Ariel University of Samaria

Department of Electrical and
Electronic Engineering

Proportional Navigation

Modern Investigation

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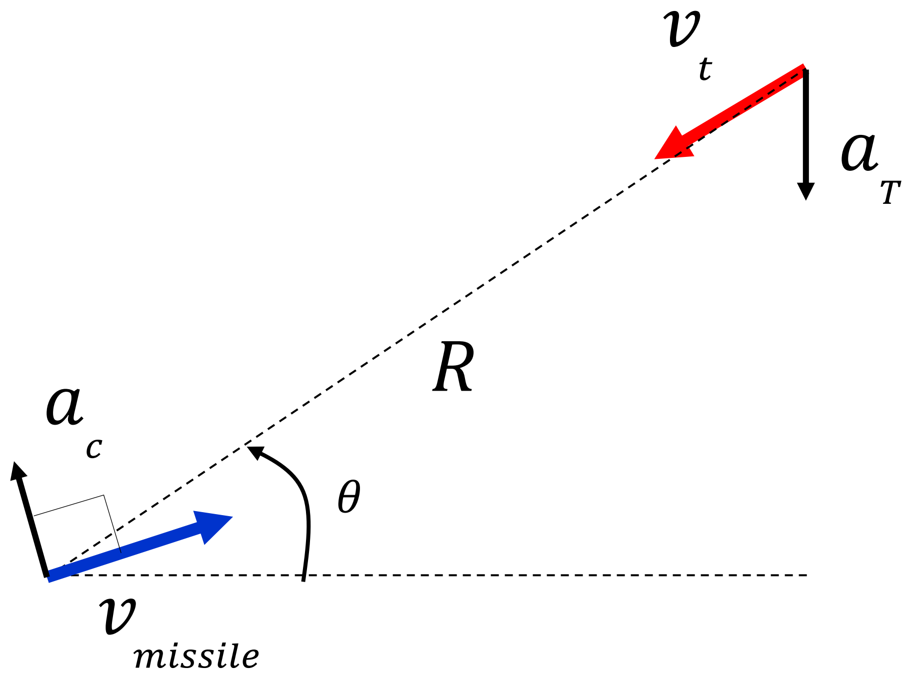
Methodology

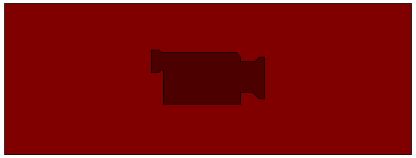
- Proportional Navigation
- Mechanization Effect
- Generalized Lyapunov Investigation
- Results
- Summary

Proportional Navigation

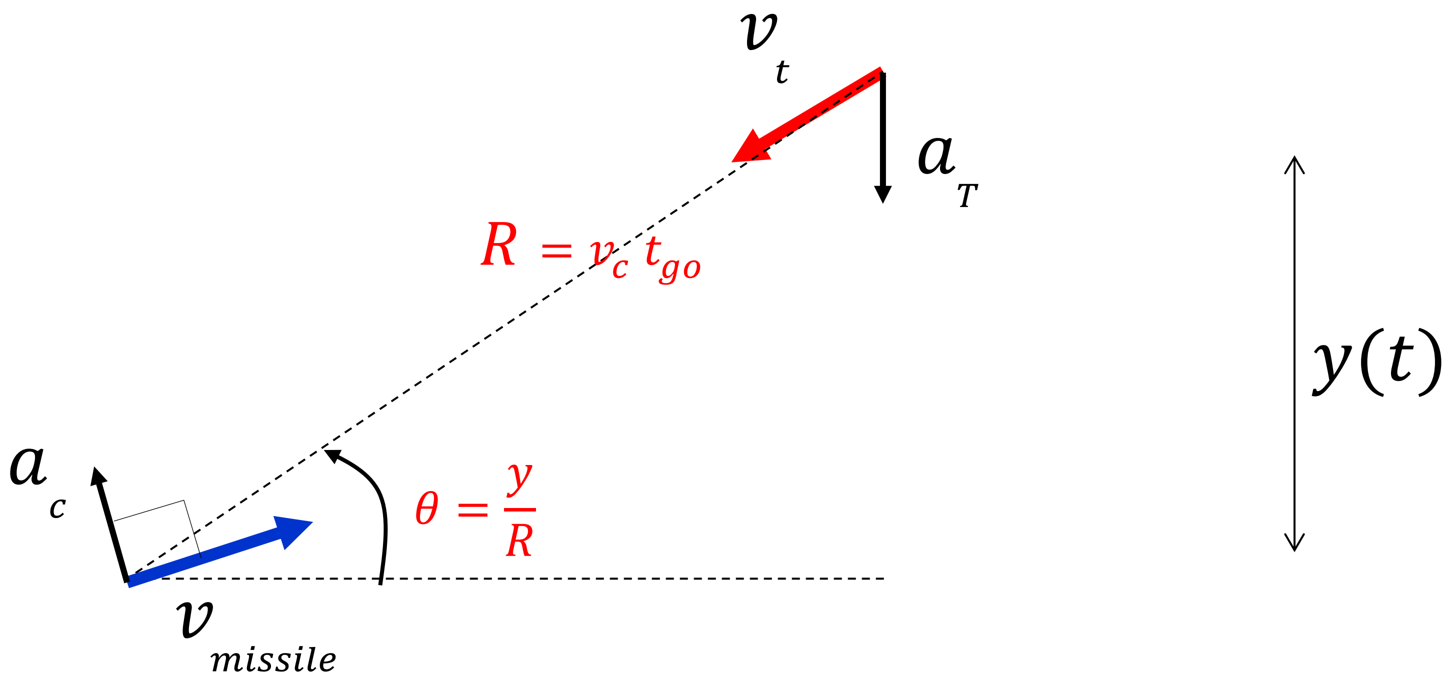


$$R\ddot{\theta} + 2\dot{R}\dot{\theta} = -a_m + a_T$$

Guidance Command $\quad a_c(t) = v_c\, N\, \dot{\theta}(t)$



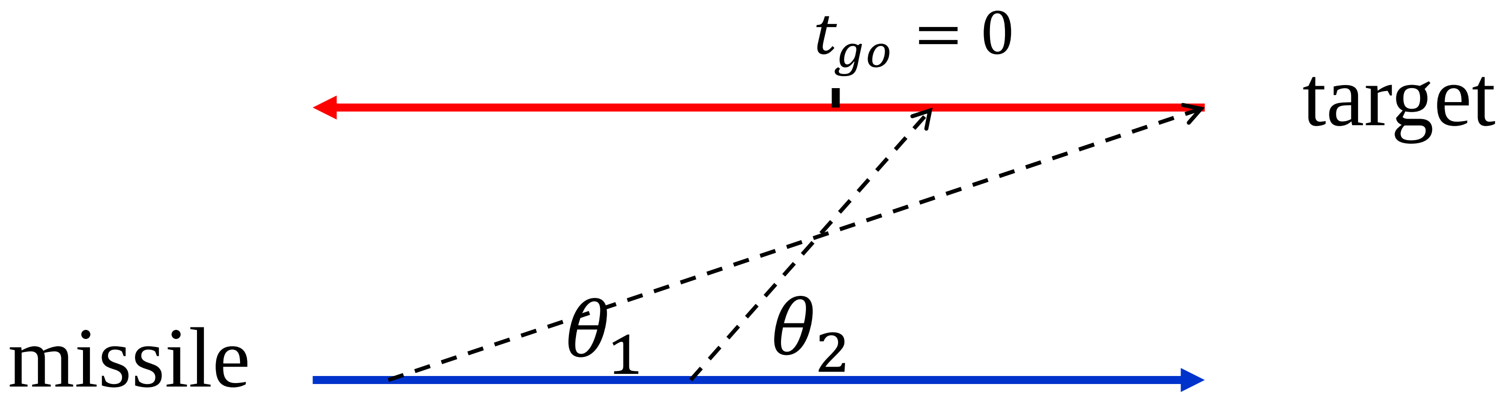
Linearization



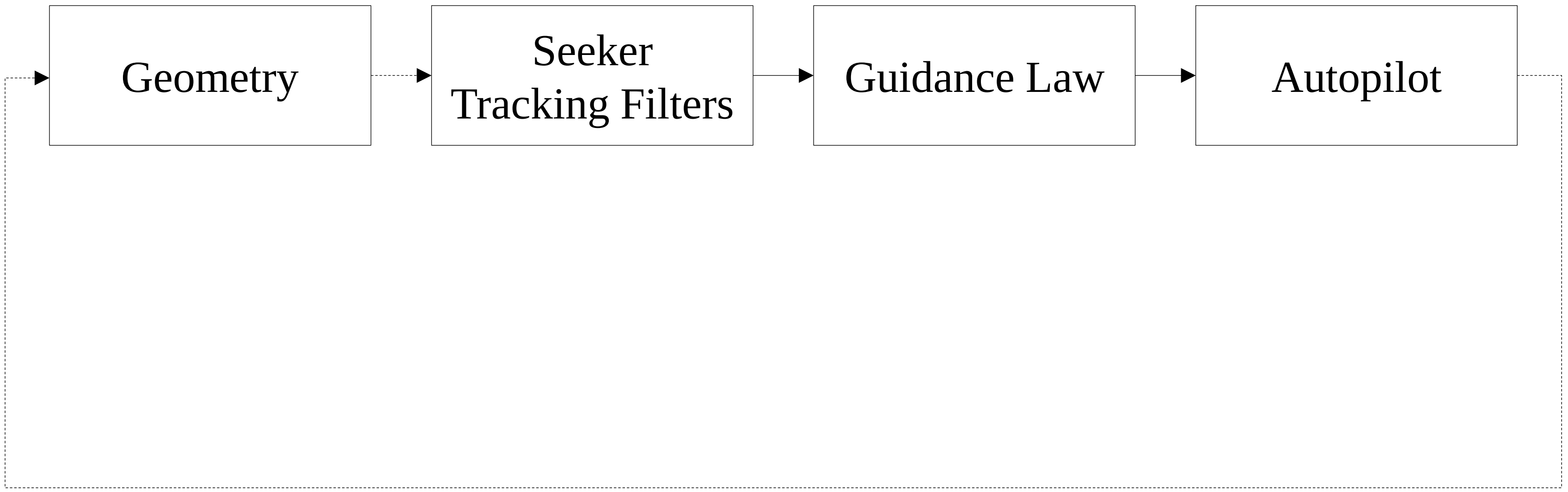
$$a_c = N \frac{y + \dot{y}\, t_{go}}{t_{go}^2}$$

$t_{go} = t_f - t$

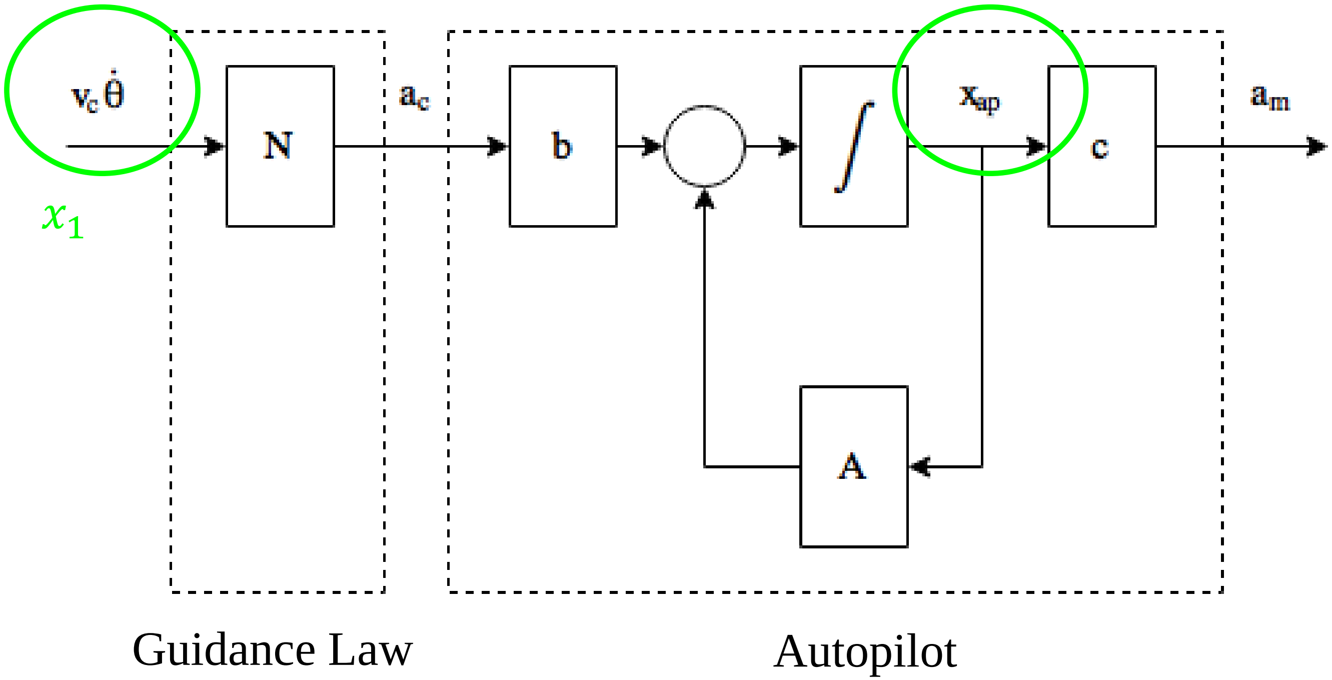
Flyby



Guidance Loop



Guidance Loop

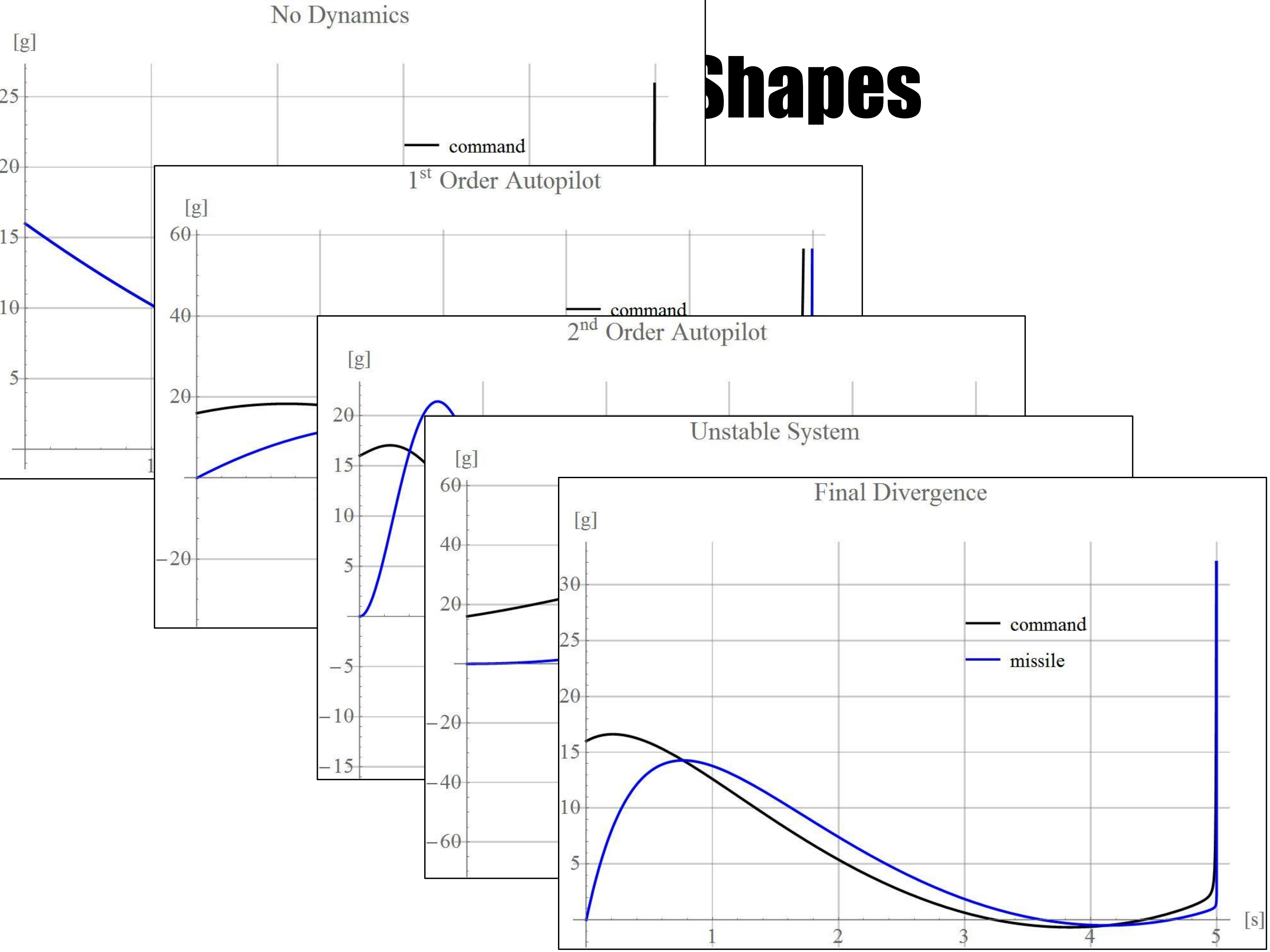


State Space model

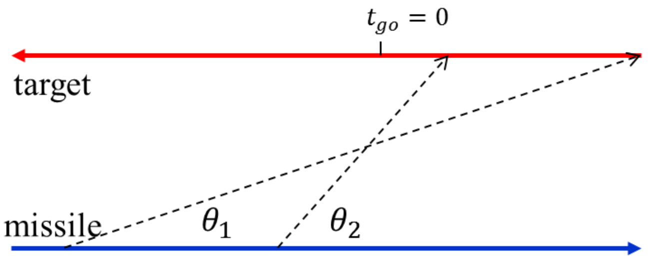
$$\dot{x}_1 = \frac{2}{t_{go}}\, x_1 - \frac{c}{t_{go}}\, x_{ap}$$

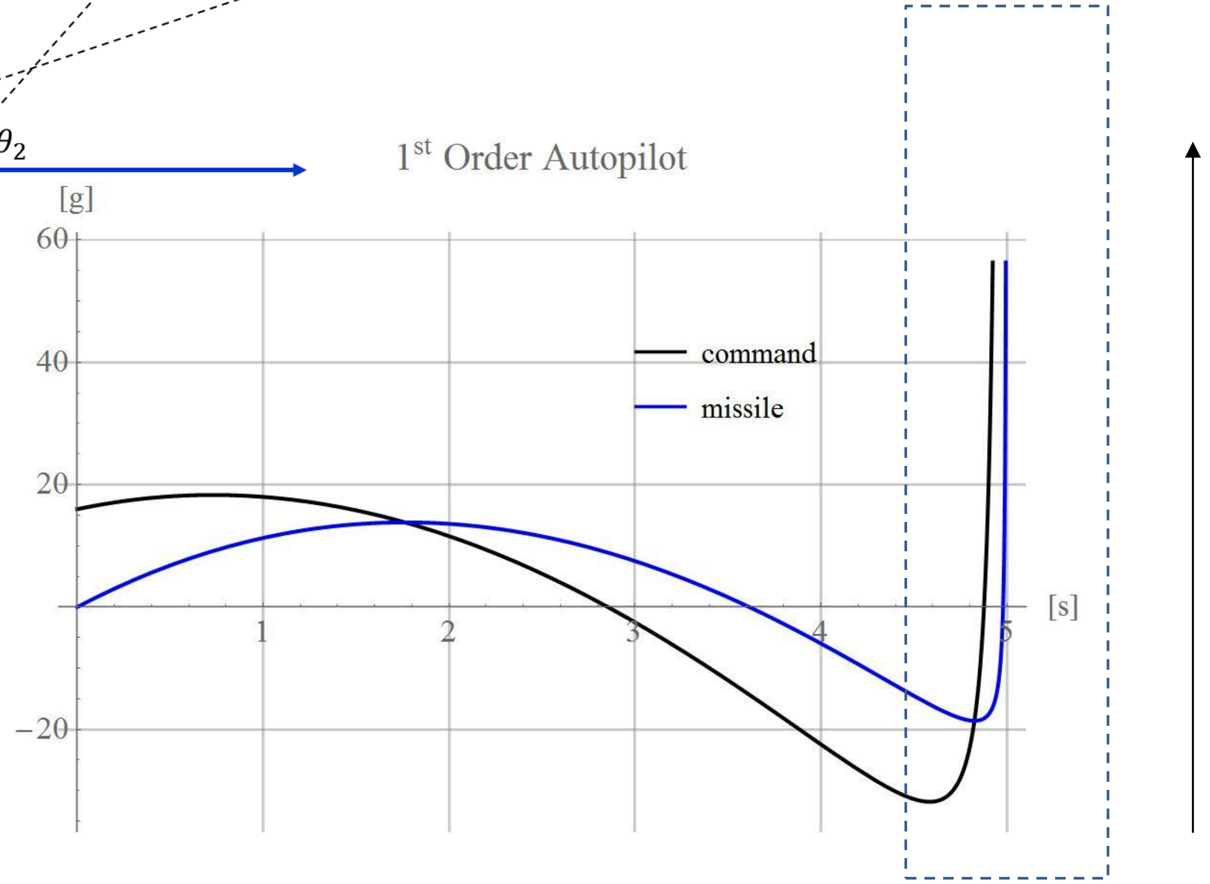
$$\dot{x}_{ap} = b\, N\, x_1 + A\, x_{ap}$$

Shapes


No Dynamics
[g]
command
1st Order Autopilot
[g]
command
2nd Order Autopilot
[g]
Unstable System
[g]
Final Divergence
[g]
command
missile
[s]

Final Divergence





$$a_c = N \frac{y + \dot{y}\, t_{go}}{t_{go}^2}$$

Analysis Method

$$\dot{x}_1 = \frac{2}{t_f - t}\, x_1 - \frac{c}{t_f - t}\, x_{ap}$$

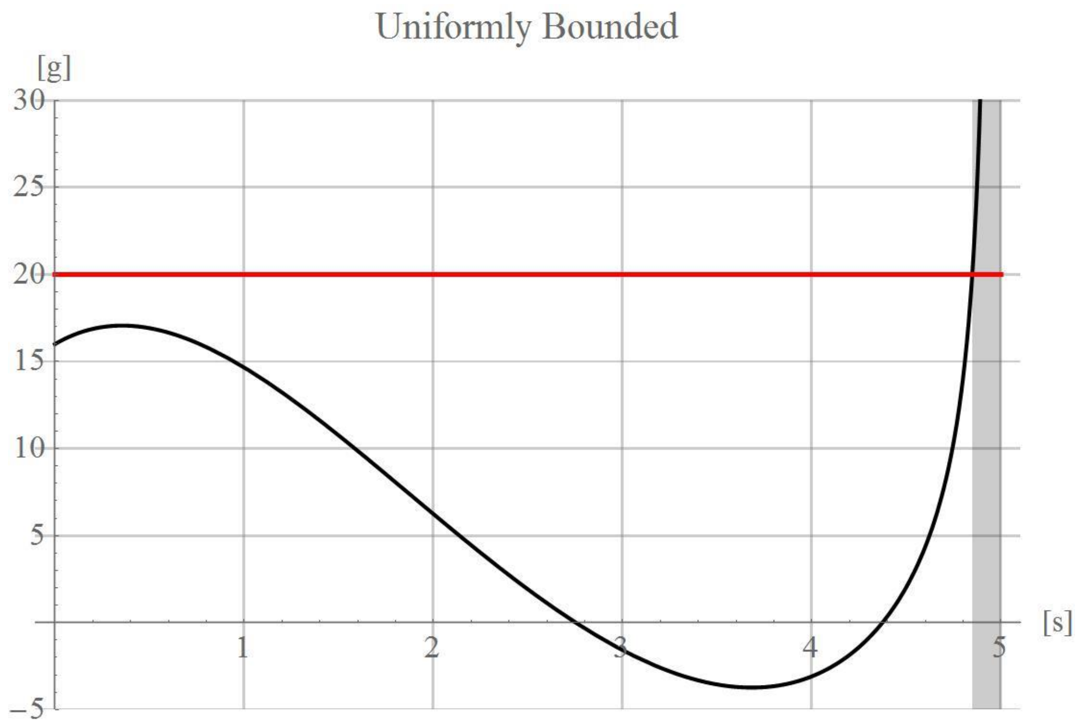
$$\dot{x}_{ap} = b\, N\, x_1 + A\, x_{ap}$$

- Stability Conditions??
  - ~~LTI theory~~
  - ~~Absolute Stability~~
  - Lyapunov Generalization

Definitions

- **Uniformly bounded stability**

$$\|x(t)\| \leq \alpha \ \|x_0\| \qquad 0 \leq t \leq t_f - \tau_1$$

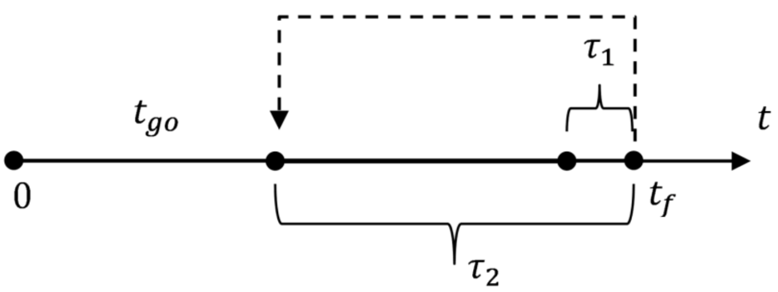


Definitions

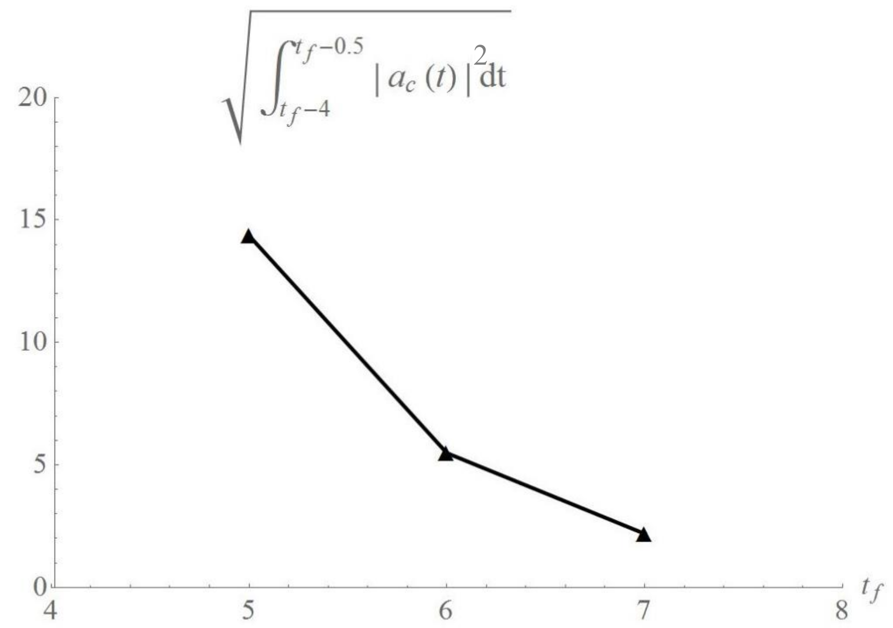
- **Asymptotic stability**

$$\|x(t)\| \to 0 \;\; for \;\; t_f \to \infty$$

$$\tau_1 \leq t_{go} \leq \tau_2$$



singular point t_f



Lyapunov Function

- **Define the Lyapunov Function**

$$v\big(z(t)\big) = x_1^2 + z_{ap}^T\, H\, z_{ap}^T$$

- **Seek for**

$$\frac{\mathrm{d}}{dt} v\big(z(t)\big) \le \left(-\frac{\alpha_1}{t_{go}} + \frac{\alpha_2}{t_{go}^2}\right) \cdot v\big(z(t)\big)$$

- **With the solution**

$$v\big(z(t)\big) \le v(z_0) \cdot \left(\frac{\mathrm{t_{go}}}{t_f}\right)^{\alpha_1} \cdot e^{\alpha_2 \cdot \left(\frac{1}{t_{go}} - \frac{1}{t_f}\right)}$$

Inequalities Manipulations

BOUNDED STABILITY

$$v\left(z(t)\right) \leq v(z_0) \cdot \left(\frac{\mathrm{t_{go}}}{t_f}\right)^{\alpha_1} \cdot e^{\alpha_2 \cdot \left(\frac{1}{t_{go}} - \frac{1}{t_f}\right)}$$

$$\tau_1 \leq \; t_{go} \rightarrow v\left(z(t)\right) \leq v(z_0) \cdot e^{\alpha_2 \cdot \left(\frac{1}{\tau_1} - \frac{1}{t_f}\right)}$$

Inequalities Manipulations

ASYMPTOTIC STABILITY

$$v\big(z(t)\big) \leq v(z_0) \cdot \left(\frac{\mathrm{t_{go}}}{t_f}\right)^{\alpha_1} \cdot e^{\alpha_2 \cdot \left(\frac{1}{t_{go}} - \frac{1}{t_f}\right)}$$

$$t_{go} \leq \tau_2 < t_f \rightarrow v\big(z(t)\big) \leq v(z_0) \cdot \left(\frac{\tau_2}{t_f}\right)^{\alpha_1} \cdot e^{\alpha_2 \cdot \left(\frac{1}{\tau_1} - \frac{1}{t_f}\right)}$$

Results

- α_1, α_2 ~ System Parameters
- Sufficient Conditions to Asymptotic Stability
  - $N > 2$
  - Hurwitz Autopilot

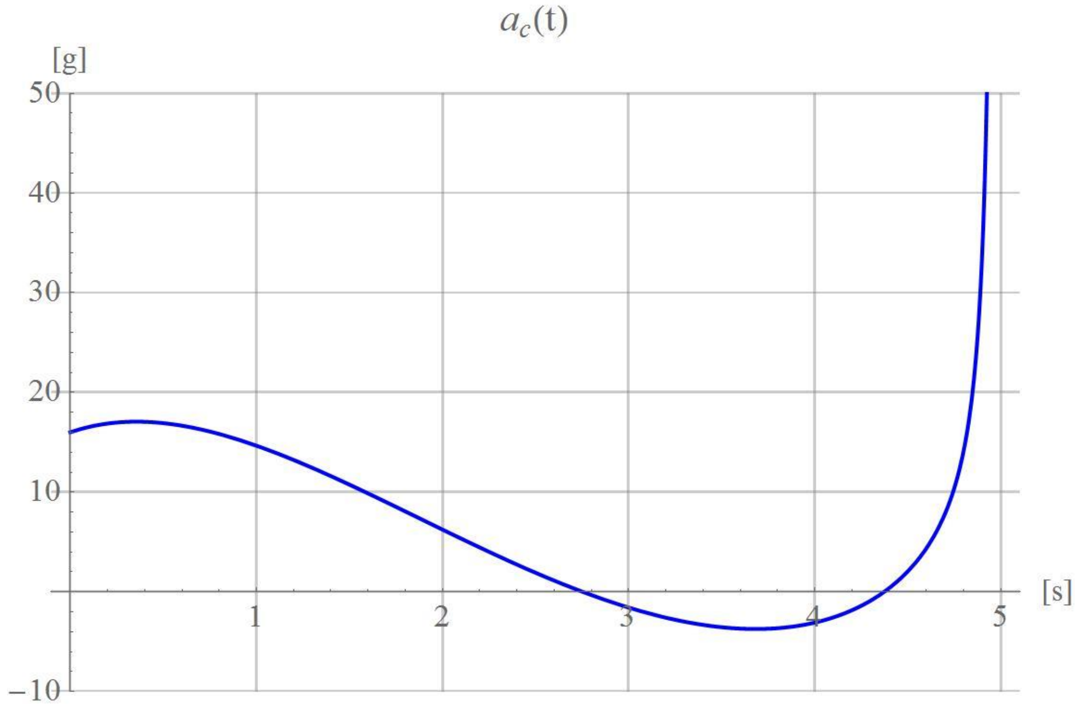
Examples – 1st order

- Autopilot $\dfrac{1}{0.5\,s+1}$

- $\boldsymbol{\lambda = -2}$ $\rightarrow$ A Hurwitz

- Let $N = 4 > 2$

$$\frac{1}{0.5\, s + 1}$$



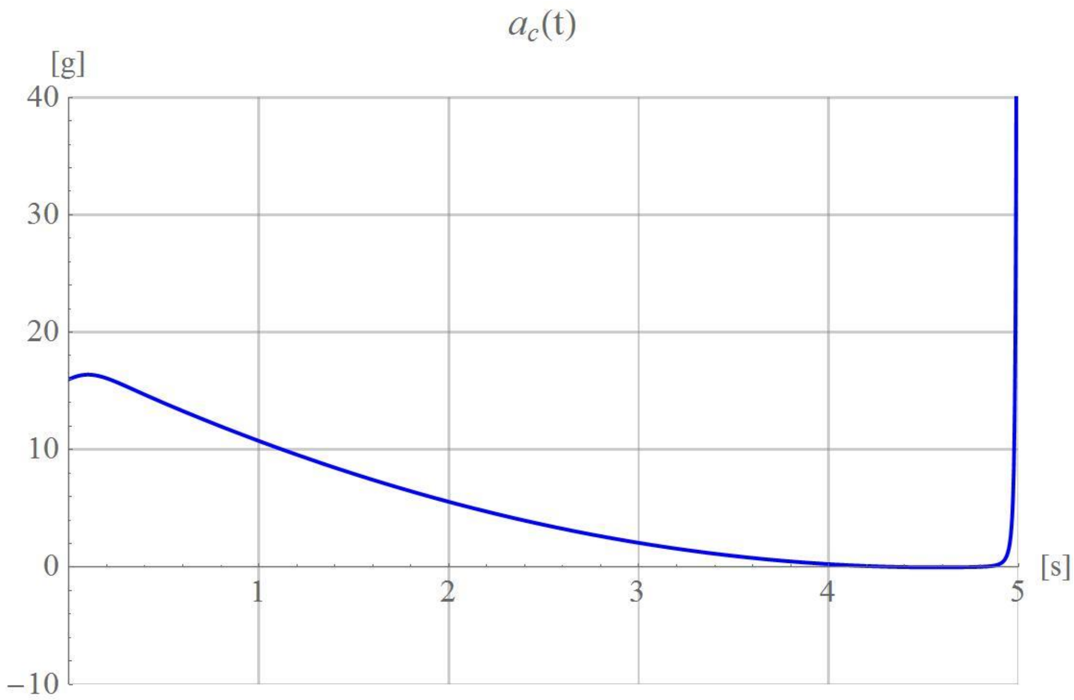
Example – 2^{nd} order

- **Autopilot** $\frac{15^2}{s^2+0.8\cdot 5\cdot s+15^2}$

- $\boldsymbol{\lambda_{1,2} = -12 \pm 9 \cdot i} \rightarrow$ A Hurwitz

- Let $N = 4 > 2$

$$\frac{15^2}{s^2 + 0.8 \cdot 5 \cdot s + 15^2}$$



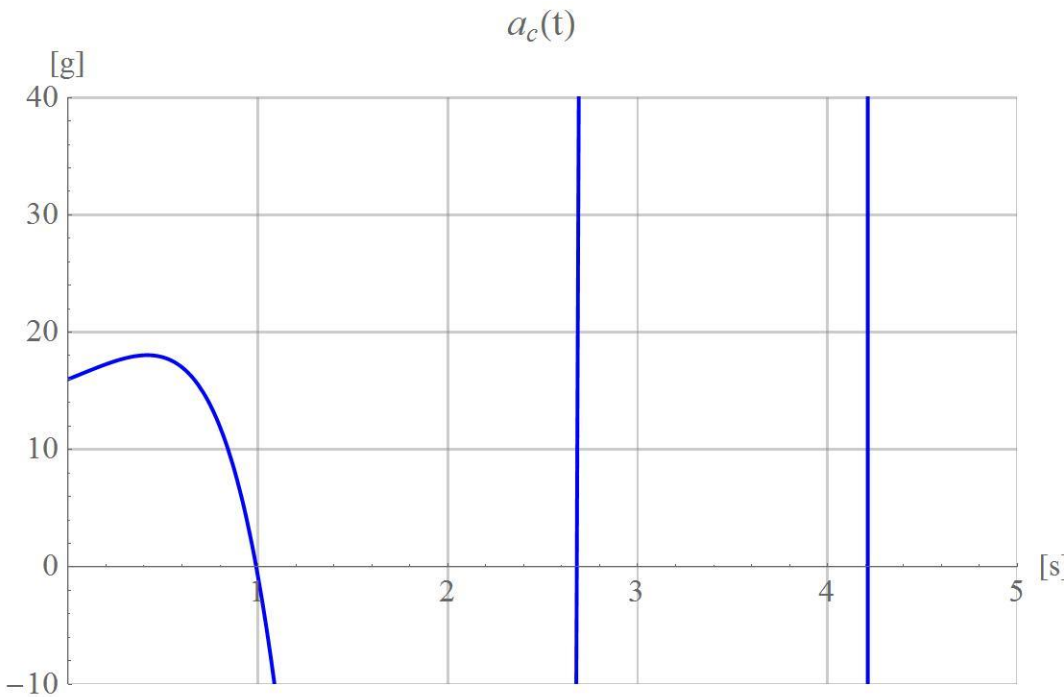
Example – 2nd order

- **Autopilot** $\dfrac{2^2}{s^2 - 0.8\cdot 2\cdot s + 2^2}$

- $\boldsymbol{\lambda_{1,2} = 1.6 \pm 0.2 \cdot i} \;\rightarrow\;$ A not Hurwitz

- Let $N = 4 > 2$

$$\frac{2^2}{s^2 - 0.8 \cdot 2 \cdot s + 2^2}$$



Unstable

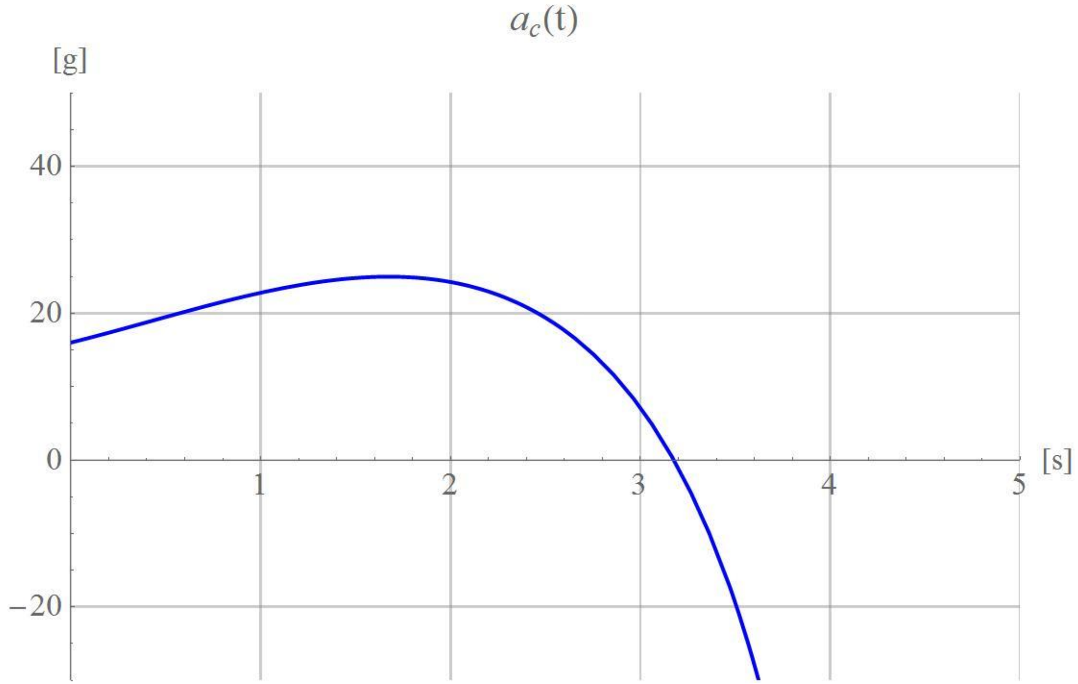
Example – 2nd order

- **Autopilot** $\frac{1}{s^2+0.7\cdot s+1}$

- $\boldsymbol{\lambda_{1,2} = -0.7 \pm 0.7 \cdot i}$ $\rightarrow$ A Hurwitz

- Let $N = 4 > 2$

$$\frac{1}{s^2 + 0.7 \cdot s + 1}$$



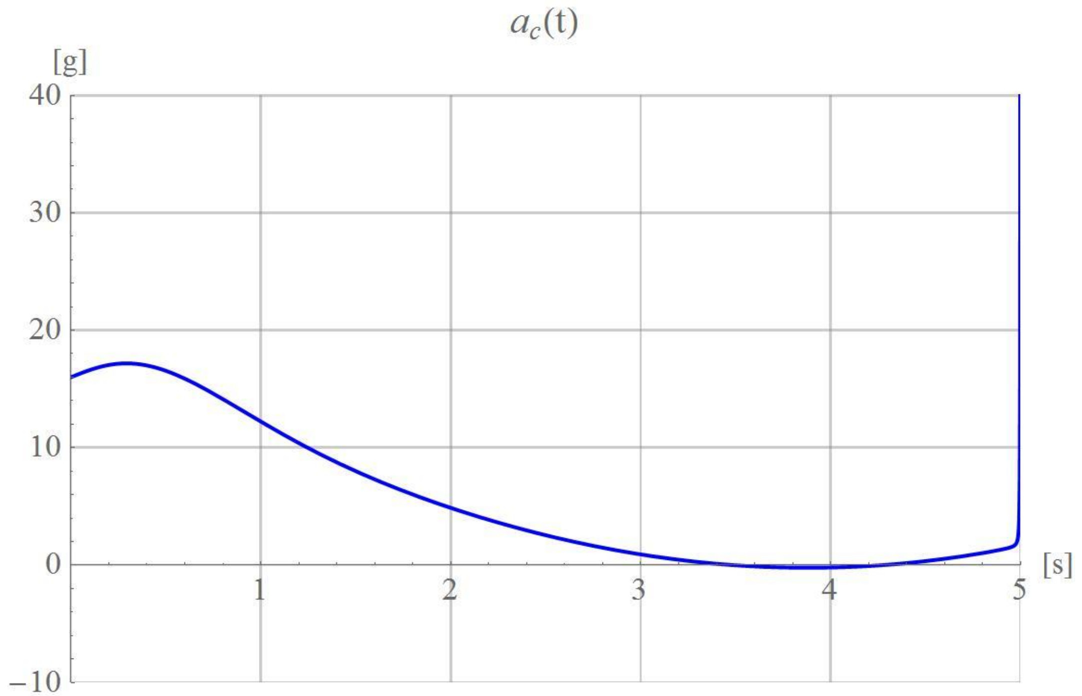
stable?

Example – 2nd order

- **Autopilot** $\frac{5^2}{s^2+0.7\cdot 5^2\cdot s+5^2}$

- $\boldsymbol{\lambda_{1,2} = -3.5 \pm 3.5 \cdot i} \rightarrow$ A Hurwitz

- Let $N = 4 > 2$



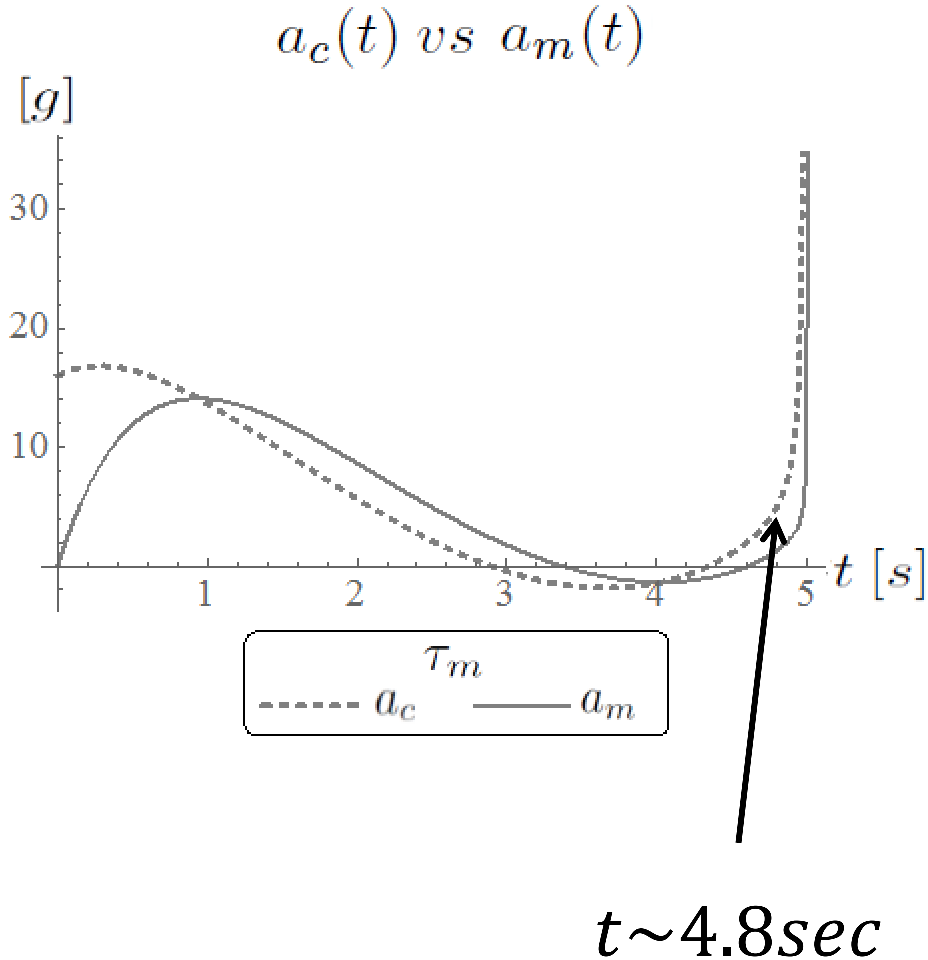
- $\omega_n \to 5\frac{r}{s} \sim \mathrm{t_f} = 25\mathrm{sec}$
- $\left\|x(t;t_{f_2})\right\|^2 < \left\|x(t;t_{f_1})\right\|^2$ - asymptotic stability
- Stability independence of time, performances not

Missile's Parameters Adjustment

$$v\big(z(t)\big) \le v(z_0) \cdot \left(\frac{\tau_2}{t_f}\right)^{\alpha_1} \cdot e^{\alpha_2 \cdot \left(\frac{1}{\tau_1} - \frac{1}{t_f}\right)}$$

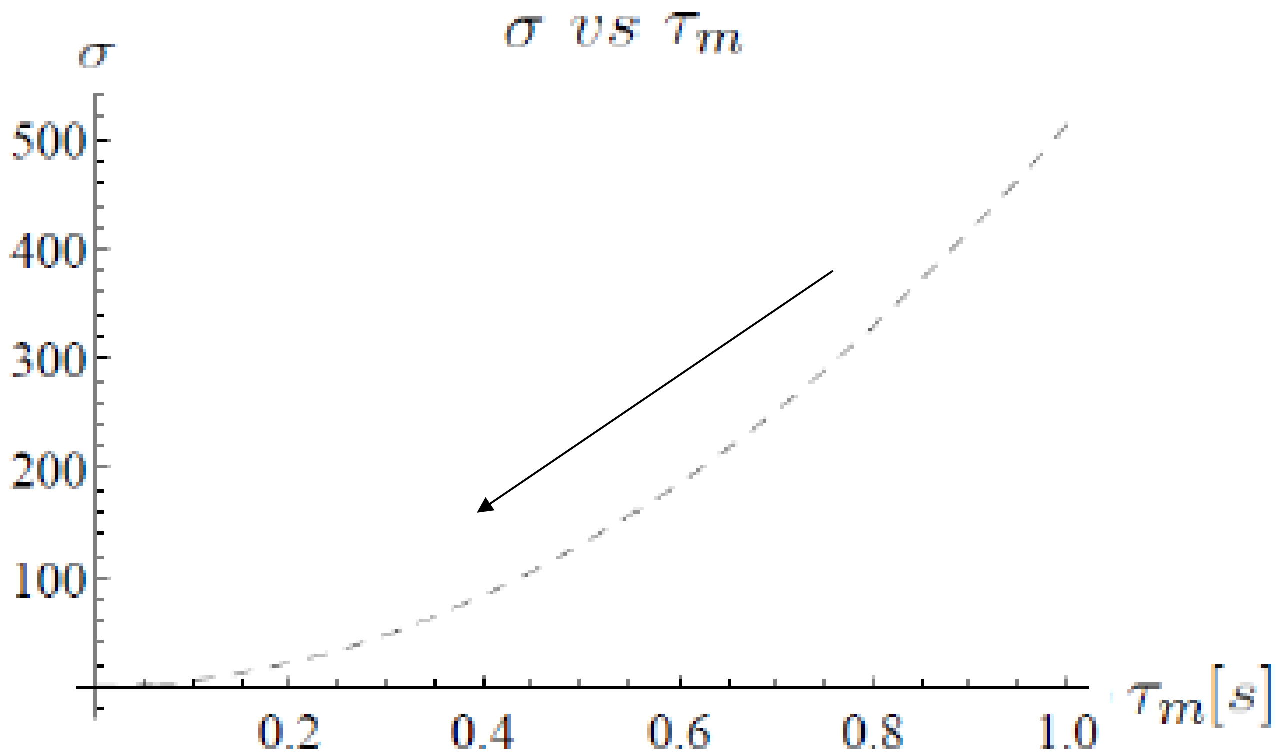
$$\rightarrow \ \sigma = \frac{\alpha_2}{\tau_1} + \alpha_1 \ln \frac{\tau_2}{t_f}$$

Example – Delay Divergence



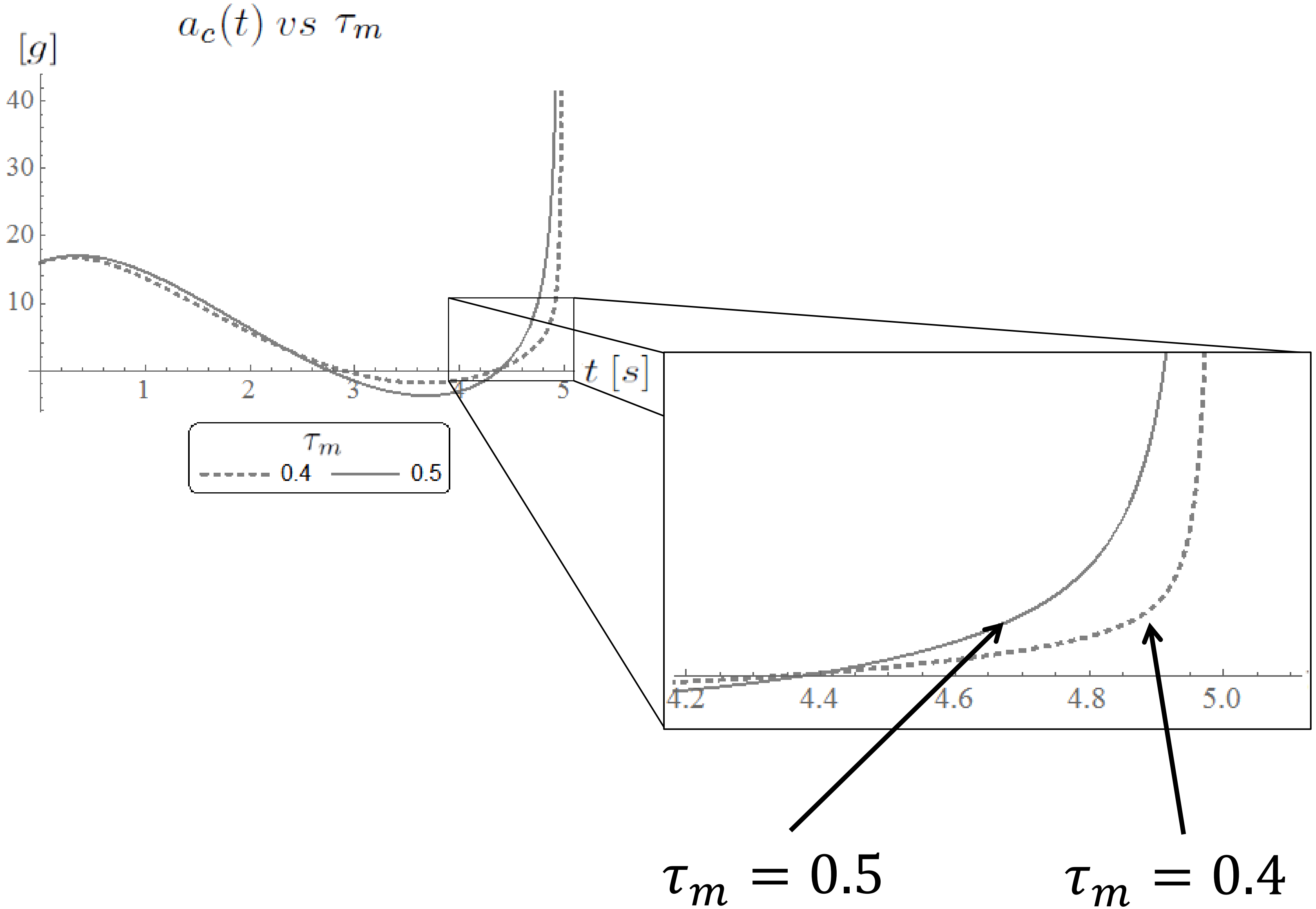
Example – Delay Divergence

$$\sigma = \frac{\alpha_2}{\tau_1} + \alpha_1 \ln \frac{\tau_2}{t_f}$$

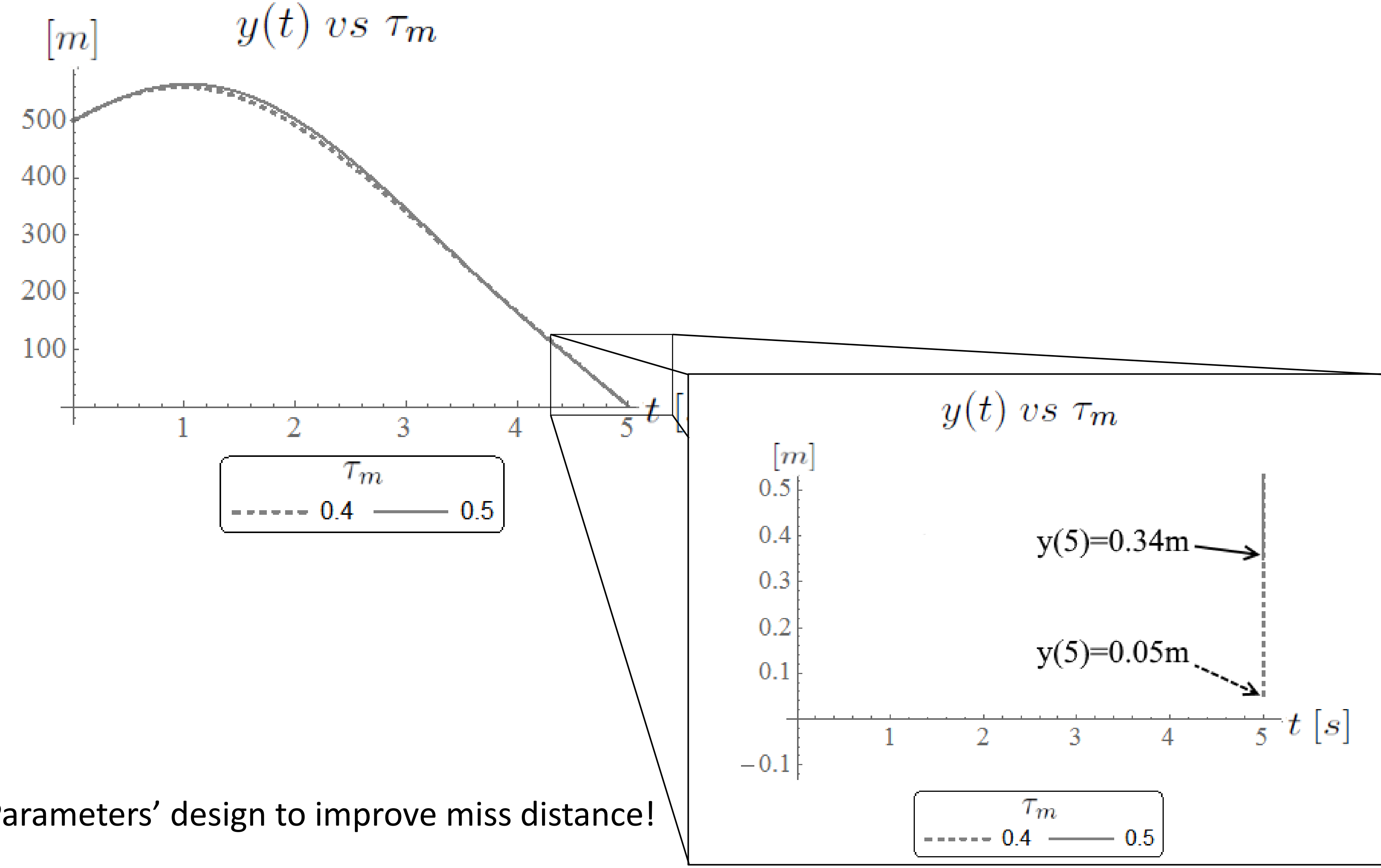


$$Let\ \tau_m = 0.4$$

Example – Delay Divergence



Example – Delay Divergence



Parameters' design to improve miss distance!

Conclusions

- Sufficient conditions to stability
  - N>2
  - Hurwitz autopilot
- Stability + Interception performances!
- Parameters' adjustment to delay divergence

Summary

- Investigation of PNG system
- Complicated t_{go} problem solution
- Conditions to asymptotic stability
- Complete tool for a designer