Adaptive Perimeter Control for Large-scale Urban Networks

PRESENTED BY Zhengfei

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- State variables with delay for two urban regions
- Distributed adaptive control in Subsystem

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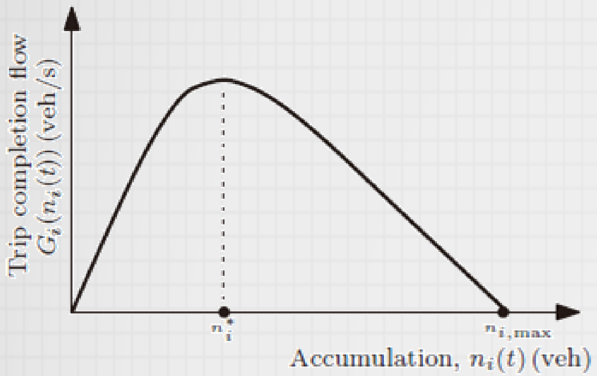
- Example 2

Concept of MFD

What is MFD



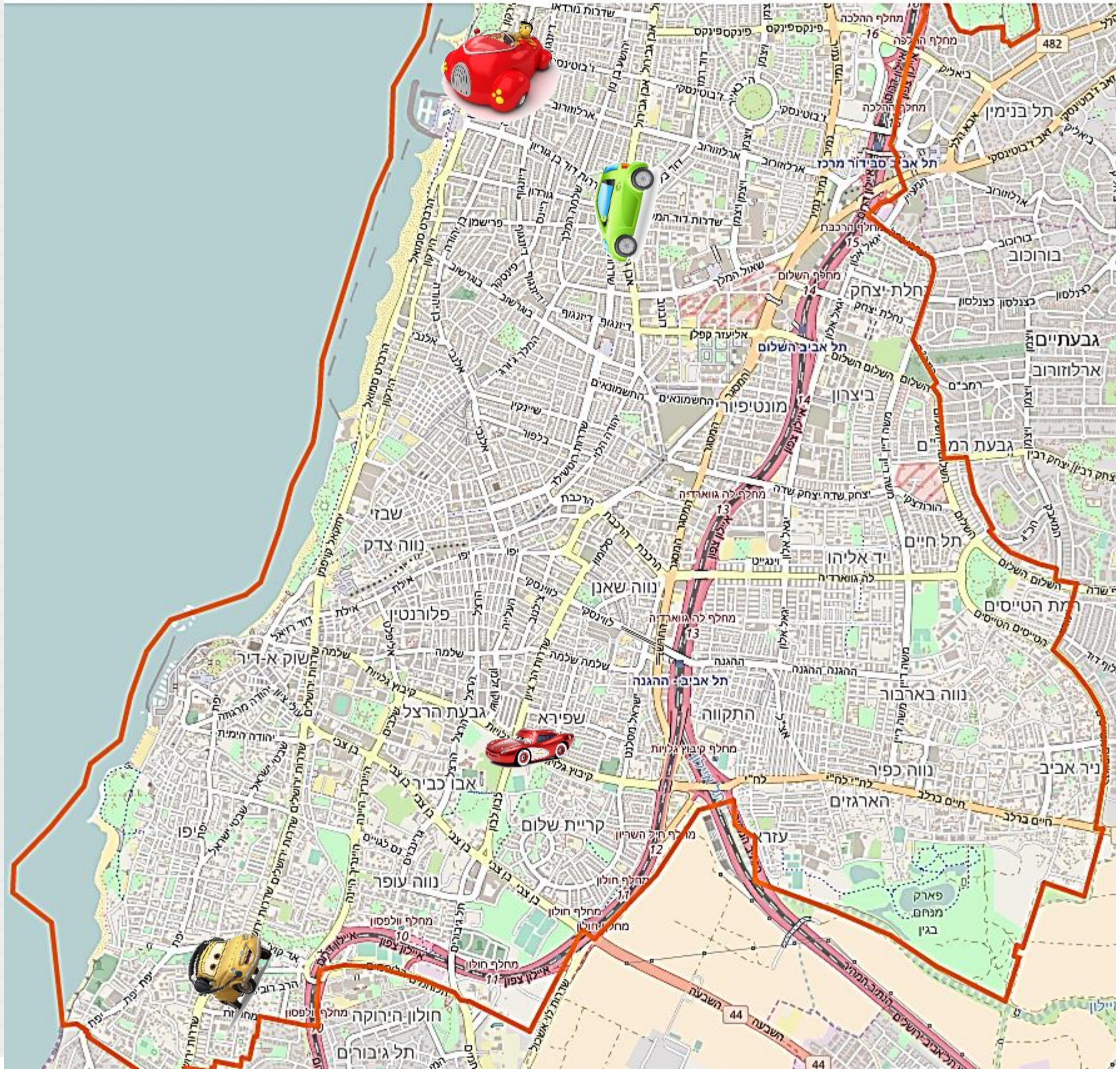
$$\bar{q} = \frac{\sum_{i=1}^{m} l_i q_i}{\sum_{i=1}^{m} l_i} \qquad \bar{k} = \frac{\sum_{i=1}^{m} l_i k_i}{\sum_{i=1}^{m} l_i}$$

A unimodal low-scatter relationship between **accumulation** and **trip completion flow**

$$G_i(n_i(t)) = a_i \cdot n_i^3(t) + b_i \cdot n_i^2(t) + c_i \cdot n_i(t).$$

MFD

Macroscopic Fundamental Diagram



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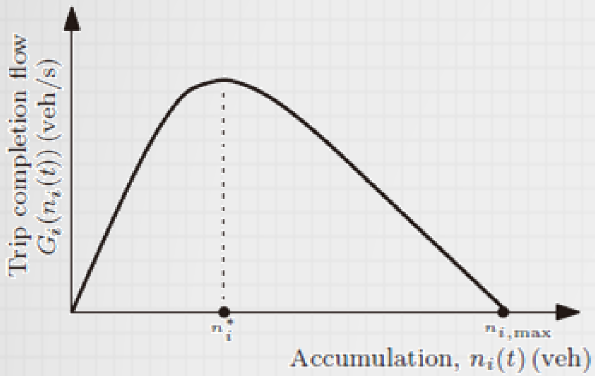
Concept of MFD

What is MFD



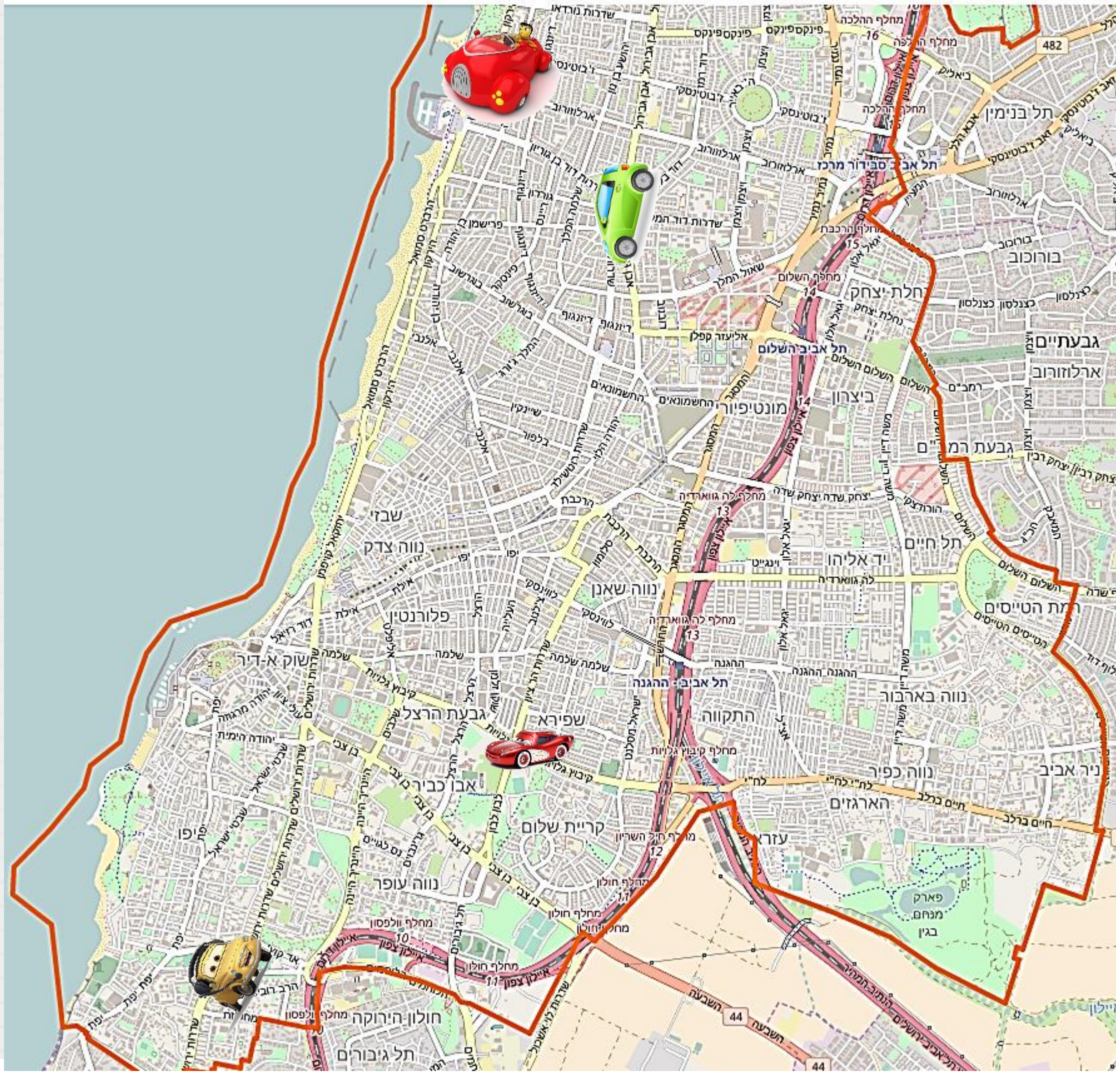
$$\bar{q} = \frac{\sum_{i=1}^{m} l_i q_i}{\sum_{i=1}^{m} l_i} \qquad \bar{k} = \frac{\sum_{i=1}^{m} l_i k_i}{\sum_{i=1}^{m} l_i}$$

A unimodal low-scatter relationship between **accumulation** and **trip completion flow**

$$G_i(n_i(t)) = a_i \cdot n_i^3(t) + b_i \cdot n_i^2(t) + c_i \cdot n_i(t).$$

MFD

Macroscopic Fundamental Diagram



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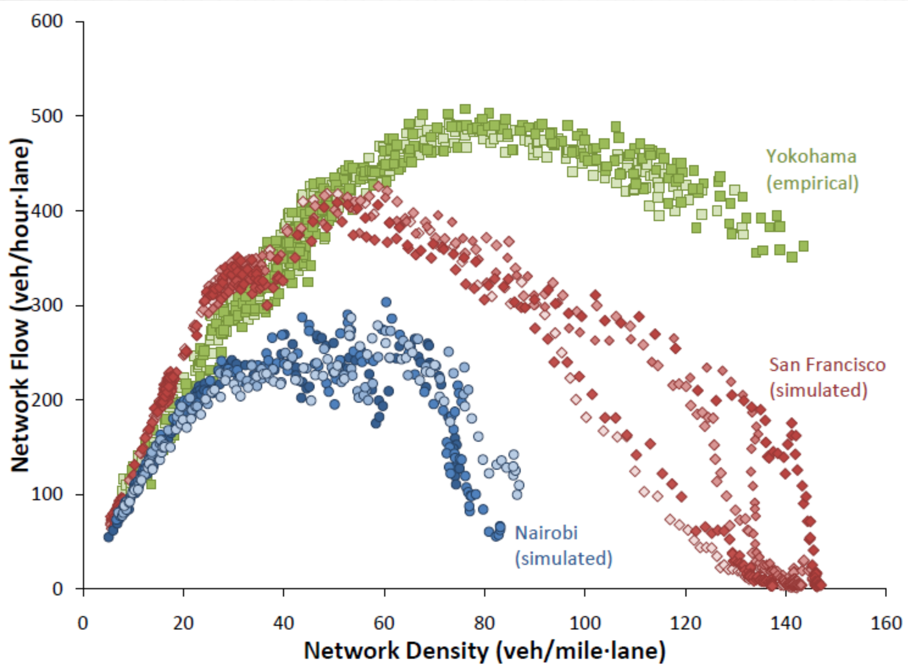
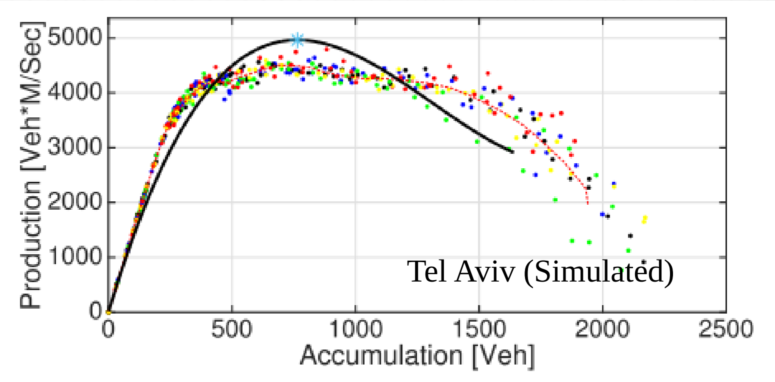
Concept of MFD

What is MFD

Verification :A field experiment in Yokohama (Japan)

a MFD linking space-mean flow, density and speed exists on a large urban area.





MFD

Macroscopic Fundamental Diagram

Macroscopic Fundamental Diagrams of

- Yokohama (measured, Geroliminis & Daganzo, 2008),
- San Francisco (simulated, Geroliminis & Daganzo, 2007)
- Nairobi (simulated, Gonzales et al., 2009).
- Tel Aviv(simulated, Jack Haddad, 2017)

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MFD model

For two regions

$$\frac{dn_{11}(t)}{dt} = q_{11}(t) + u_{21}(t)\cdot\frac{n_{21}(t)}{n_2(t)}\cdot G_2(n_2(t)) - \frac{n_{11}(t)}{n_1(t)}\cdot G_1(n_1(t))$$

$$\frac{dn_{12}(t)}{dt} = q_{12}(t) - \frac{n_{12}(t)}{n_1(t)}\cdot G_1(n_1(t))\cdot u_{12}(t)$$

$$M_{ii}(n_{ii}(t), n_i(t)) = \frac{n_{ii}(t)}{n_i(t)} G_i(n_i(t)),$$

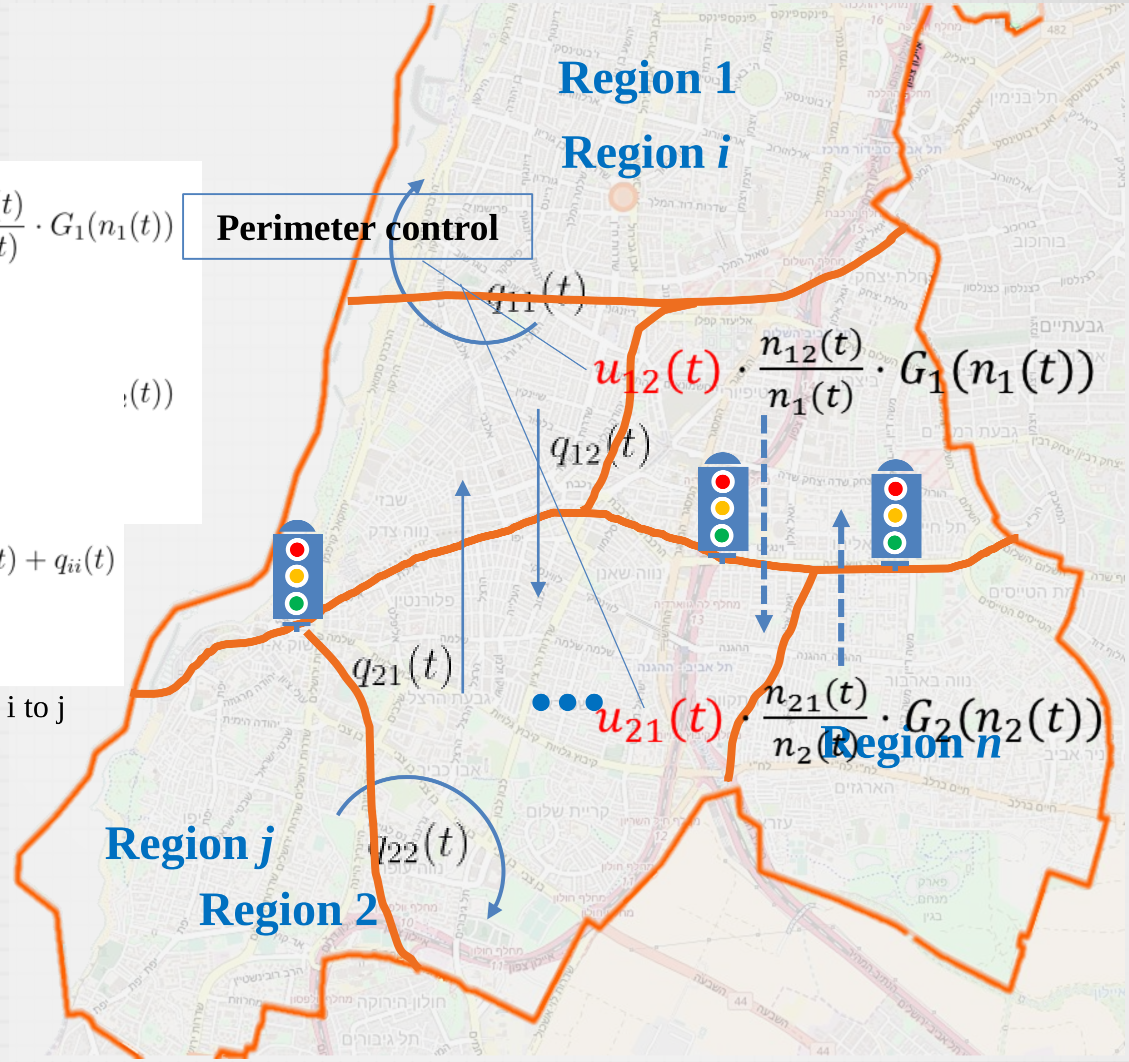
$$M_{ij}(n_{ij}(t), n_i(t)) = \frac{n_{ij}(t)}{n_i(t)} G_i(n_i(t)),$$

$$\dot{n}_{ii}(t) = -M_{ii}(n_{ii}(t), n_i(t)) + \sum_{j\in S_i} M_{ji}(n_{ji}(t), n_j(t))u_{ji}(t) + q_{ii}(t)$$

$$\dot{n}_{ij}(t) = -M_{ij}(n_{ij}(t), n_i(t))u_{ij}(t) + q_{ij}(t), j \in S_i$$

$q_{ij}(t)$ (veh/s) traffic demand generated from region i to j

Extend to n regions



MFD

Macroscopic Fundamental Diagram

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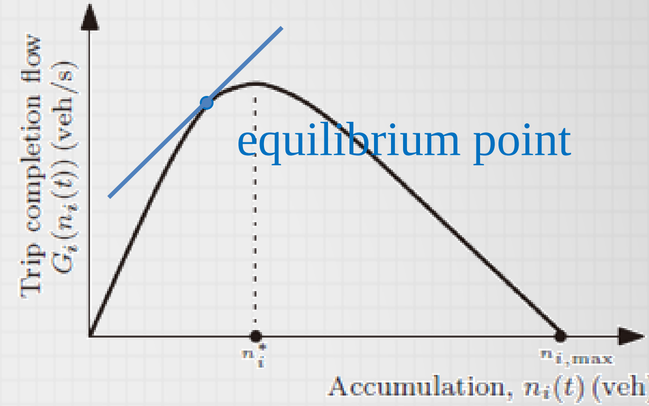
Model Linearization



Given an equilibrium point

$$(n_{11,eq}, n_{12,eq}, q_{11,eq}, q_{12,eq}, u_{12,eq})$$
$$(n_{21,eq}, n_{22,eq}, q_{21,eq}, q_{22,eq}, u_{21,eq})$$

Denote

$$F_{11}(u_{21}, n_{11}, n_{12}, n_{21}, n_{22}) = q_{11}(t) + u_{21}(t) \cdot \frac{n_{21}(t)}{n_2(t)} \cdot G_2(n_2(t)) - \frac{n_{11}(t)}{n_1(t)} \cdot G_1(n_1(t))$$

$$F_{12}(u_{12}, n_{11}, n_{12}) = q_{12}(t) - \frac{n_{12}(t)}{n_1(t)} \cdot G_1(n_1(t)) \cdot u_{12}(t)$$

$$F_{22}(u_{12}, n_{11}, n_{12}, n_{21}, n_{22}) = q_{22}(t) + u_{12}(t) \cdot \frac{n_{12}(t)}{n_1(t)} \cdot G_1(n_1(t)) - \frac{n_{22}(t)}{n_2(t)} \cdot G_2(n_2(t))$$

$$F_{21}(u_{21}, n_{21}, n_{22}) = q_{21}(t) - \frac{n_{21}(t)}{n_2(t)} \cdot G_2(n_2(t)) \cdot u_{21}(t)$$

$$\dot{x} = Ax + Bu$$

Perimeter control：State of the art

Traffic model

- For single-region network(1-3)
- Multi-region network (4-9)

References:

1. Daganzo(2007), 2. Geroliminis and Daganzo (2008), 3. Keyvan-Ekbatani et al. (2012), 4. Haddad and Shraiber (2014), 5. Haddad and Geroliminis (2012), 6. Geroliminis et al. (2013), 7. Aboudolas and Geroliminis (2013), 8. Mahmassani et al. (2013), 9. Ramezani et al. (2015).

Perimeter control：State of the art

Control approaches for different model

01. Classical feedback control (3.7)

- Feed information without the need to predict the near future dynamics or demand
- Do not consider the constraints directly, impose them after the design

02. Model Predictive Control (6)

- Performed well for different levels of demand and errors in the MFD shape
- Nonlinear

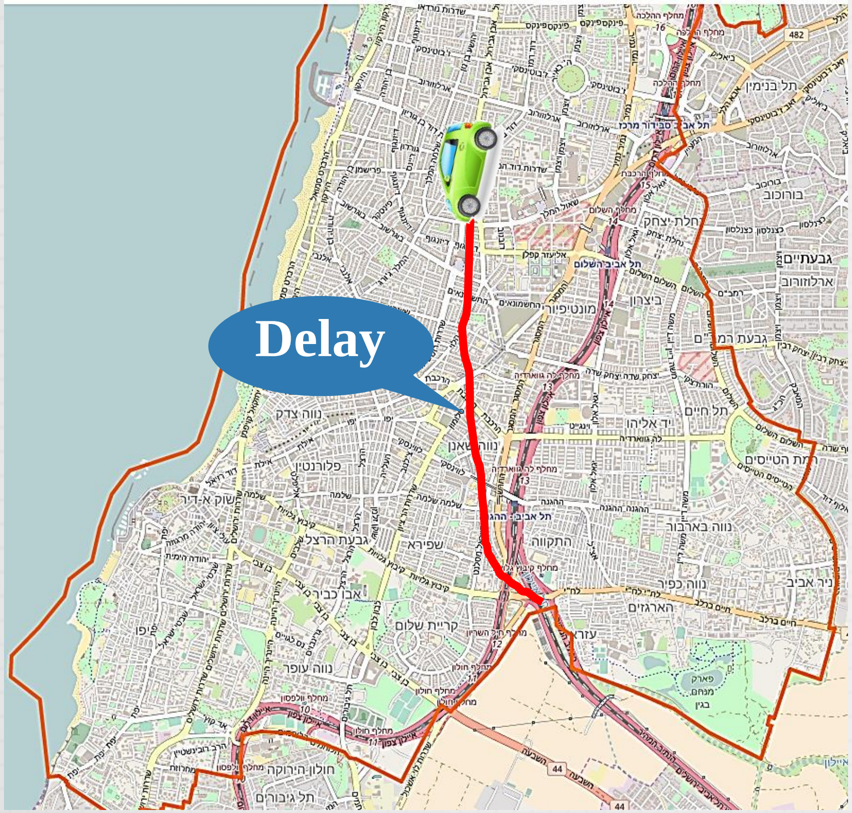
03. Classical robust feedback (4)

- Handles uncertainty in the MFD
- Deal with the control constraints within the design level.
- Linear

References:
1. Daganzo(2007), 2. Geroliminis and Daganzo (2008), 3. Keyvan-Ekbatani et al. (2012), 4. Haddad and Shraiber (2014), 5. Haddad and Geroliminis (2012), 6. Geroliminis et al. (2013), 7. Aboudolas and Geroliminis (2013), 8. Mahmassani et al. (2013), 9. Ramezani et al. (2015).

Delay in the model

Time delay



Travel time needed for vehicles to travel from the urban region to its border

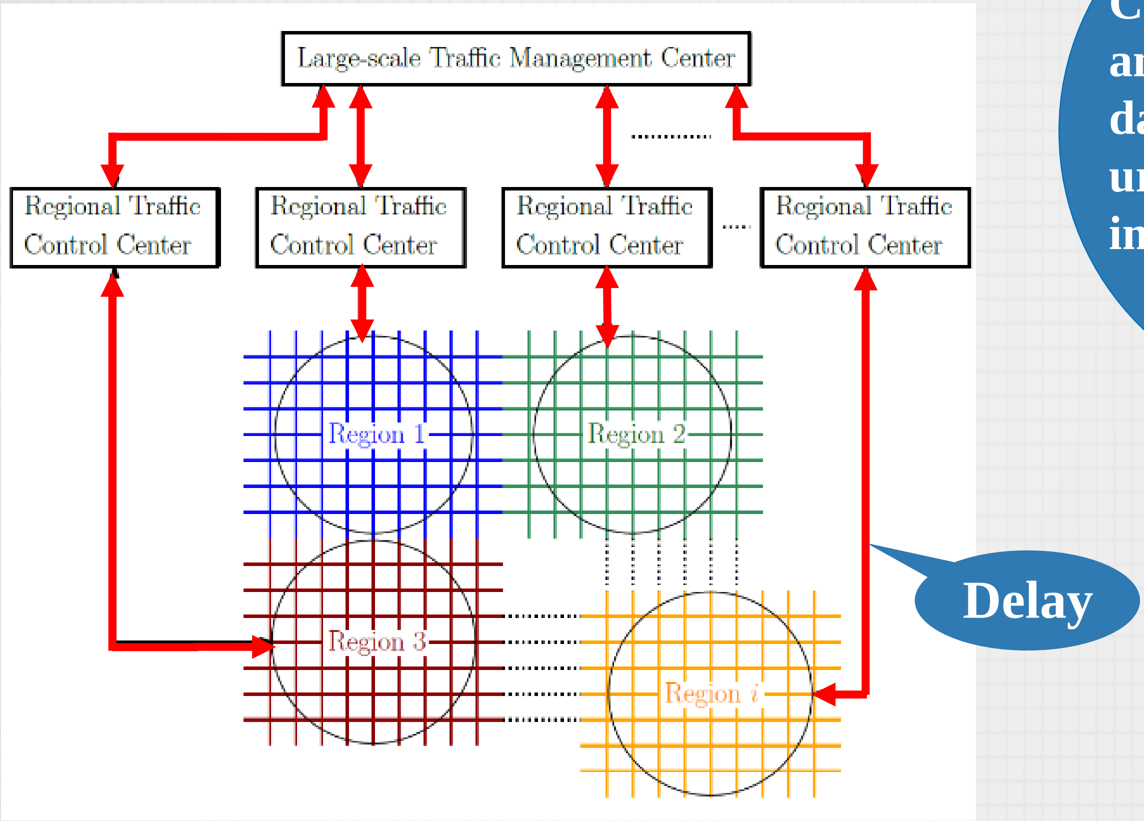
$$\tau_i(n_i) = \frac{L_i}{V_i} = \frac{n_i}{G_i(n_i)}$$

L_i (km) The average trip length

$V_i = G_i(n_i) \cdot L_i / n_i$ (km/hr) The average spee

Delay in the model

Interconnection delay



Collecting, processing, and uploading traffic data in a large-scale urban road networks impose time delays

Contribution

WHAT IS **NEW**

Advanced Model:

Introduce state delay based on MFD model

No such research has been done before

Improved perimeter control

Model Reference Adaptive Control (MRAC)

Deal with uncertainties in MFD

Coordinated control for multi-regions

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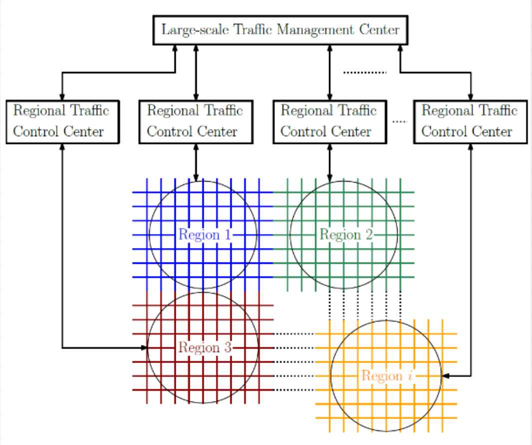
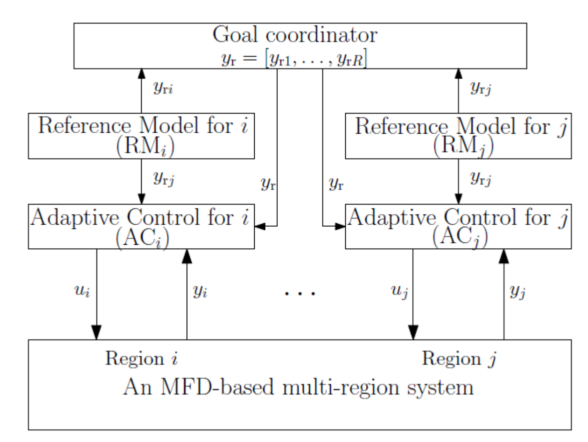
Adaptive control

Why reference model

Problem definition:
How to maintain the number of vehicles in a region



A large-scale traffic management center



Model reference adaptive control

Coordination by providing the reference signals

Picture: Haddad, Jack, and Boris Mirkin. "Coordinated distributed adaptive perimeter control for large-scale urban road networks." *Transportation Research Part C: Emerging Technologies* 77 (2017): 495-515.

Controller design

Case 1: The new MFD model with state delay

The delays are added to the MFD

The linearized state space form model for two regions

$$
\begin{aligned}
\dot{x}_1(t) &= A_1 x_1(t) + \tilde{A}_1 x_1(t-\tau_1) + b_1 u_1(t) + A_{12} x_2(t) + \tilde{A}_{12} x_2(t-\tau_2)\,, \\
y_1(t) &= c_1^T x_1(t),\; c_1 = [1,1]^T, \\
\dot{x}_2(t) &= A_2 x_2(t) + \tilde{A}_2 x_2(t-\tau_2) + b_2 u_2(t) + A_{21} x_1(t) + \tilde{A}_{21} x_1(t-\tau_1)\,, \\
y_2(t) &= c_2^T x_2(t),\; c_2 = [1,1]^T\,,
\end{aligned}
$$

$$
x_1(t) = \begin{bmatrix} \Delta n_{11}(t) \\ \Delta n_{12}(t) \end{bmatrix} = \begin{bmatrix} n_{11}(t) - n_{11,\mathrm{eq}} \\ n_{12}(t) - n_{12,\mathrm{eq}} \end{bmatrix} \in \mathbb{R}^2
$$

$$
x_2(t) = \begin{bmatrix} \Delta n_{21}(t) \\ \Delta n_{22}(t) \end{bmatrix} = \begin{bmatrix} n_{21}(t) - n_{21,\mathrm{eq}} \\ n_{22}(t) - n_{22,\mathrm{eq}} \end{bmatrix} \in \mathbb{R}^2
$$

$$
\begin{aligned}
u_1(t) &= \Delta u_{12}(t) = u_{12}(t) - u_{12,\mathrm{eq}} \in \mathbb{R} \\
u_2(t) &= \Delta u_{21}(t) = u_{21}(t) - u_{21,\mathrm{eq}} \in \mathbb{R}
\end{aligned}
$$

$$
y_i(t) = \Delta n_i(t) \in \mathbb{R}
$$

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Controller design

Case 1: The new MFD model with state delay

Give reference models

$$\dot{x}_{mi}(t) = A_{mi}x_{mi}(t) + b_{mi}r_i(t)\,,$$
$$y_{mi}(t) = c_i^T x_{mi}(t), \quad c_i = [1,1]^T, \quad i = 1,2$$

$$n_i(t) \to n_i^* \text{ if } t \to \infty \text{ for } i = 1,2.$$

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Controller design

Case 1: The new MFD model with state delay

$$u_i(t) = u_{fi}(t) + u_{gi}(t)\,, \quad i = 1, 2\,,$$

$u_{gi}(t)$: Coordinated signals

$u_{fi}(t)$: Local signals

$u_{fi}(t)$ is based only on *local* signals

$u_{fi}(t) =$

$\dot{x}_{fi}(t) =$

$w_{fi}(t) =$

$e_i(t) = y_i$

$$\dot{z}_{mij}(t) = F_{di} z_{mij}(t) + g_{di} x_{mj}(t)\,,$$

$$\dot{z}_{m\tau ij}(t) = F_{di} z_{m\tau ij}(t) + g_{di} x_{mj}(t - \tau_{ij})\,,$$

$$\omega_{mij}(t) = \left[x^T_{mj}(t) x^T_{mj}(t - \tau_{ij})\right]^T,$$

$$\omega_{zij}(t) = \left[z^T_{mij}(t) z^T_{m\tau ij}(t)\right]^T,$$

$$u_{gi}(t) = -\sum^{M} \left[K^T_{mij}(t)\omega_{mij}(t) - K^T_{zij}(t)\omega_{zij}(t)\right]$$

(F_i, g_i) sta ... reference model

$F_{di} = \mathrm{diag}\{F_i\}$ $g_{di} = \mathrm{diag}\{g_i\}$

B. M. Mirkin and P.O. Gutman. "Decentralized output-feedback MRAC of linear state delay systems." IEEE Transactions on Automatic Control 48.9 (2003): 1613-1619.

Controller design

Case 1: The new MFD model with state delay

Adaptation algorithms

$$\frac{\mathrm{dK_i}}{\mathrm{d}t} = -\mathrm{sign}\left(\frac{k_{pi}}{k_{mi}}\right)\Gamma_{ii}e_i\omega_{fi}\,,$$
$$\frac{\mathrm{dK_{mij}}}{\mathrm{d}t} = \mathrm{sign}\left(\frac{k_{pi}}{k_{mi}}\right)\Gamma_{mi}e_i\omega_{mij}$$
$$\frac{\mathrm{dK_{zij}}}{\mathrm{d}t} = -\mathrm{sign}\left(\frac{k_{pi}}{k_{mi}}\right)\Gamma_{zi}e_i\omega_{zij}$$

Γ_{ii}, Γ_{mi}, Γ_{zi} are from the Lyapunov function

k_{pi} is a known gain of the plant k_{mi} is a constant gain from the reference model

Detailed Lyapunov Stability analysis, please refer to

B. M. Mirkin and P.O. Gutman. "Decentralized output-feedback MRAC of linear state delay systems." IEEE Transactions on Automatic Control 48.9 (2003): 1613-1619.

Controller design

Case 2: The new MFD model with interconnection delay

$$\dot{n}_{ii}(t) = q_{ii}(t) + \sum \frac{n_{ji}(t-\tau_j)}{} \cdot G_j(n_j(t-\tau_j)) \cdot u_{ji}(t) - \frac{n_{ii}(t)}{} \cdot G_i(n_i(t)),$$

$$\dot{x}_i(t) = A_i x_i(t) + B_i u_i(t) + \sum_{j \neq i} \tilde{A}_{ji} \tilde{x}_j(t-\tau_j).$$

$\dot{n}_{ij}(t) = q_{ij}(t)$ -

$$x_i(t) = [\Delta n_{ii}(t), \ldots, \Delta n_{ij}(t), \ldots]^{\mathrm{T}} \in \mathrm{R}^{(\dim(S_i)+1)\times 1}$$

$$\tilde{x}_j(t-\tau_j) = [\ldots, \Delta n_{ji}(t-\tau_j), \ldots]^{\mathrm{T}} \in \mathrm{R}^{\dim(S_i)\cdot(\dim(S_j)+1)\times 1}$$

$$u_i(t) = [\ldots, \Delta u_{ij}(t), \ldots, \Delta u_{ji}(t), \ldots]^{\mathrm{T}} \in \mathrm{R}^{2\dim(S_i)\times 1}$$

$$\Delta n_{ij}(t) = n_{ij}(t) - n_{ij,\mathrm{eq}}, j \in S_i$$

$$\Delta u_{ij}(t) = u_{ij}(t) - u_{ij,\mathrm{eq}}, j \in S_i$$

$$\Delta n_{ji}(t-\tau_j) = n_{ji}(t-\tau_j) - n_{ji,\mathrm{eq}}, i, j = 1, 2, \cdots, R, i \neq j$$

S_i the set of regions that are directly reachable from region i

Controller design

Case 2: The new MFD model with interconnection delay

$$u_i(t) = u_{li}(t) + u_{ci}(t)$$

Coordinated feedforward control component

Control component on local information $u_{li}(t)$

$$u_{li}(t) =$$

$$u_{ci}(t) = \sum_{j=1, j\neq i}^{R} \theta_{ij}^{*\mathrm{T}} \omega_{ij} ,$$

$$\omega_{fi}(t) = \begin{bmatrix} I_{mi} & 0 & 0 \\ 0 & \Phi_i(s) & 0 \\ 0 & 0 & \Phi_i(s) \end{bmatrix}$$

$$\dot{Z}_{rij}(t) = A_{\phi ij} Z_{rij}(t) + B_{\phi ij} y_{rj}(t - \tau_j) ,$$

$$z_{rij}(t) = C_{\phi ij} Z_{rij}(t) ,$$

$$\omega_i(t) = \begin{bmatrix} I_{mi} \\ W_{ri}(s) \end{bmatrix} r_i($$

$$\omega_{ij} = \begin{bmatrix} z_{rij}^{\mathrm{T}}(t) & y_{rj}^{\mathrm{T}}(t - \tau_j) \end{bmatrix}^{\mathrm{T}} ,$$

…nstant θ^*_{ei}, $\theta^{*\mathrm{T}}_{1i}$ $\theta^{*\mathrm{T}}_{2i}$, θ^*_{r1i}, θ^*_{r2i}

$W_{ri}(s)$ TF of reference model

$W_i(s)$ TF of the plant

$$\Phi_i(s) = \frac{[I_{mi} s^{v_i - 1}, \cdots, I_{mi} s, I_{mi}]^{\mathrm{T}}}{\Lambda_i(s)}$$

$(A_{\phi ij}, B_{\phi ij}, C_{\phi ij})$ is a minimal state space realization for the stable transfer matrix

Mirkin, B. M., & Gutman, P. O. (2005). Output-feedback co-ordinated decentralized adaptive tracking: The case of MIMO subsystems with delayed interconnections. International Journal of Adaptive Control and Signal Processing, 19(8), 639.

Controller design

Case 2: The new MFD model with interconnection delay

Adaptation algorithms

$$\dot{\theta}_{li}(t) = -\eta_i(t) - \dot{\eta}_i(t) - \dot{\eta}_i(t - h_i)$$

$$\eta_i^{\mathrm{T}}(t) = \gamma_i S_{pi} e_i(t) \omega_{li}^{\mathrm{T}}(t) \, ,$$

$$\dot{\theta}_{ij}(t) = -\eta_{ij}(t) \, ,$$

$$\eta_{ij}^{\mathrm{T}}(t) = \gamma_{ij} S_{pi} e_i(t) \omega_{ij}^{\mathrm{T}}(t) \, .$$

$e_i(t) = x_i(t) - x_{ri}(t)$

$h_i, \gamma_i, \gamma_{ij}$ are some constant traditional gains

S_{pi} is constant matrix, meet the condition

Mirkin, B. M., & Gutman, P. O. (2005). Output-feedback co-ordinated decentralized adaptive tracking: The case of MIMO subsystems with delayed interconnections. International Journal of Adaptive Control and Signal Processing, 19(8), 639.

Contents

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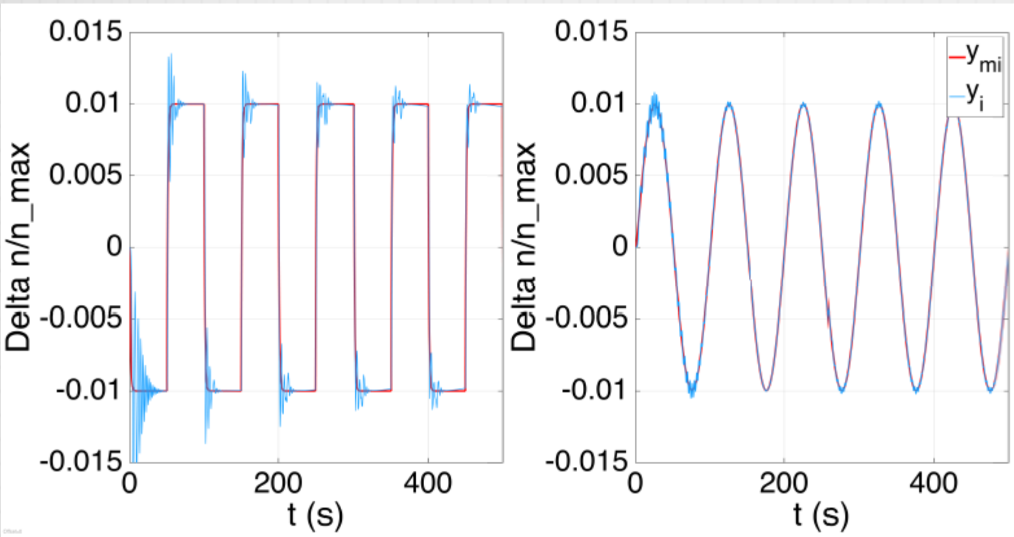
Numerical examples



Case 1

MFD_1=MFD_2=MFD_Yok

$$b_1 = b_2 = \begin{bmatrix} -0.6636 \\ -1.3273 \end{bmatrix} \cdot 10^{-4}.$$

$$A_1 = \begin{bmatrix} -4.1504 & 2.4939 \\ 1.2692 & -0.7615 \end{bmatrix} \cdot 10^{-4}, \quad \tilde{A}_1 = \begin{bmatrix} -2.4937 & -2.4937 \\ -1.2691 & -1.2691 \end{bmatrix} \cdot 10^{-4},$$

$$A_{12} = \begin{bmatrix} -2.2927 & -0.9968 \\ 0 & 0 \end{bmatrix} \cdot 10^{-4}, \quad \tilde{A}_{12} = \begin{bmatrix} 0.9968 & 0.9968 \\ 0 & 0 \end{bmatrix} \cdot 10^{-4},$$

$$A_2 = \begin{bmatrix} -2.2927 & 0.9968 \\ 4.635 & -2.015 \end{bmatrix} \cdot 10^{-4}, \quad \tilde{A}_2 = \begin{bmatrix} -0.9968 & -0.9968 \\ -4.6348 & -4.6348 \end{bmatrix} \cdot 10^{-4},$$

$$A_{21} = \begin{bmatrix} 0 & 0 \\ -0.1269 & 0.0762 \end{bmatrix} \cdot 10^{-4}, \quad \tilde{A}_{21} = \begin{bmatrix} 0 & 0 \\ 1.2691 & 1.2691 \end{bmatrix} \cdot 10^{-4},$$

$$(n_{11,\text{eq}}, n_{12,\text{eq}}, n_{21,\text{eq}}, n_{22,\text{eq}}, u_{12,\text{eq}}, u_{21,\text{eq}}) = (1200, 2000, 1000, 2300, 0.31, 0.49)$$

$$A_{m1} = A_{m2} = \begin{bmatrix} -0.03 & 0 \\ 0 & -0.03 \end{bmatrix}, \; b_{m1} = b_{m2} = [1 \quad 1]^T, \; \tau_1 = \tau_2 = 5, \; \Gamma_{11} = 0.4 I_{4\times4},$$

$$\Gamma_{22} = 0.6 I_{4\times4}, \; \Gamma_{m1} = 4 \cdot 10^6 I_{8\times8}, \; \Gamma_{m2} = 2.5 \cdot 10^6 I_{8\times8}, \Gamma_{z1} = 5 \cdot 10^6 I_{8\times8},$$

$$\Gamma_{z2} = 3.5 \cdot 10^6 I_{8\times8}, \; g_{d1} = g_{d2} = \text{diag}\{\, 1 \quad 1 \,\}, F_{d1} = F_{d2} = \text{diag}\{\, -1 \quad -1 \,\}$$

MFD_{Yok}

$n_{cr} = 3400 \text{ (veh)}$

$n_{max} = 10021 \text{ (veh)}$

$G(n_{cr}) = 6.3 \text{ (veh/s)}$

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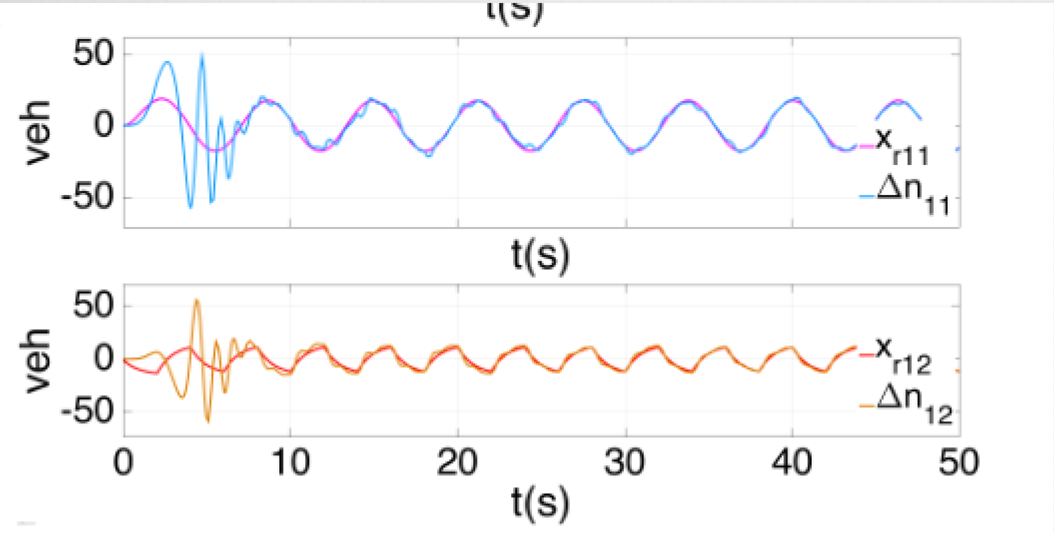
Numerical examples



Case 2

MFD_1=MFD_2=MFD_Yok

$$A_{r1} = A_{r2} = \begin{bmatrix} -1 & 0 \\ 0 & -1 \end{bmatrix},\ B_{r1} = B_{r2} = [1 \quad 1]^{\mathrm{T}},$$

$$\tau_1 = 4, \tau_2 = 6,$$

$$S_{pi} = \begin{bmatrix} 0.2 & -0.03 \\ -0.03 & -0.2 \end{bmatrix} \cdot 10^{-3},$$

$$\gamma_1 = \gamma_2 = 1, \gamma_{21} = \gamma_{12} = 1, h_1 = h_2 = 1,$$

$$K_I = 0.05,\ K_P = 4, K_D = 0.005\,.$$

$$A_1 = \begin{bmatrix} -1.19 & -0.72 \\ 0.73 & -0.52 \end{bmatrix} \cdot 10^{-3}, \tilde{A}_{21} = \begin{bmatrix} 0.37 & -0.28 \\ 0 & 0 \end{bmatrix} \cdot 10^{-3},$$

$$B_1 = \begin{bmatrix} 2.95 & 0 \\ 0 & -3.82 \end{bmatrix}, A_2 = \begin{bmatrix} -0.50 & 0.37 \\ 0.96 & -1.01 \end{bmatrix},$$

$$\tilde{A}_{12} = \begin{bmatrix} 0 & 0 \\ -0.54 & 0.38 \end{bmatrix} \cdot 10^{-3}, B_2 = \begin{bmatrix} -2.95 & 0 \\ 0 & 3.82 \end{bmatrix}.$$

A linearized MFD-based model of the two regions is obtained at the equilibrium point for region 1: $n_{11,\mathrm{eq}} = 2000$, $n_{12,\mathrm{eq}} = 1300$, $q_{11,\mathrm{eq}} = 1.5$, $q_{12,\mathrm{eq}} = 2.5$, $u_{11,\mathrm{eq}} = 0.33$, $u_{12,\mathrm{eq}} = 0.65$, and for region 2: $n_{21,\mathrm{eq}} = 1500$, $n_{22,\mathrm{eq}} = 1700$, $q_{21,\mathrm{eq}} = 1.3$, $q_{22,\mathrm{eq}} = 1.5$, $u_{21,\mathrm{eq}} = 0.44$ $u_{22,\mathrm{eq}} = 0.48$.

Conclusion

1

the reference model adaptive control (MRAC) approach has been implemented to allow us designing distributed adaptive perimeter (DAP) control laws

2

This contribution does not only enhance the MFD modeling, but it also improves the perimeter control algorithms.

3

the paper focused only on two urban regions, the current control scheme is general and can be applied for multiple regions

THANKS FOR YOUR TIME

Zhengfei Supervised by Jack Haddad
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