Auxiliary Command Input Shaping Technique to Reduce Disturbance Induced Vibration

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Outline

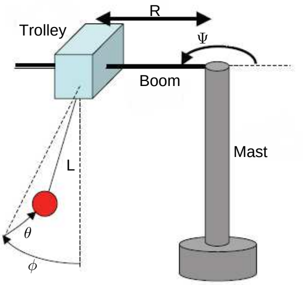
Motivation

- Flexible structures would be easier to control when vibrational modes are attenuated. Thus, there is motivation to combine a pre-filter and feedback.
- **Input Shaping**: shape the input in such a way that the vibration modes are not excited
- Traditional **input shaping** posses a weak point: Inability to suppress vibrations caused by external disturbances. Only command-incurred vibrations are attenuated
- Thus, we will focus on:

Applying input shaping approaches to flexible systems to reduce disturbance-incurred residual vibrations

Problem Definition

Consider a crane conveying a load through a rest-to-rest manoeuver. Once the manoeuver is accomplished, the load is dropped to its final location. Dropping the load incurs a sudden change in the external force induced by the load on the crane. The sudden change in the external force may be seen as an external, impulse-like, disturbance.

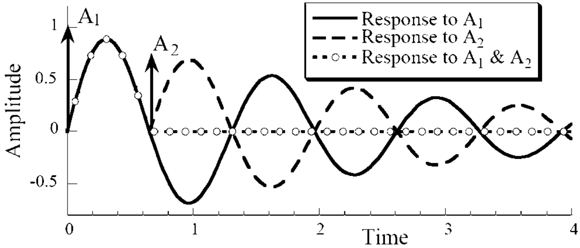


Introduction: Input Shaping

The term **Input Shaping**© (introduced by *Singer et al, 1990*):

The operation of convolving a desired input command with an impulse sequence

1. Applying an impulse, A_1, will cause system to vibrate
2. Applying second impulse, A_2, at a later time cancels vibration
3. The second impulse must be applied at the correct time and must have the appropriate magnitude for complete cancelation



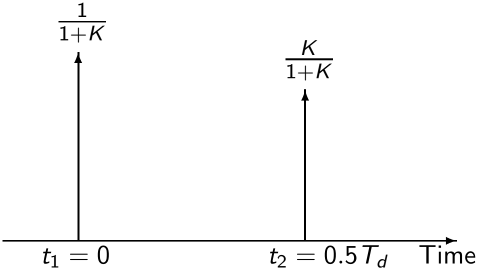
Introduction: Input Shaping - Cont.

The amplitudes and time instances of the impulses in an input shaper are determined by solving a set of constraint equations. A variety of constraints are used:

- Robustness constraints (*Singer et al. 1990*)
- Residual vibration constraints (*Singhose et al. 1996, 1997*)
- Impulse amplitude constraints (*Singhose et al. 1994*)
- Optimal time requirement (*Pao & Singhose, 1995*)

Introduction: Input Shaping - Cont.

For example, the ZV shaper (*Singer et al, 1990*), for 2^{nd} Ord. linear system, with damping ratio ζ, and damped natural frequency, $\omega_d = \omega_n\sqrt{1-\zeta^2}$:



$K = e^{\frac{-\zeta\pi}{\sqrt{1-\zeta^2}}}$

$T_d = \frac{2\pi}{\omega_d}$ - Damped period, in [*sec*]

Introduction: Input Shaping - Cont.

In the ZV-shaper, t_2 depends on T_d, and cannot be shortened arbitrarily. This is a major weakness of the ZV-shaper.

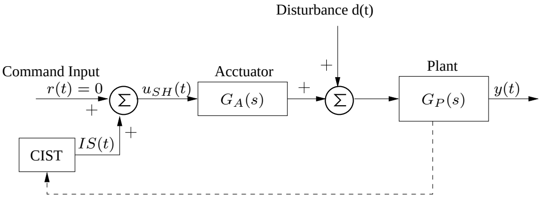
- A limited attempt to overcome this drawback was presented by *Singhose et al. (1994)*, where negative impulses are allowed in the filter. Yet, impulse instances still depends on T_d.
- A different approach is taken by *Magee & Book (1998)* where the *Optimal Arbitrary Time-delay Filter* (OATF) is introduced:

$$IS_{OATF}(t) = \sum_{j=1}^{3} A_j \delta(t - (j-1) \cdot \Delta) \quad (1)$$

$\delta(t)$ - Dirac delta function
Δ - Shaper time delay, arbitrarily chosen

Introduction: Input Shaping - Cont.

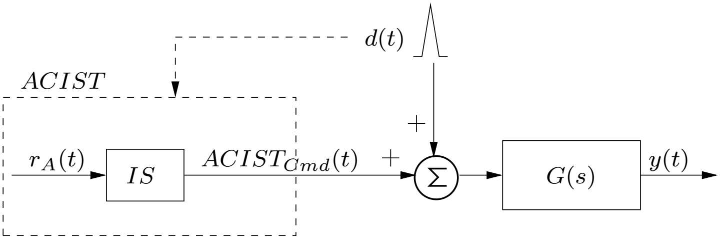
Chang et al (2004) introduce the **CIST** - Commandless Input Shaping Technique:



- When a disturbance is induced, a set of two impulse is directly applied to the system via an actuator
- An analytic solution for the shaper parameters for different actuator dynamics and 2 types of disturbances was proposed
- Yet, dynamic systems can not be controlled by direct impulses, thus, **CIST** seems impractical !!.

ACIST - Concept

Consider:



$y(t)$ - Output
$r_A(t)$ - Auxiliary reference command
IS - Impulse set which convolves $r_A(t)$
The disturbance:

$$d(t) = A_d\delta(t - t_d) \tag{2}$$

We seek to bring the total response, $y(t)$, to zero after some finite time $t \geq t_n$ by a well designed **ACIST**

Simple **ACIST**

Assume:

$$IS(t) = A_1\delta(t - t_1) + A_2\delta(t - t_2 \tag{3}$$
$$r_A(t) = R_A \cdot \mathbf{1}(t_r)$$

The output, $y(t)$, is

$$y(t) = R_A(A_1 + A_2) + R_A e^{-\zeta\omega_n t}\sqrt{C^2 + S^2}\sin\left(\omega_d t + \psi\right) \tag{4}$$

where ψ is a phase shift and (after setting $A_2 = -A_1$)

$$C = \frac{\frac{A_d\omega_n}{R}}{\sqrt{1-\zeta^2}} + \frac{\frac{A_1\zeta}{\sqrt{1-\zeta^2}}}{2\zeta^2 - 1} - \frac{A_1 e^{\omega_n\zeta t_2}}{2\zeta^2 - 1}\left[\cos\left(\frac{\pi}{2} - \omega_d t_2\right) + \frac{\zeta\cos(\omega_d t_2)}{\sqrt{1-\zeta^2}}\right]$$
$$S = \frac{A_1}{2\zeta^2 - 1} - \frac{A_1 e^{\omega_n\zeta t_2}}{2\zeta^2 - 1}\left[\sin\left(\frac{\pi}{2} - \omega_d t_2\right) + \frac{\zeta\sin(\omega_d t_2)}{\sqrt{1-\zeta^2}}\right] \tag{5}$$

Simple **ACIST** - Cont.

For zero vibration we set $C = 0$ and $S = 0$, which yields:

$$\frac{A_d\omega_n(2\zeta^2 - 1)}{R_A A_1\sqrt{1-\zeta^2}} + \frac{\zeta}{\sqrt{1-\zeta^2}} = e^{\omega_n\zeta t_2}\left[\sin\left(\omega_d t_2\right) + \frac{\zeta\cos(\omega_d t_2)}{\sqrt{1-\zeta^2}}\right] \quad (6)$$

$$1 = e^{\omega_n\zeta t_2}\left[\cos(\omega_d t_2) - \frac{\zeta\sin(\omega_d t_2)}{\sqrt{1-\zeta^2}}\right] \quad (7)$$

For $\zeta = 0$ we get

$$t_2 = \frac{n\pi}{\omega_d}, \qquad n = 0, 2, 4, ..., \infty \quad (8)$$

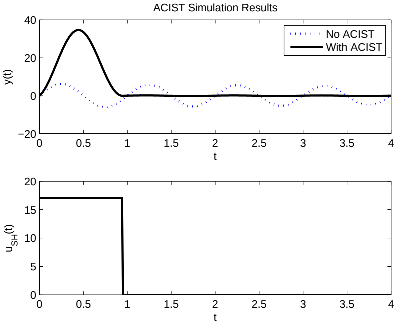
$$A_1 \to \infty \quad (9)$$

For $\zeta \neq 0$ we solve numerically to get t_2, and

$$A_1 = \frac{A_d\omega_n(3\zeta^2 - 2\zeta^4 - 1)e^{-\omega_n\zeta t_2}}{R\sqrt{1-\zeta^2}\sin(\omega_d t_2)} \quad (10)$$

Simple **ACIST** - Cont.

Solution for the damped case via simulation



The simple **ACIST** reduces residual vibration drastically for $t > t_2$. However, the magnitude of the transient response is much higher than the response without **ACIST** !!

ACIST with OATF

Enhance the **ACIST** by using the 3-impulse OATF

$$IS_{OATF}(t)=\sum_{j=1}^{3} A_j\delta(t-(j-1)\cdot\Delta) \tag{11}$$

The total response is

$$y(t)=R_A(A_1+A_2+A_3)+R_A e^{-\zeta\omega_n t}\sqrt{C^2+S^2}\sin(\omega_d t+\psi) \tag{12}$$

where

$$\begin{aligned}
C=&\frac{A_d\omega_n}{R_A\sqrt{1-\zeta^2}}+\frac{\frac{A_1\zeta}{\sqrt{1-\zeta^2}}}{2\zeta^2-1}+\frac{A_2e^{\omega_n\zeta\Delta}}{2\zeta^2-1}\left[\sin(\omega_d\Delta)+\frac{\zeta\cos(\omega_d\Delta)}{\sqrt{1-\zeta^2}}\right]+\\
&+\frac{A_3e^{2\omega_n\zeta\Delta}}{2\zeta^2-1}\left[\sin(2\omega_d\Delta)+\frac{\zeta\cos(2\omega_d\Delta)}{\sqrt{1-\zeta^2}}\right]\\
S=&\frac{A_1}{2\zeta^2-1}+\frac{A_2e^{\omega_n\zeta\Delta}}{2\zeta^2-1}\left[\cos(\omega_d\Delta)-\frac{\zeta\sin(\omega_d\Delta)}{\sqrt{1-\zeta^2}}\right]+\\
&+\frac{A_3e^{2\omega_n\zeta\Delta}}{2\zeta^2-1}\left[\cos(2\omega_d\Delta)-\frac{\zeta\sin(2\omega_d\Delta)}{\sqrt{1-\zeta^2}}\right]
\end{aligned} \tag{13}$$

ACIST with OATF - Cont.

For zero vibration we set

$$
\begin{aligned}
0 &= A_1 + A_2 + A_3 \\
C &= 0 \\
S &= 0
\end{aligned}
\tag{14}
$$

which yields:

$$A_3 = -\mathbb{K} K_\zeta \frac{e^{-\omega_n \zeta \Delta} - \cos(\omega_d \Delta) + \grave{\zeta} \sin(\omega_d \Delta)}{2\cos(\omega_d \Delta) - (e^{\omega_n \zeta \Delta} + e^{-\omega_n \zeta \Delta})} \tag{15}$$

$$A_2 = -A_3 \frac{1 - e^{2\omega_n \zeta \Delta}\left(\cos(2\omega_d \Delta) - \grave{\zeta} \sin(2\omega_d \Delta)\right)}{1 - e^{\omega_n \zeta \Delta}\left(\cos(\omega_d \Delta) - \grave{\zeta} \sin(\omega_d \Delta)\right)} \tag{16}$$

$$A_1 = -A_2 - A_3 \tag{17}$$

where: $\mathbb{K} = \frac{A_d \omega_n}{2R_A \sin(\omega_n \Delta)}$, $K_\zeta = \frac{2\zeta^2 - 1}{\sqrt{1-\zeta^2}}$, $\grave{\zeta} = \frac{\zeta}{\sqrt{1-\zeta^2}}$

ACIST via Optimization

Same results can be obtained by the following: Consider the dynamic state-space equations:

$$\dot{\mathbf{x}}(t) = \overbrace{\begin{bmatrix} 0 & 1 \\ -\omega_n^2 & -2\zeta\omega_n \end{bmatrix}}^{\Gamma} \mathbf{x}(t) + \underbrace{\begin{bmatrix} 0 \\ \omega_n^2 \end{bmatrix}}_{B} u(t)$$

$$y(t) = \underbrace{\begin{bmatrix} 1 & 0 \end{bmatrix}}_{C} \mathbf{x}(t) \tag{18}$$

The solution $x(t)$ is

$$\mathbf{x}(t) = \phi(t,0)\mathbf{x}(0) + R_A \int_0^t \phi(t,\tau) B \sum_{j=1}^{3} A_j \delta(t-(j-1)\Delta) d\tau \tag{19}$$

$\phi(t,\tau) = e^{\Gamma(t-\tau)}$ - state transition matrix from τ to t.

ACIST via Optimization - Cont.

Solving the integral in (19) for $t \geq 2\Delta$ gives

$$\mathbf{x}(t) = \phi(t,0)\mathbf{x}(0) + \Psi(t) f_A \quad \forall t \geq 2\Delta \tag{20}$$

where

$$\Psi(t) = \left[\Phi_{[2\Delta]}(t) - \Phi_{[0]}(t), \Phi_{[2\Delta]}(t) - \Phi_{[\Delta]}(t)\right] \tag{21}$$

$$f_A = R_A[A_1, A_2]^T \tag{22}$$

and

$$\Phi_{[t_k]}(t) = e^{-\zeta\omega_n(t-t_k)} \begin{bmatrix} \grave{\zeta} \sin(\omega_d(t-t_k)) + \cos(\omega_d(t-t_k)) \\ -\frac{\omega_d}{1-\zeta^2} \sin(\omega_d(t-t_k)) \end{bmatrix}, \quad t_k = 0, \Delta, 2\Delta \tag{23}$$

ACIST via Optimization - Cont.

For zero residual vibration at $t \geq 2\Delta$ we set $\mathbf{x}(2\Delta) = [0,0]^T$.
Minimize

$$J = \frac{1}{2}\mathbf{x}(2\Delta)^T W \mathbf{x}(2\Delta) \tag{24}$$

with W some weighting matrix. Substituting (20) into (24) and solving for $t = 2\Delta$

$$\begin{aligned} J = {} & \tfrac{1}{2}(\phi(2\Delta,0)\mathbf{x}(0))^T W \phi(2\Delta,0)\mathbf{x}(0) + \\ & + \tfrac{1}{2}(\Psi(2\Delta)f_A)^T W \Psi(2\Delta)f_A + (\phi(2\Delta,0)\mathbf{x}(0))^T W \Psi(2\Delta)f_A \end{aligned} \tag{25}$$

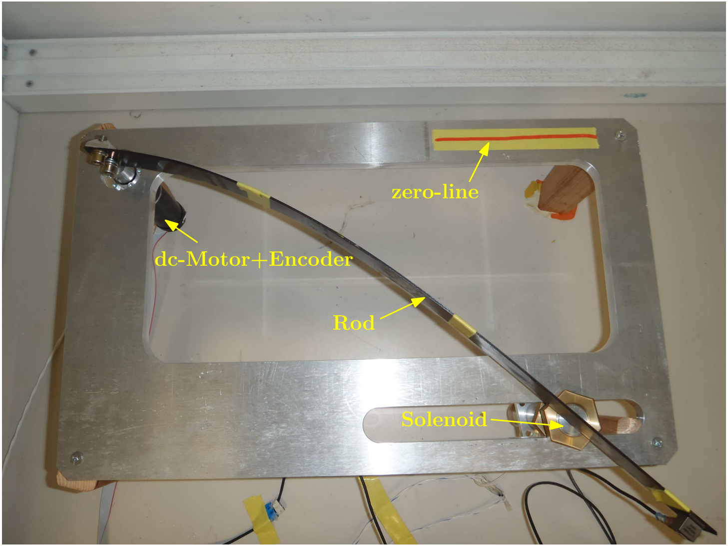
Differentiating (25) with respect to f_A and equating to zero:

$$f_A = -\left[\Psi(2\Delta)^T W \Psi(2\Delta)\right]^{-1}\left[(\phi(2\Delta,0)\mathbf{x}(0))^T W \Psi(2\Delta)\right] \tag{26}$$

A direct solution for (26) with $W = I$ will give the same results as in (15) and (16). Furthermore, it can be shown that the second optimality condition $\frac{\partial^2 J}{\partial f_A^2}$ is fulfilled and $\Psi(2\Delta)^T W \Psi(2\Delta)$ is invertible when $\sin(\omega_n \Delta) \neq 0$.

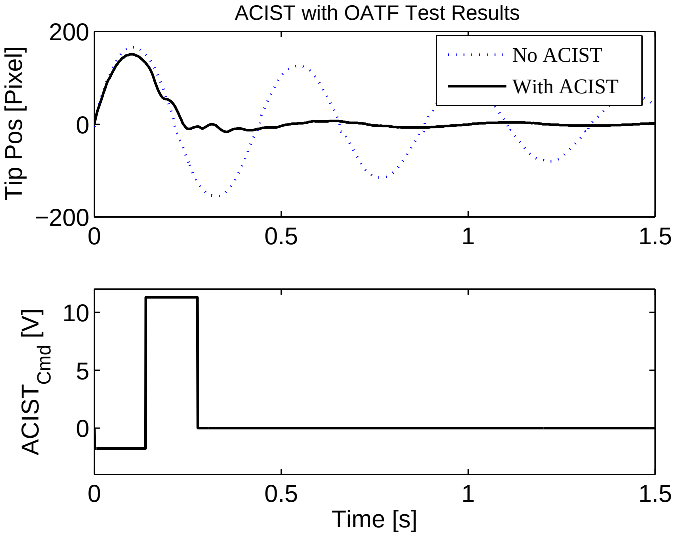
Test Results

To validate the results we used a simple test-bed



Test Results - Cont.

Test results of the **ACIST** with *OATF* shaper



Conclusions

- A technique to reduce the residual vibration of a flexible system was presented
- The shaper parameters are obtained via algebraic solution based on the disturbance and system parameters
- It was shown that the shaper is optimal in terms of residual vibration
- Hardware experiments verified the technique performance