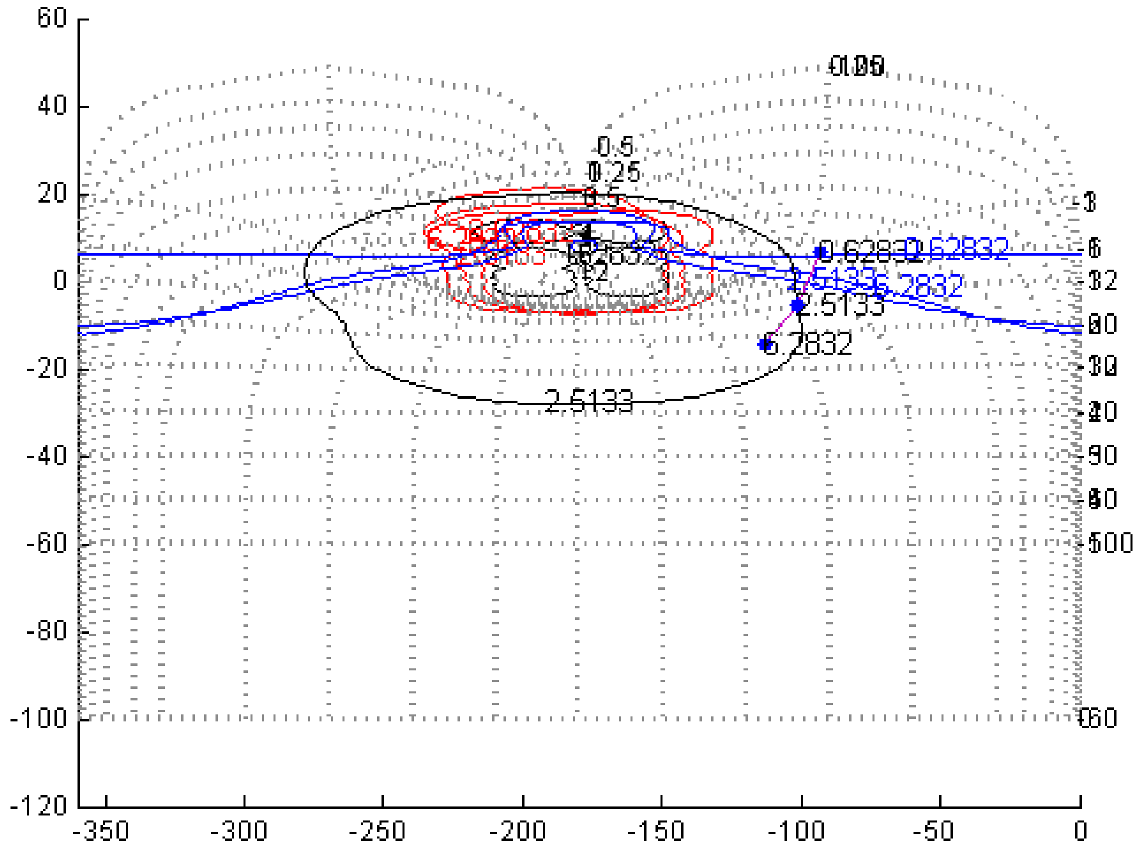
$$|S| = \left|\frac{1}{1+PG}\right| \le c$$

- Cross-Coupling bounds (only for MIMO)

$$\left|\frac{\frac{W_{12}}{W_{11}} b_{22}}{1+\frac{L_{10}}{W_{11}}}\right| \le b_{12}$$

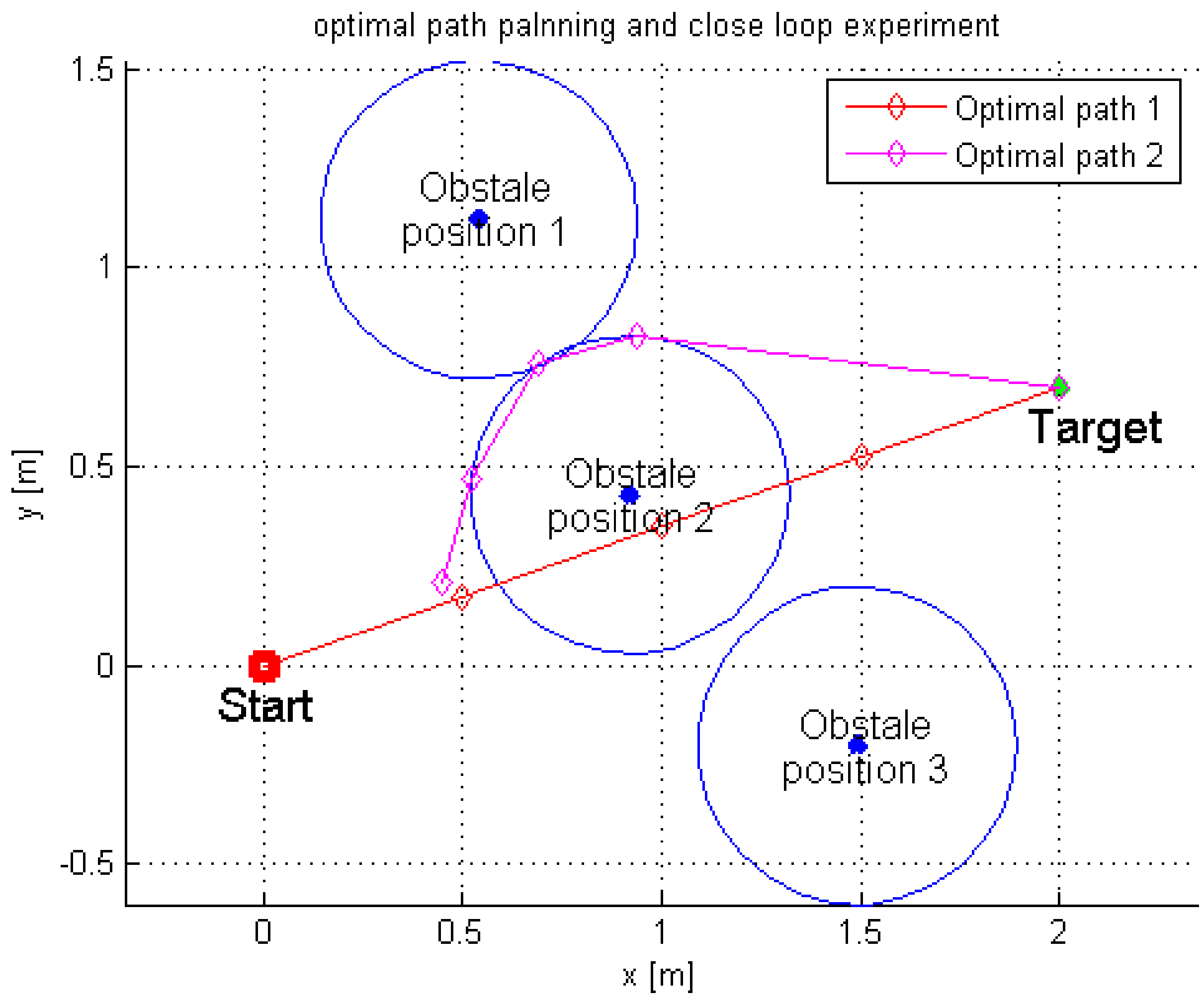
Control Design

- Loop Shaping with the Horowitz-Sidi bounds
- Performance
- Sensitivity
- Cross coupling



Experiments

- Optimal Path Planner, using DIDO solver
- The obstacle is moving
- Optimal Path 2 is the updated path



Experiments

- Reference command tracking
- In this example