Flight Course Maneuver Optimization for a Fighter Jet in a Threatened Area

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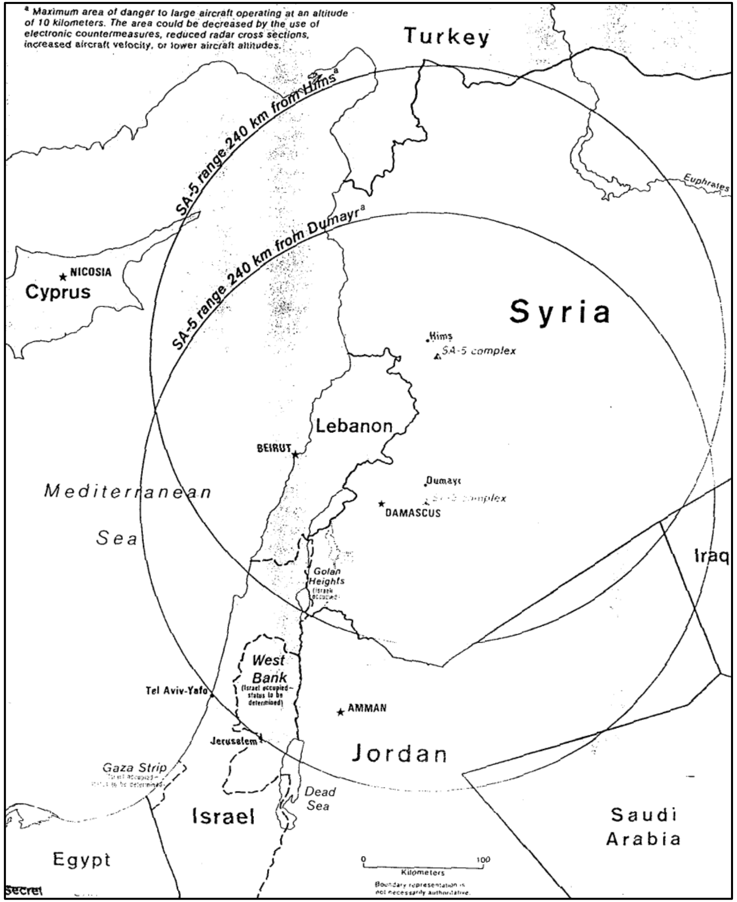
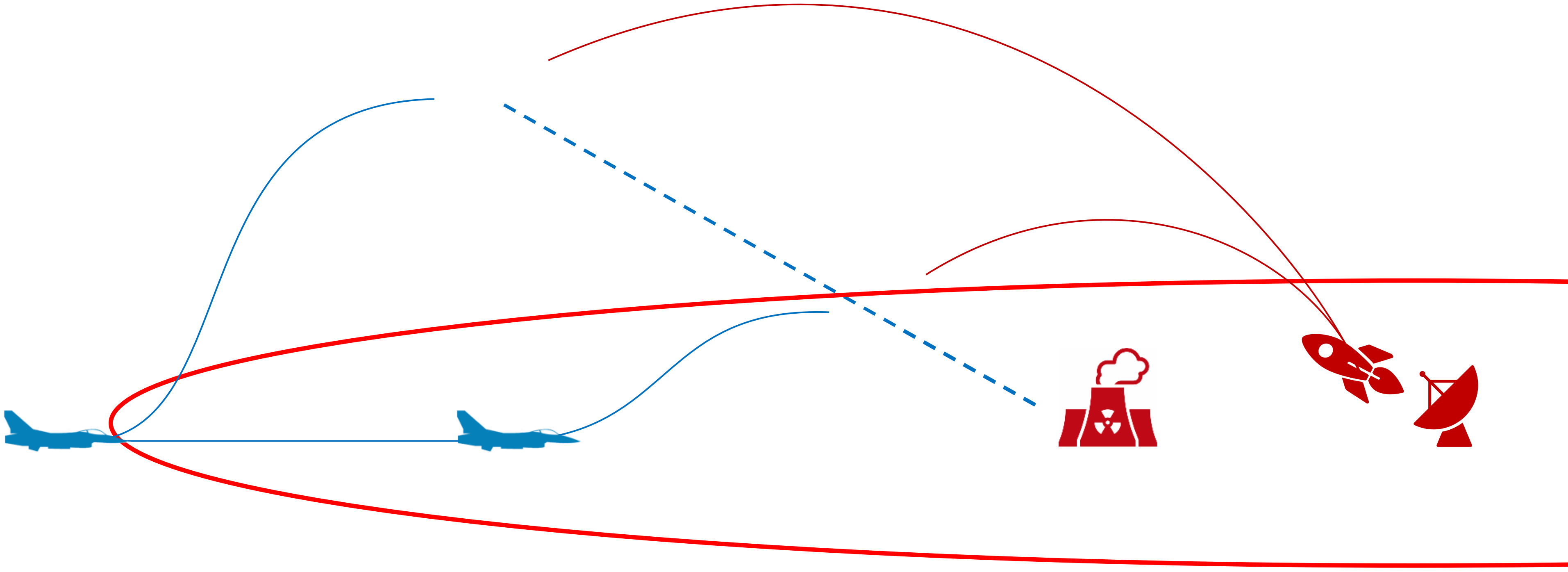
Problem Statement

- Deploy Munition on Target
- Missile-Protected Environment
- Optimize Survivability



Problem Statement - Illustration

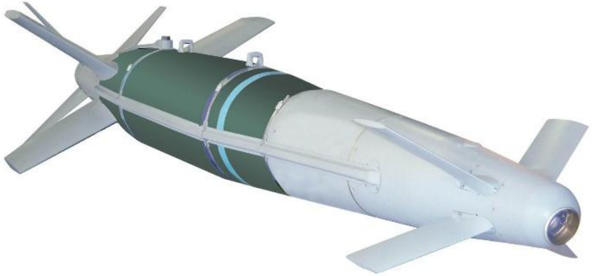


Physical Model

Missile

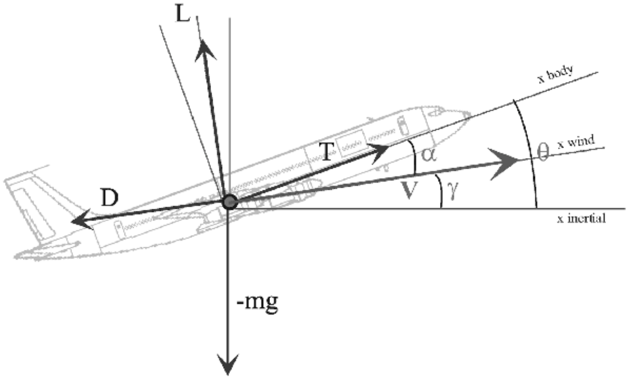
Aircraft

Munition



Aircraft Model

- 3DOF – Kinematics, Dynamics & Control
- Decoupled Engine & Aerodynamic Models
- Pilotable Control Inputs:
  - Load Factor
  - Roll Rate
  - Throttle



Quaternions

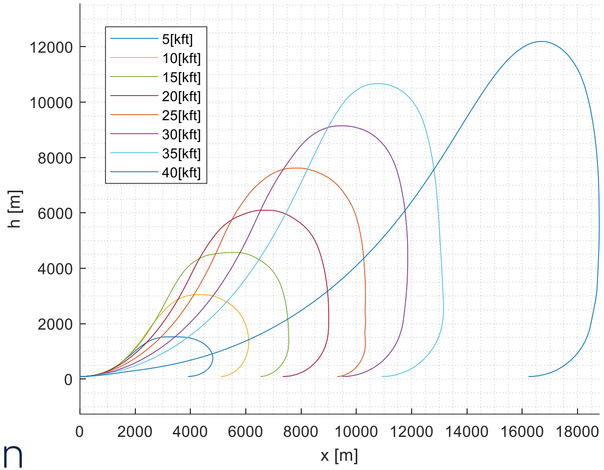
$$\begin{cases} \frac{d\underline{r}_E}{dt} = \underline{v}_{g\,E} \\ \dot{v} = \frac{g}{W}\underline{F}_v \cdot \hat{\underline{i}}_v \\ \dot{\underline{e}} = \frac{1}{2}\begin{bmatrix} 0 & -P & -Q & -R \\ P & 0 & R & -Q \\ Q & -R & 0 & P \\ R & Q & -P & 0 \end{bmatrix}\underline{e} \\ \dot{W} = -TSFC \cdot T \\ \dot{P} = -\frac{1}{\tau_P}(P - P_c) \\ \dot{J}_s = -\frac{1}{\tau_J}(J_s - J_{s,c}) \\ \dot{n} = -\frac{1}{\tau_n}(n - n_c) \end{cases}$$

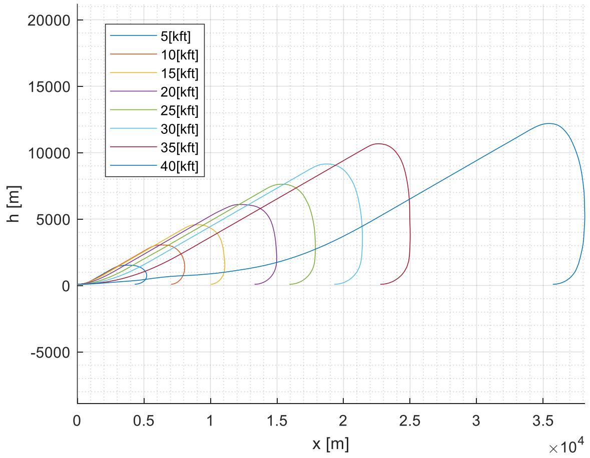
Euler Angles

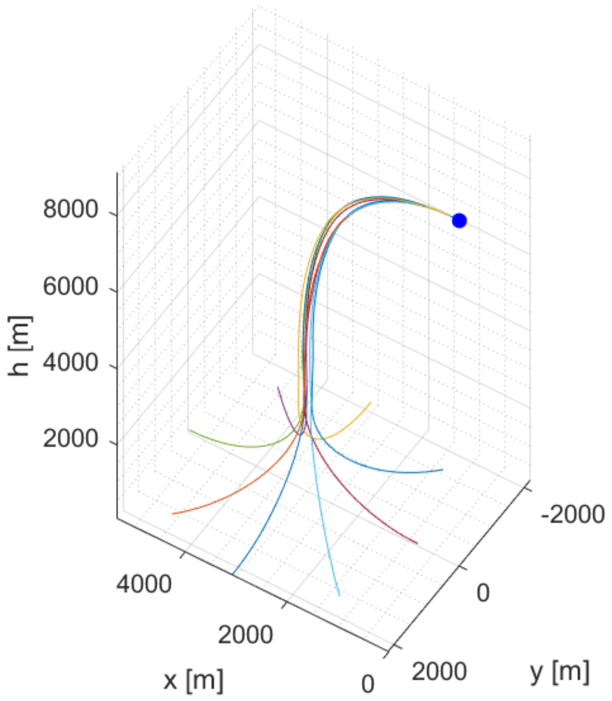
$$\begin{cases} \dot{x} = v\cos\gamma\cos\psi + v_w\cos(\psi_w + \pi) \\ \dot{y} = v\cos\gamma\sin\psi + v_w\sin(\psi_w + \pi) \\ \dot{h} = v\sin\gamma \\ \dot{v} = g\frac{T\cos\alpha - D}{W} - g\sin\gamma \\ \dot{\phi} = P + \dot{\psi}\sin\gamma \\ \dot{\gamma} = \frac{g}{v}(n_g\cos\phi - \cos\gamma) \\ \dot{\psi} = n_g\frac{g}{v\cos\gamma}\sin\phi \\ \dot{W} = -TSFC \cdot T \\ \dot{P} = -\frac{1}{\tau_P}(P - P_c) \\ \dot{J}_s = -\frac{1}{\tau_J}(J_s - J_{s,c}) \\ \dot{n} = -\frac{1}{\tau_n}(n - n_c) \end{cases}$$

Flight Course

- 2 Phase Maneuver:
  - Climb
  - Evasion Maneuver
- Minimum Time Optimization
- GPOPS (& FALCON)
- "Open-Loop"
- Parameter Inputs:
  - Munition Toss Altitude
  - Munition Toss Mach
  - Maneuver Turn Angle







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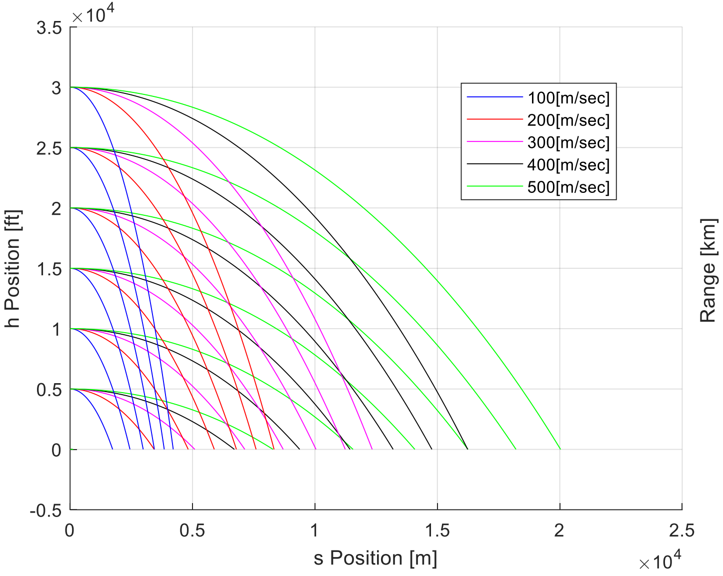
Munition Model

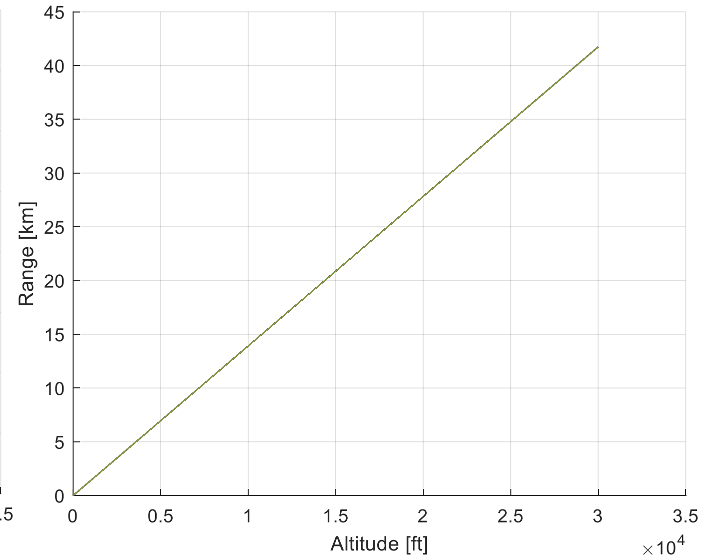
$$\begin{cases} \dot{x} = v\cos\gamma \\ \dot{y} = v\sin\gamma \\ \dot{v} = -g\sin\gamma - kgv^2, \\ \dot{\gamma} = -\frac{g}{v}\cos\gamma \end{cases} \qquad k = \frac{\rho S C_D}{2mg}$$

$$\gamma = \frac{C_D^*}{C_L^*}, \qquad C_L^* = \sqrt{\frac{C_{D_0}}{K}}, \qquad C_D = C_{D_0} + K{C_L}^2$$

- 2 Types
- Determines Aircraft Range From Target



Ballistic



Gliding

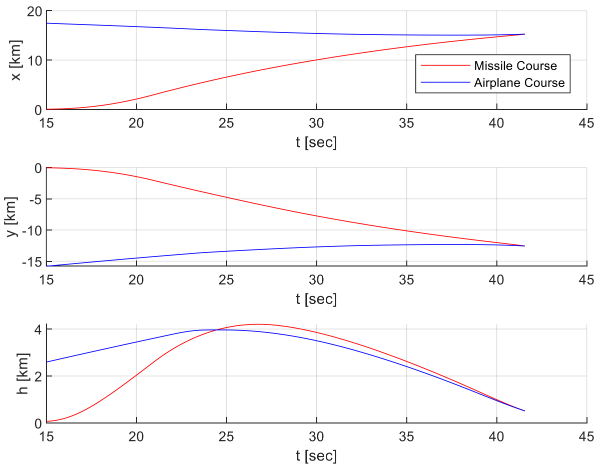
Missile Model

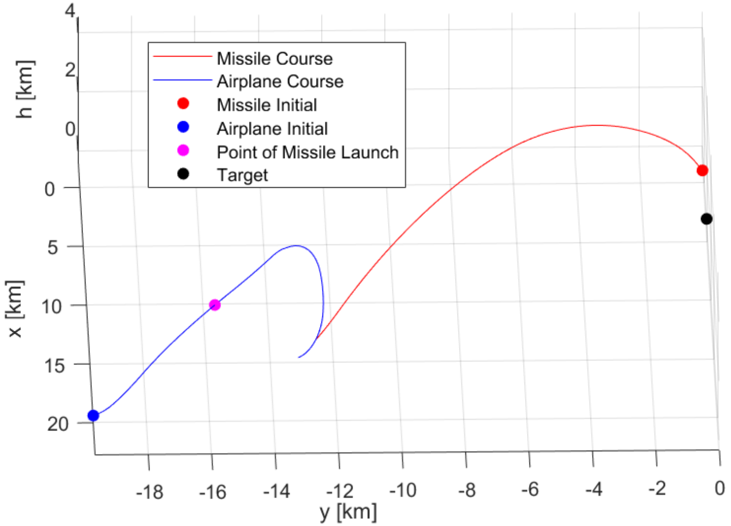
- 3DOF – Kinematics, Dynamics & Control
- Proportional Navigation Guidance Law
- Radar Detection & Delay
- "Red Side" Uncertainty

$$\begin{cases} \ddot{r}_M = \dot{\lambda}_M{}^2 r_M + \dot{\psi}_M{}^2 r_M \cos^2 \lambda_M - g \sin \lambda_M - k_M v_M \dot{r}_M + u_{M_x} \\ \ddot{\lambda}_M = -\dfrac{2\dot{\lambda}_M \dot{r}_M}{r_M} - \dfrac{1}{2} \dot{\psi}_M{}^2 \sin 2\lambda_M - \dfrac{g}{r_M} \cos \lambda_M - k_M v_M \dot{\lambda}_M - \dfrac{u_{M_z}}{r_M} \\ \ddot{\psi}_M = 2\dot{\psi}_M \dot{\lambda}_M \tan \lambda_M - \dfrac{2\dot{\psi}_M \dot{r}_M}{r_M} - k_M v_M \dot{\psi}_M + \dfrac{u_{M_y}}{r_M \cos \lambda_M} \\ \dot{\underline{u}}_M = -\dfrac{1}{\tau_M} \left(\underline{u}_M - \underline{u}_{M,c} \right) \end{cases}$$

Interception Simulation

- Simulating the Aircraft as target in open-loop
- Missile ODE Solution
- Output – Miss Distance
- Aircraft Flight Azimuth





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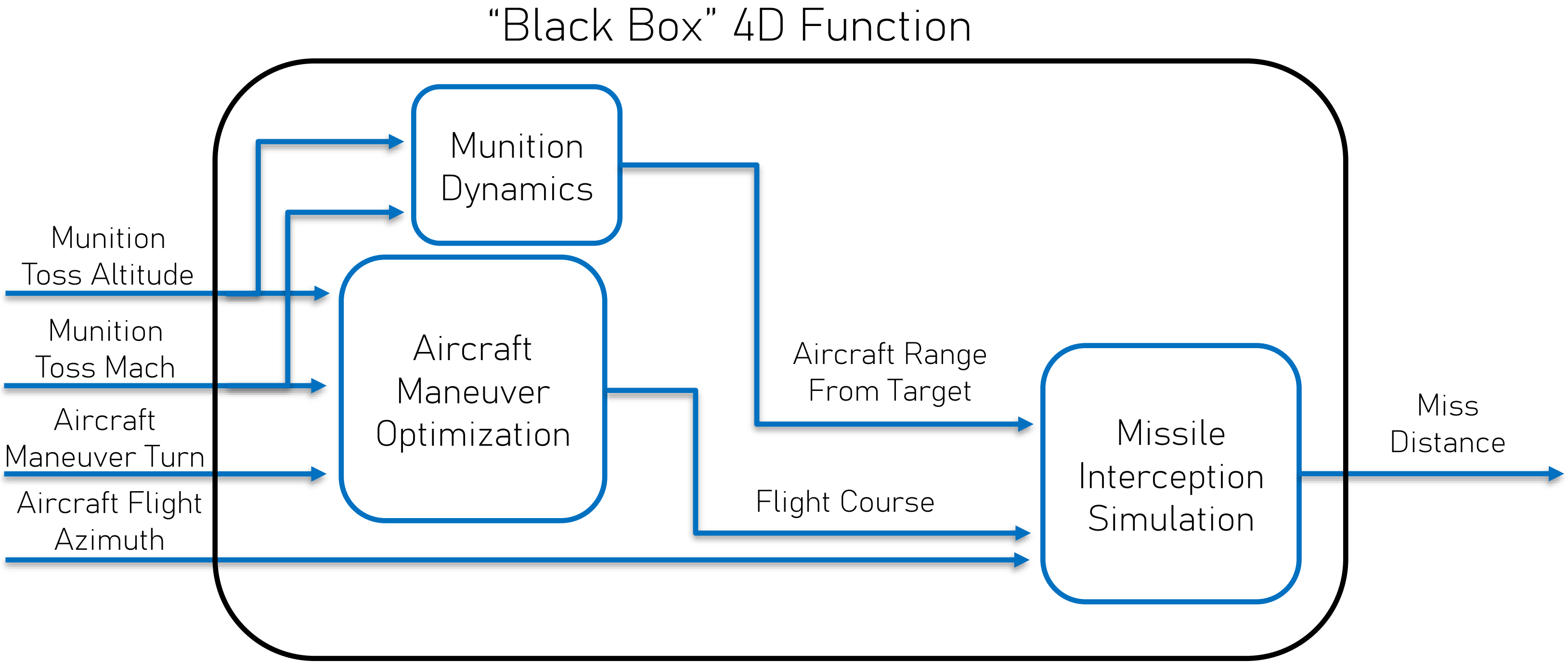
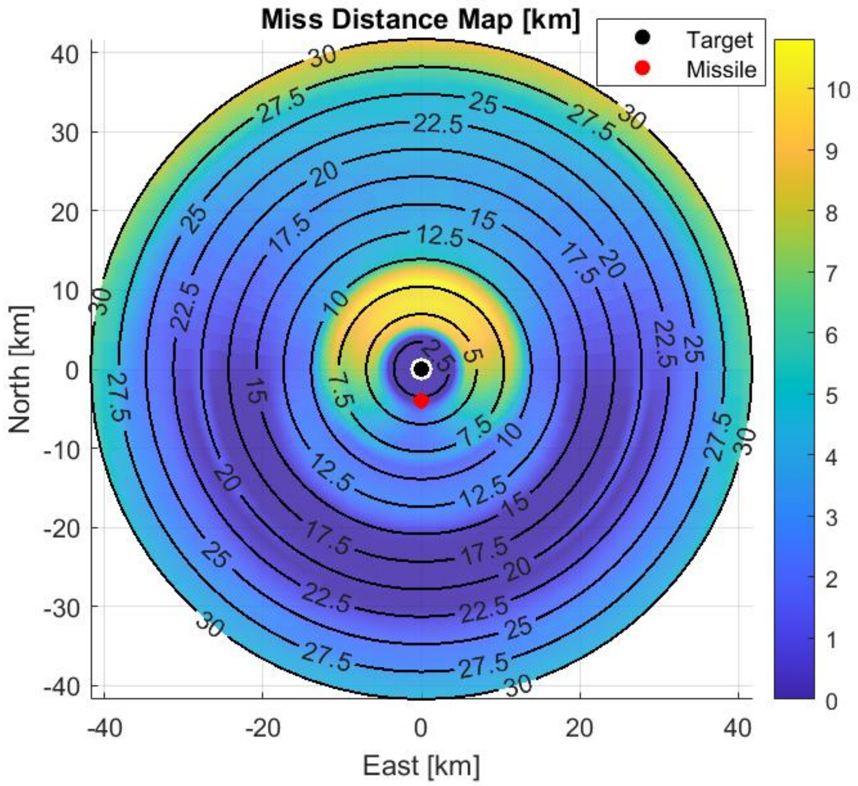
Bi-Level Optimization



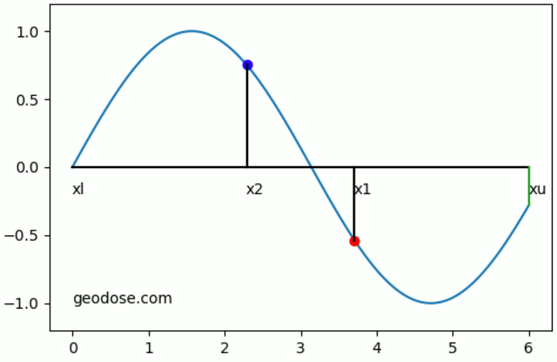
Survivability Envelope



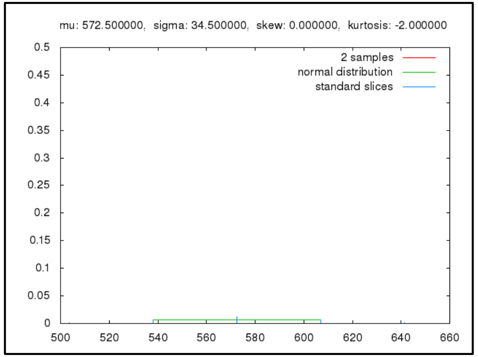
Optimization Methods

- Multivariable Non-Convex "Black Box" Function
- Numerically Heavy
- Iterative Methods

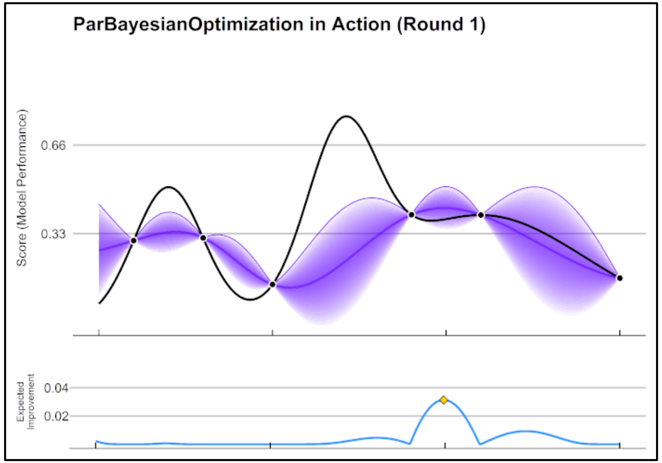
Golden-Section Search



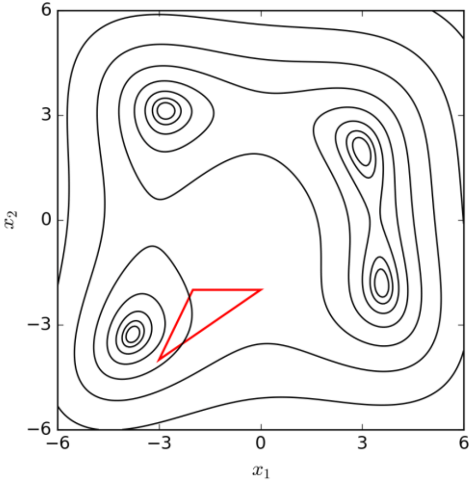
Monte-Carlo



Bayesian Optimization



Nelder-Mead



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Thank You!