Minimal Controllability of Conjunctive Boolean Networks

Eyal Weiss

Supervisor: Prof. Michael Margaliot

Joint work with: Prof. Guy Even

School of Electrical Engineering

Tel Aviv University, Israel

TEL AVIV UNIVERSITY אוניברסיטת תל אביב

Outline

- **Introduction:**
  - **Boolean Networks (BNs), and their role in Systems Biology**
- **Main part:**
  - **Conjunctive Boolean Control Networks (CBCNs): definition and representation**
  - **Minimal Controllability of CBCNs: necessary & sufficient conditions; hardness result**
- **Conclusions and further research**

Boolean Networks in Systems Biology

- **General formulation of a Boolean Network (BN):**

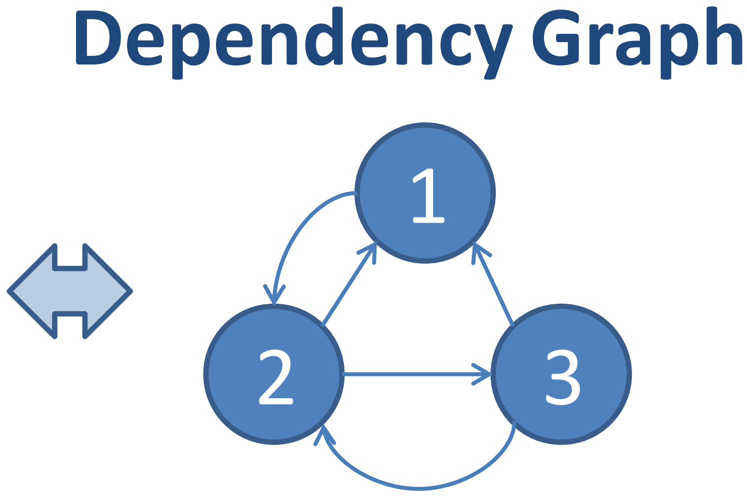
$$X_1(k+1) = f_1(X_1(k), \ldots, X_n(k))$$
$$\vdots$$
$$X_n(k+1) = f_n(X_1(k), \ldots, X_n(k))$$
$$f_1, \ldots, f_n \quad are\ Boolean\ functions$$

- **Genes are logically expressed => BNs are used as models of gene regulating networks**
- **Model of underlying structure for systems with non-linear dynamics**

Conjunctive Boolean Networks and Dependency Graph

- **Every f_i is a logical AND operator**
- **Example:**

$$X_1(k+1) = X_2(k)X_3(k)$$
$$X_2(k+1) = X_1(k)X_3(k)$$
$$X_3(k+1) = X_2(k)$$

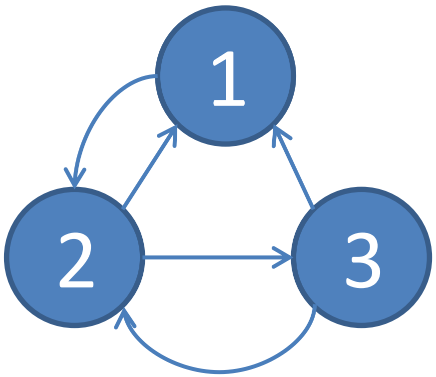


- **Dominant attribute:** $\mathrm{AND}(0, X_1, \ldots, X_k) = 0$

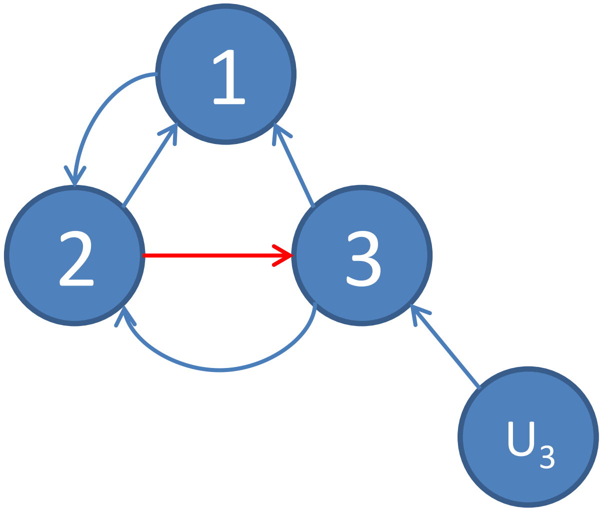
Conjunctive Boolean Control Networks

- **Same example:**

$$X_1(k+1) = X_2(k)X_3(k)$$
$$X_2(k+1) = X_1(k)X_3(k)$$
$$X_3(k+1) = X_2(k)$$



- **Add control input to** X_3



$$X_1(k+1) = X_2(k)X_3(k)$$
$$X_2(k+1) = X_1(k)X_3(k)$$
$$X_3(k+1) = U_3(k)$$

The Minimal Controllability Problem

- **Reminder: definition of Controllability**
- **Problem: given a CBN, determine a minimal number of control inputs to add, such that the resulting system is Controllable**
- **Practical implication - minimum resources**
- **Theoretical aspect – recognizing key nodes**

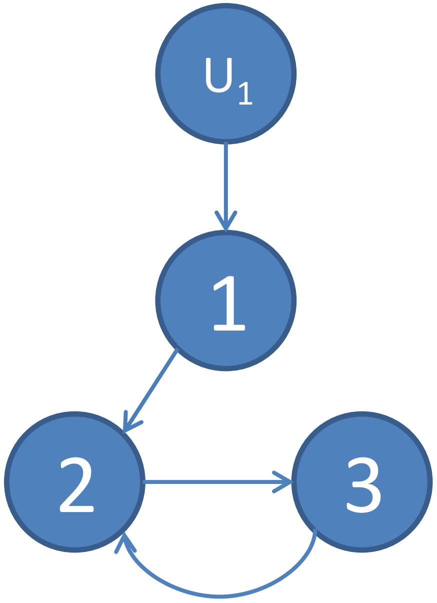
Main Results

1. **A necessary and sufficient condition for Controllability of a CBCN**
2. **An $O(n^2)$ algorithm for testing Controllability**
3. **A proof that the minimal Controllability problem for CBNs is NP-complete**

Controllability of CBCNs: Necessary Condition #1

- **The dependency graph is *acyclic***

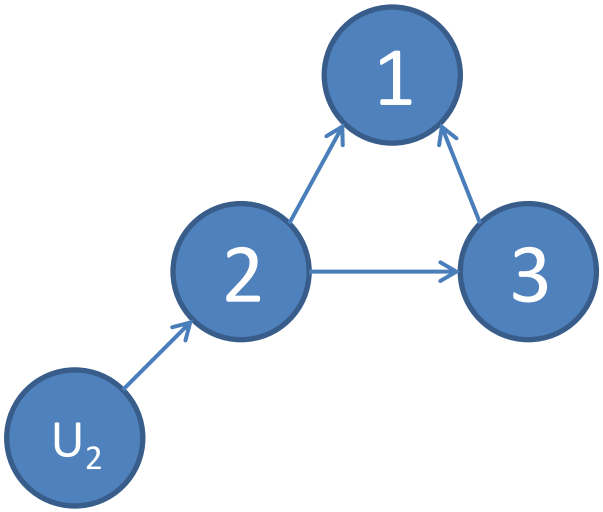
$$X_1(k+1) = U_1(k)$$
$$X_2(k+1) = X_1(k)X_3(k)$$
$$X_3(k+1) = X_2(k)$$



Controllability of CBCNs: Necessary Condition #2

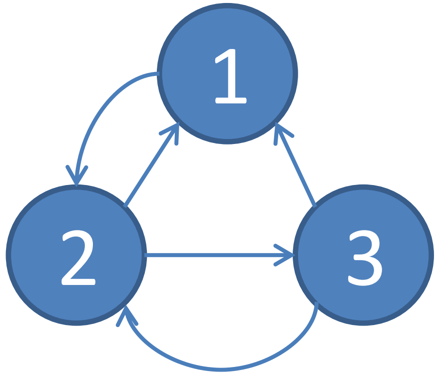
- **Every state-variable has a "special dedicated" input**

$$X_1(k+1) = X_2(k)X_3(k)$$
$$X_2(k+1) = U_2(k)$$
$$X_3(k+1) = X_2(k)$$

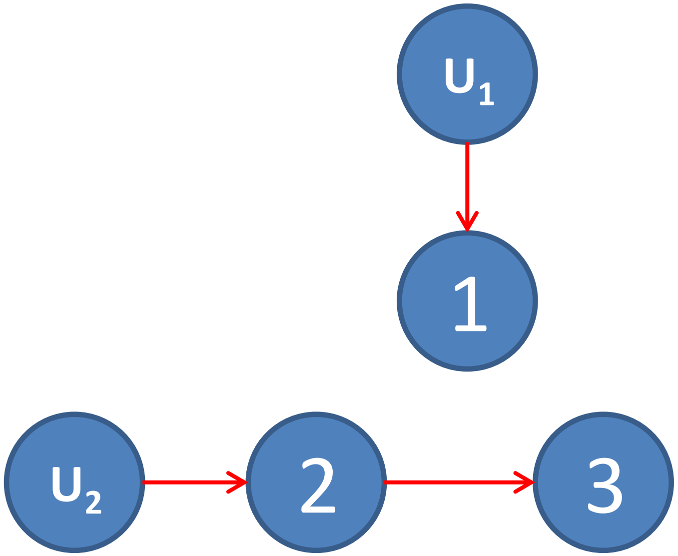


Controllability of CBCNs: Control Paths

$$X_1(k+1) = X_2(k)X_3(k)$$
$$X_2(k+1) = X_1(k)X_3(k)$$
$$X_3(k+1) = X_2(k)$$



$$X_1(k+1) = U_1(k)$$
$$X_2(k+1) = U_2(k)$$
$$X_3(k+1) = X_2(k)$$



Complexity Analysis

- The minimal Controllability problem for CBNs is NP-complete
- Proof based on reduction to minimum dominating set problem
- Implications on finding an exact solution

Conclusions

- **CBNs have a strong structure**
- **Novel method – Control Paths**
- **The minimal controllability problem is computationally hard, even for highly constrained BNs like CBNs**

Further Research

- **Finding approximate solutions to the problem presented**
- **Minimal Observability of CBNs**
- **Minimal Controllability of other special classes of BNs**