Coupled Oscillator Models for Multi-legged Robots

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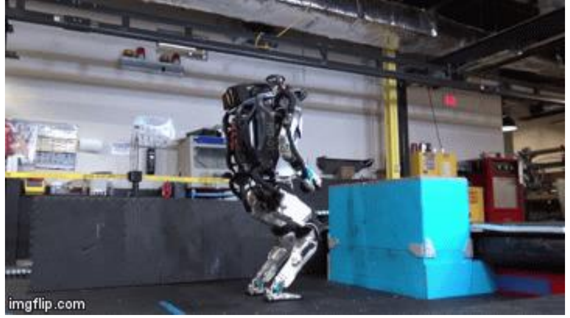
Outline

Motivation & Background

- Legged robots are constantly developing and are used in many fields to aid human work and safety.
- These are usually inspired by animals, from bipedal humans and birds to multi-legged insects and other crawlers.



From top to bottom: Boston Dynamics' Atlas, Agility Robotics' Cassie, Boston Dynamics' Spot and CSRIO Data61's Gizmo.

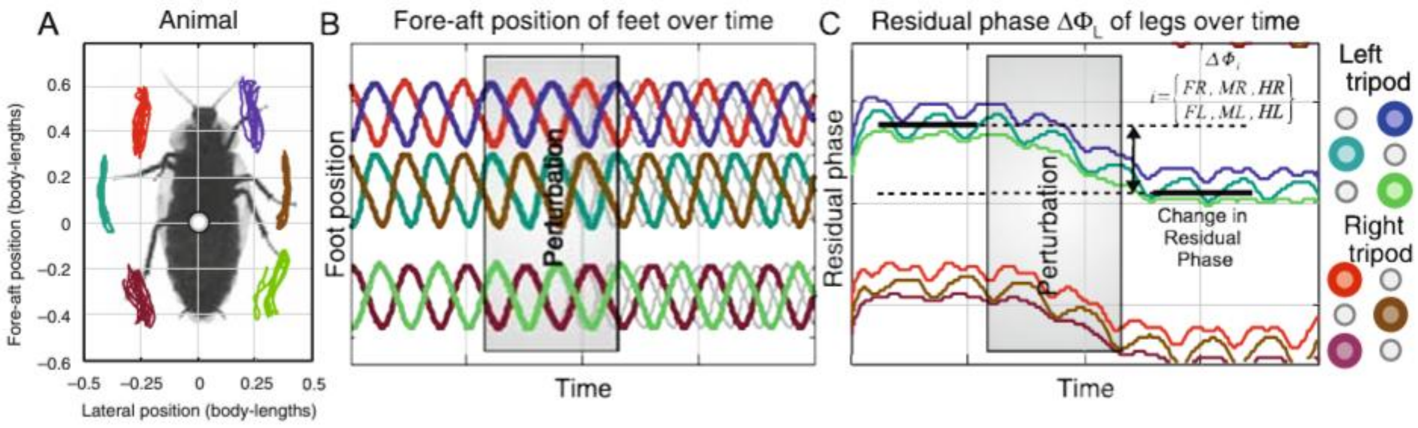
Key Ideas

- Multi-legged animals have “rhythmic” (almost periodic) gaits.
    - Gaits – repeated shape changes of the body that move it in the world.
- A rhythmic gait can be modelled as an oscillator (which decays into a **limit cycle** that is governed by a **phase**); with system noise (defined by **Floquet theory**).

1. Revzen S, Koditschek DE, Full RJ. Towards testable neuromechanical control architectures for running. Progress in motor control: a multidisciplinary perspective. 2009:25-55.
2. Revzen S. and Kvalheim M. Locomotion as an Oscillator. Bioinspired Legged Locomotion. 2017:97-110

Key Ideas

- The system is an oscillator; its variables also oscillate. Therefore, we can find subsystems that are also oscillators.
- Complicated mechanical coupling between DOFs reduces to coupling between oscillators which further reduces to co.
- In small deviations about a cycle, there is coupling between **phases**.



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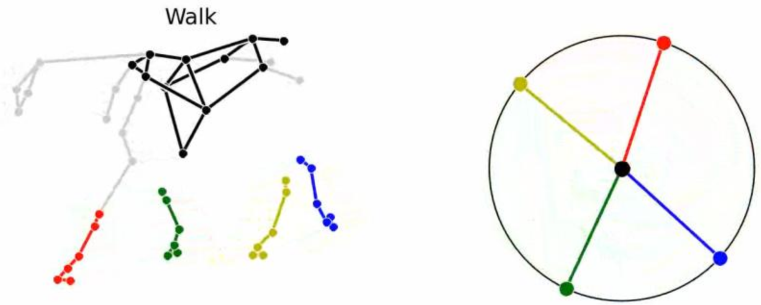
Key Ideas – Oscillators as Framework

- We can use **coupled phase oscillators** as a framework for a family of models for rhythmic motion.
- Mechanical systems' dynamics are complicated
- Measurements are partial and noisy.
- Thus, we chose to develop data-driven tools for this framework.



Revzen S. and Kvalheim M. Locomotion as an Oscillator. Bioinspired Legged Locomotion. 2017:97-110

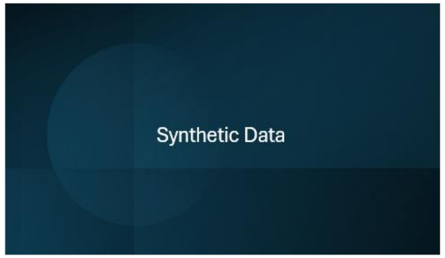
Use **coupled phase oscillators** as models for gaits instead of classical mechanical dynamics.

Develop data driven tools that can perform this task.

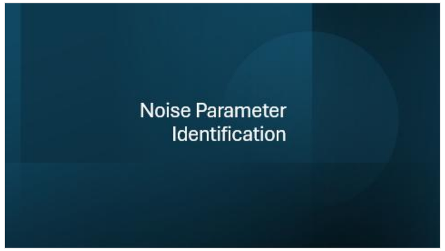
Research Goals

Main Research Contributions

Produced a synthetic dataset of motions based on coupled Hopf oscillators.



Chose the random perturbation parameters to mimic statistics from real robot’s noisy data.

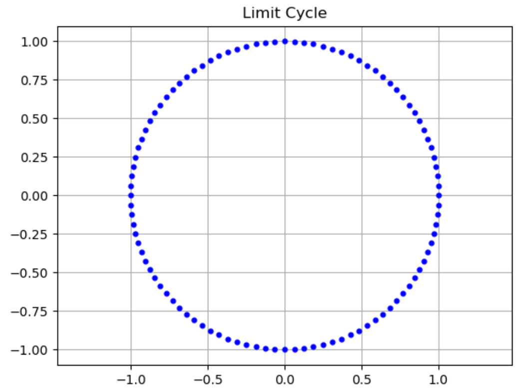


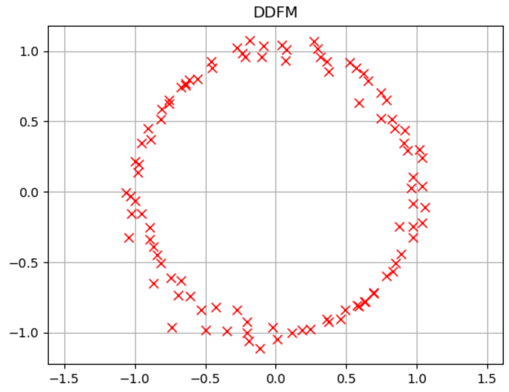
Compared our coupled-oscillators model to a phase oscillator driving a limit cycle; and a Data-Driven Floquet Analysis model.

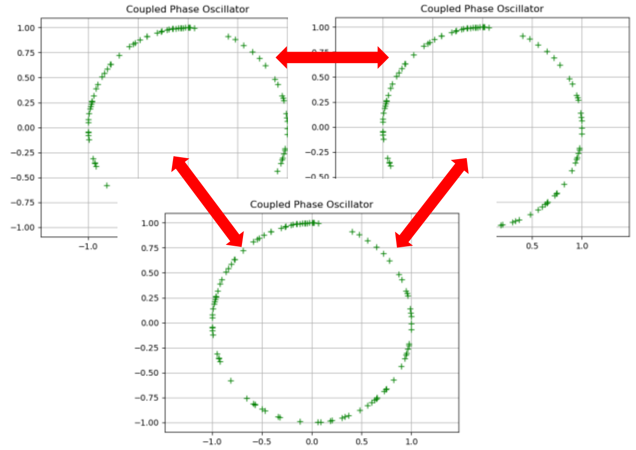
The 3 Models

The 3 Models

1. A phase oscillator driving a limit cycle
2. Data-Driven Floquet Analysis (DDFA)
3. A coupled phase oscillator Model







Why Have 3 models?

- We know the limit cycle cannot model perturbation but easy to calculate.
- DDFA models need lots of data to calculate and predict things other than phase.
- Coupling Oscillator model – is a middle ground between ease of calculation and prediction power.

1. The Limit Cycle

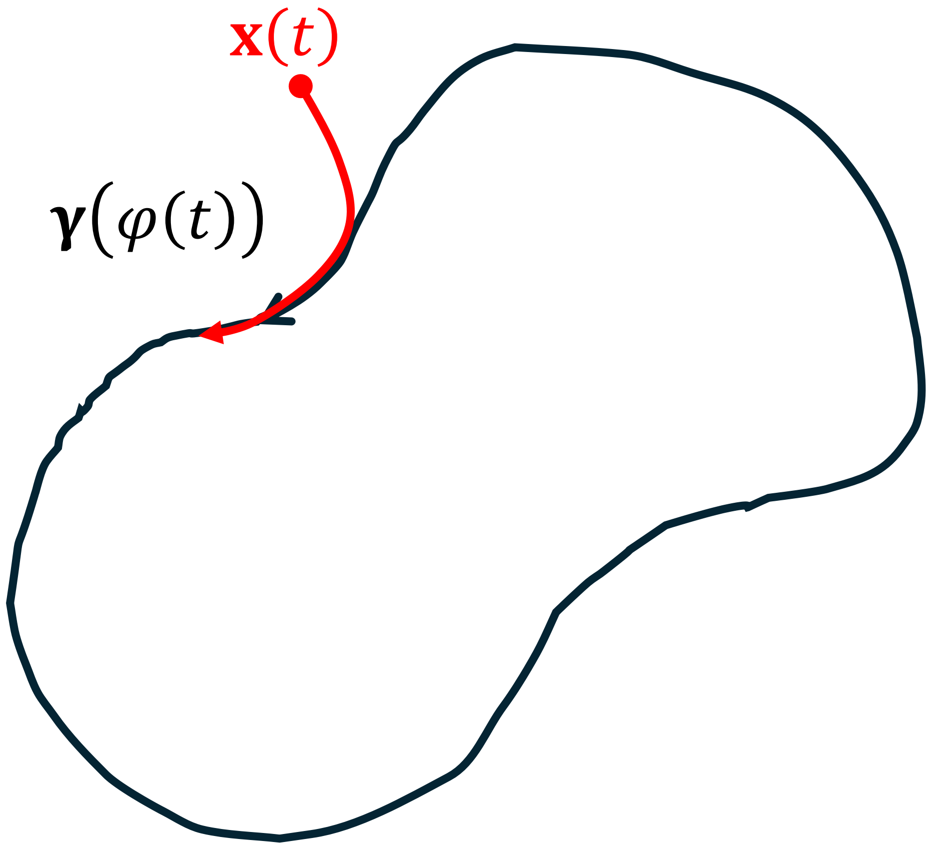
- We model our system $\mathbf{x}(t)$ as the periodic solution.
- Our system has a **global phase** $\varphi(t)$ such that $\varphi(t) = e^{j\omega t}\varphi(0)$.

$$\mathbf{x}(t) \approx \boldsymbol{\gamma}\big(\varphi(t)\big)$$

- Defining the limit cycle $\boldsymbol{\gamma}(\varphi)$ which we estimate as a Fourier series.

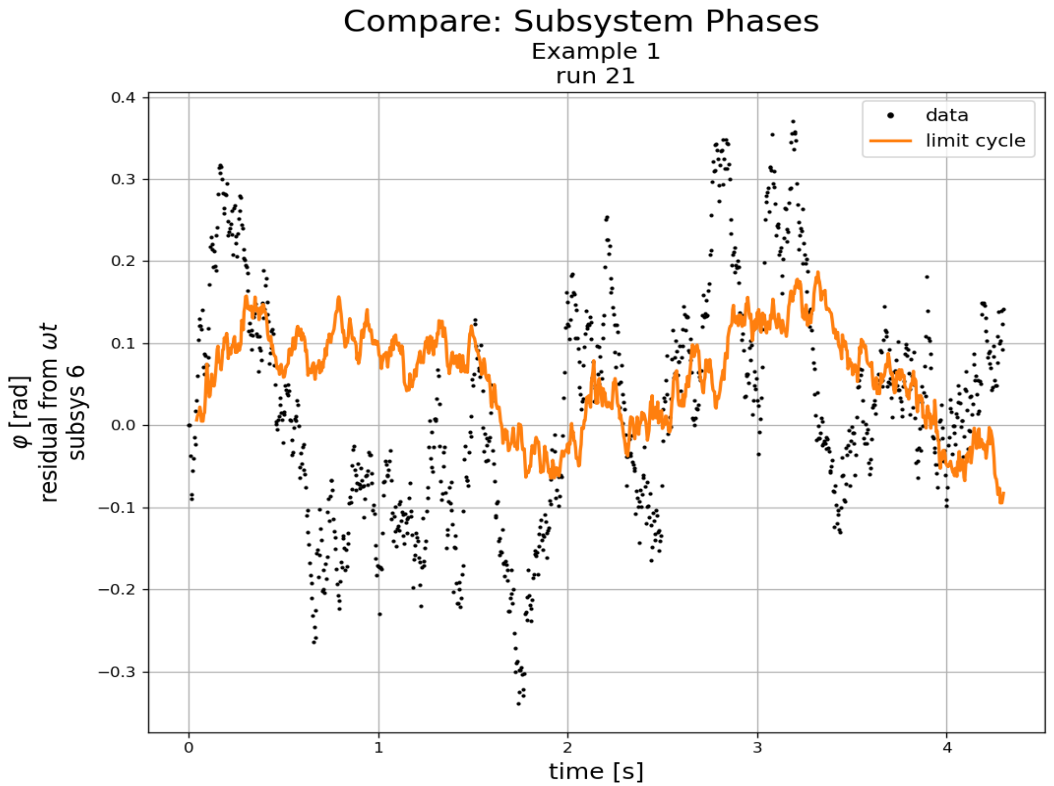
$$\hat{\boldsymbol{\gamma}}(\theta) := \sum_{m=-N_{ord}}^{N_{ord}} c_m e^{jm\theta}$$

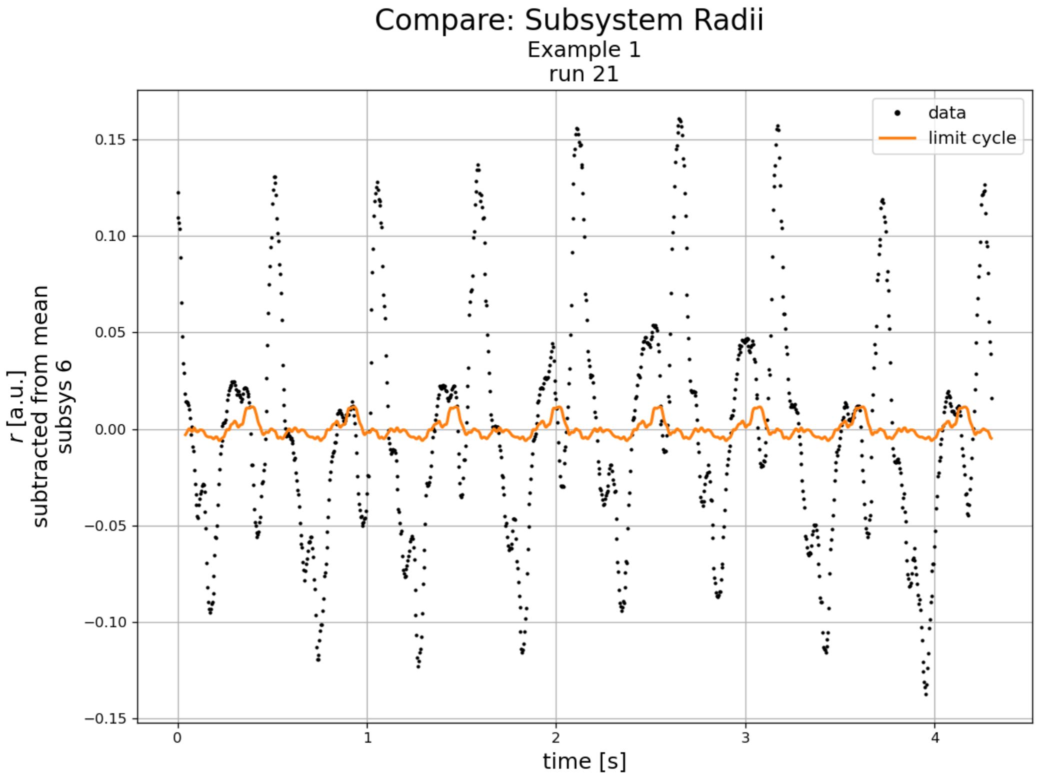
- With the condition that $\varphi(0)$ is the phase of $\mathbf{x}(0)$.



1. The Limit Cycle

- The prediction $\tilde{\mathbf{x}}_0^k[n+s]$ from some initial state and phase $\hat{\mathbf{x}}^k[n], \hat{\varphi}^k[n]$ by advancing s sample points into the future, consists of advancing the limit cycle by the phase difference. $$\tilde{\mathbf{x}}_0^k[n+s] = \hat{\boldsymbol{\gamma}}\left(e^{j\omega s\Delta t}\hat{\varphi}^k[n]\right)$$





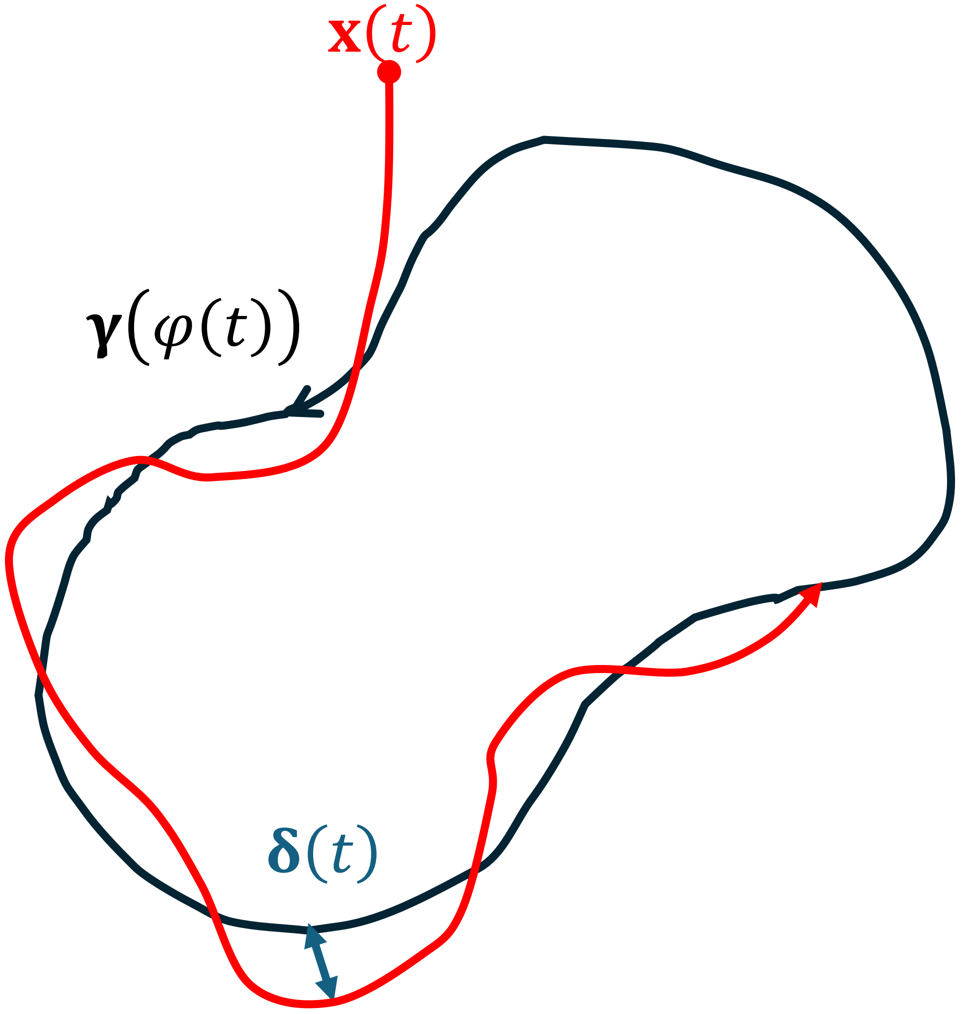
2. Data-Driven Floquet Analysis

- In our case the system $\mathbf{x}(t)$ isn't always at the limit cycle so we assume some perturbations $\boldsymbol{\delta}(t)$ from the limit cycle $\boldsymbol{\gamma}(\varphi)$:

$$\mathbf{x}(t) \approx \boldsymbol{\gamma}(\varphi) + \boldsymbol{\delta}(t)$$

- In practice we estimated perturbations $\{\widehat{\boldsymbol{\delta}}[n]\}^k$ as the difference from the limit cycle:

$$\widehat{\boldsymbol{\delta}}^k[n] \coloneqq \hat{\mathbf{x}}^k[n] - \hat{\boldsymbol{\gamma}}(\hat{\varphi}^k[n])$$



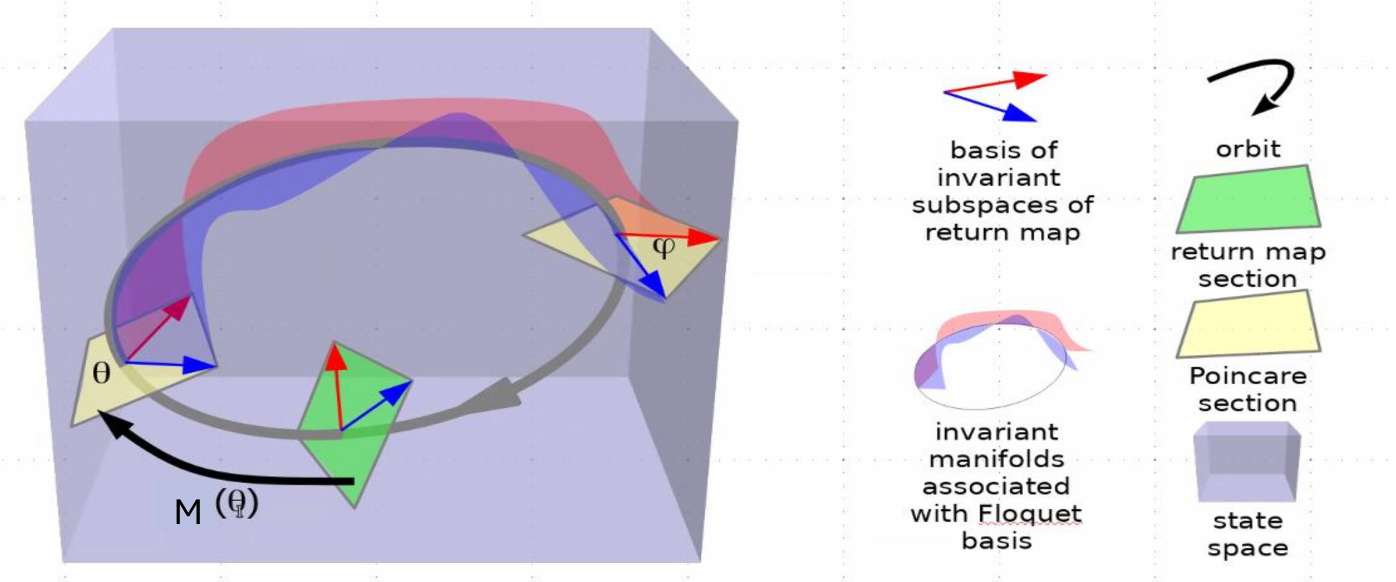
[2] Guckenheimer J, Holmes P. Nonlinear oscillations, dynamical systems, and bifurcations of vector fields. Springer Science & Business Media; 2013 Nov 21. 24-25

2. Floquet Theory

- Floquet theory governs linear time periodic (LTP) differential equations.
- It defines a **flow matrix** that determines the behavior of solutions near the limit cycle [2].

$$\begin{cases} \boldsymbol{\delta}(t) = \mathbf{M}_{\theta}(t)\boldsymbol{\delta}(0) \\ \varphi(t) = e^{j\omega t}\underbrace{\theta}_{\varphi(0)} \end{cases}$$

S. Revzen

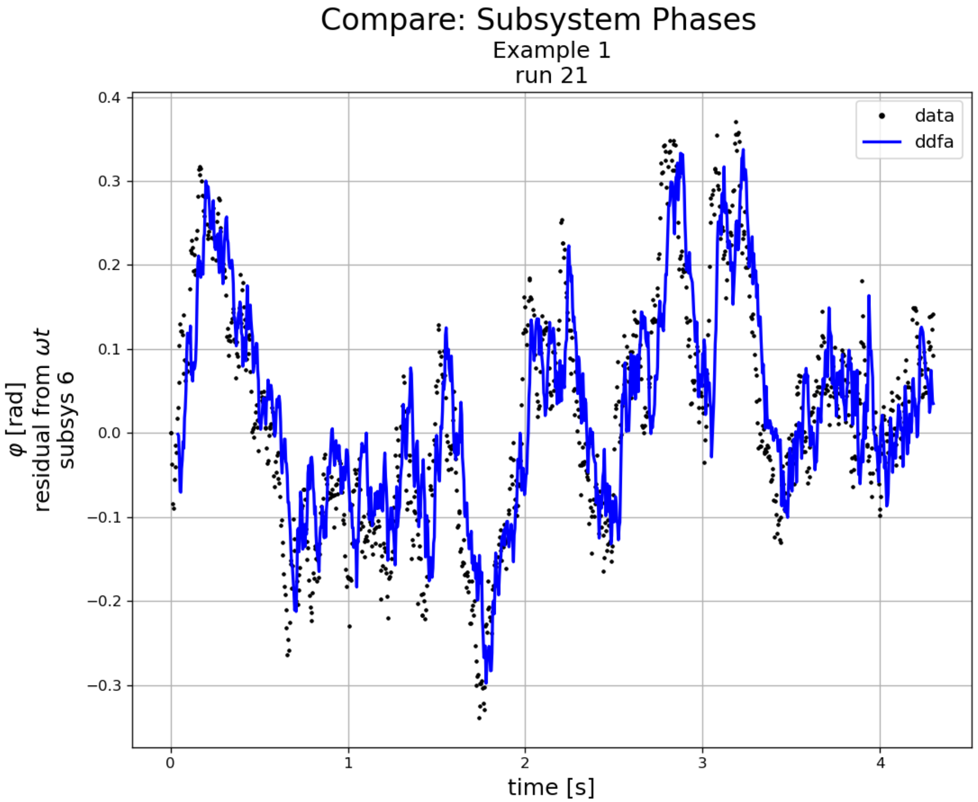


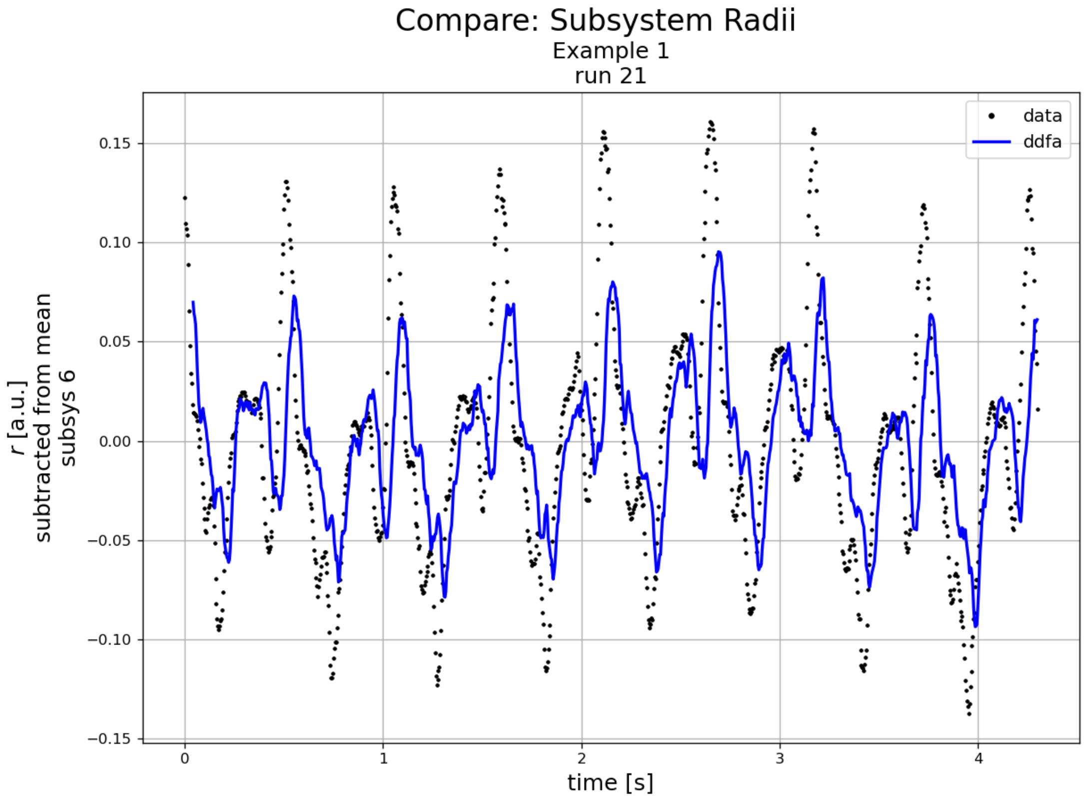
- The flow matrix is estimated as $\widehat{\mathbf{M}}[\theta, n]$ using LLS of pairs of perturbations $\{\widehat{\boldsymbol{\delta}}[n], \widehat{\boldsymbol{\delta}}[n+s]\}^{k}$ for multiple paths s samples apart.

2. Data-Driven Floquet Analysis

- The propagating perturbation $\widehat{\mathbf{M}}[\theta,s]\widehat{\boldsymbol{\delta}}^k[n]$ is now added to the prediction from before:

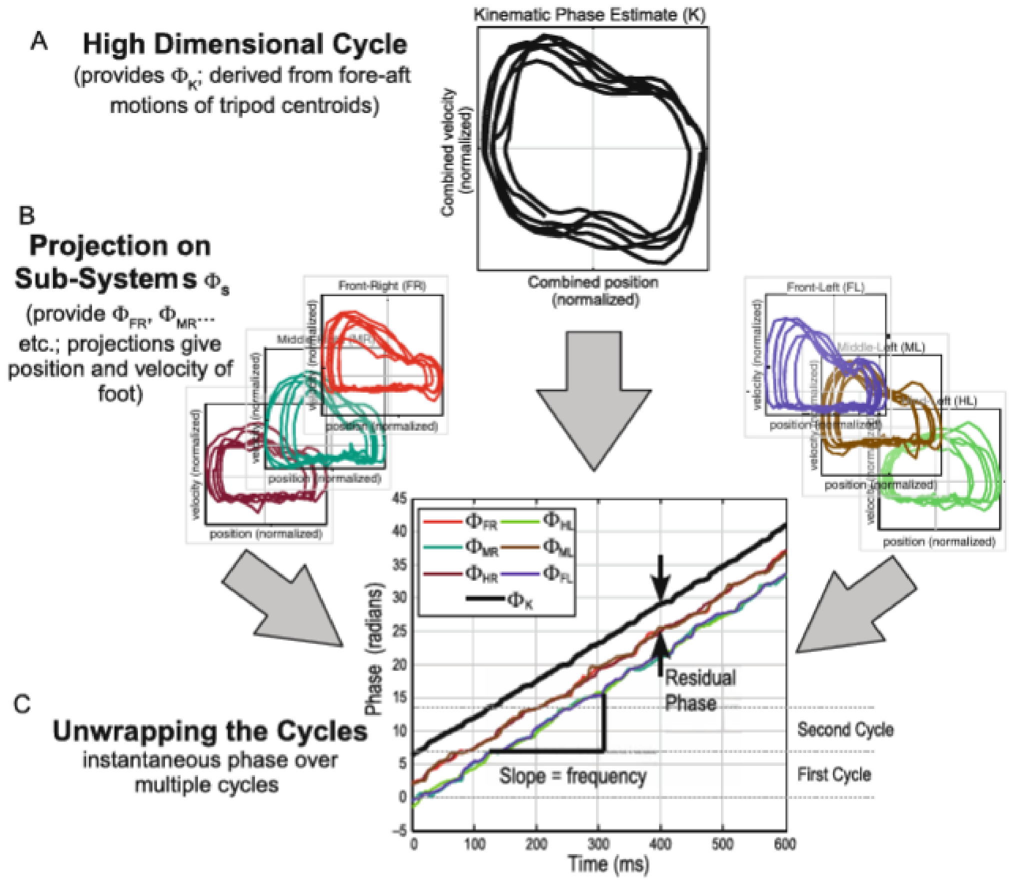
$$\tilde{\mathbf{x}}_1^k[n+s] = \hat{\boldsymbol{\gamma}}\left(e^{j\omega s\Delta t}\hat{\varphi}^k[n]\right) + \widehat{\mathbf{M}}[\theta,s]\widehat{\boldsymbol{\delta}}^k[n]$$





3. Subsystem Phase

- When looking at data and modeling as an oscillator we can model partial data as an oscillating subsystem.
- We chose a set of natural subsystems – the robot legs.
- Whether this is a sufficient model is the main question.



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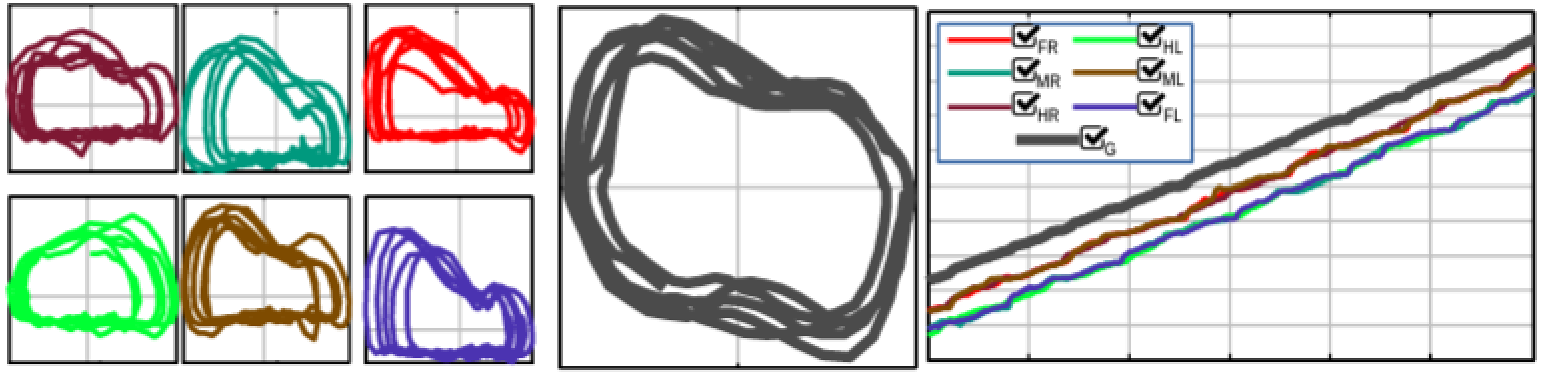
3. Coupled Phase Oscillator Model

- Instead of modeling the entire path we only model the phase states of each subsystem using a coupling term and stochastic noise:

$$\dot{\phi}_i(t) = \omega + \sum_{j=1}^{N_s} c_{ji}(\varphi)\left(\phi_j(t) - \phi_i(t)\right) + d\nu$$

- For simplicity we use phases as real numbers from now on.
- N_s is the number of subsystems (number of legs).



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3. Coupled Phase Oscillator Model

- Using a bit of algebra, we can convert the equation into vector form for all legs where each element is the "relative phase" $\beta_i(\mathrm{t}) = \phi_i(t) - \angle\varphi$: $\dot{\boldsymbol{\beta}}(t) \approx \mathbf{A}(\varphi)\boldsymbol{\beta}(t)$
- With Floquet solution: $\boldsymbol{\beta}(t) = \mathbf{F}_\theta[\mathrm{t}]\boldsymbol{\beta}(0)$.
- $\mathbf{F}$ represents a flow matrix of phase perturbation from global phase, which we can estimate similarly to DDFA as $\mathbf{F}[\theta, s]$.

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3. Coupled Phase Oscillator Model

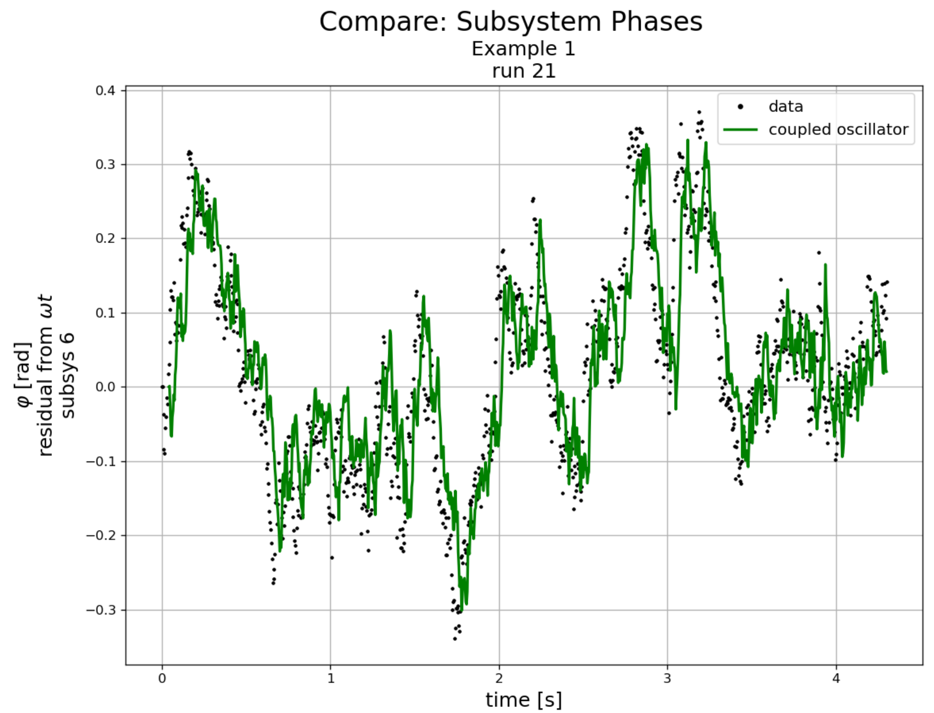
- The leg phases are then estimated as:

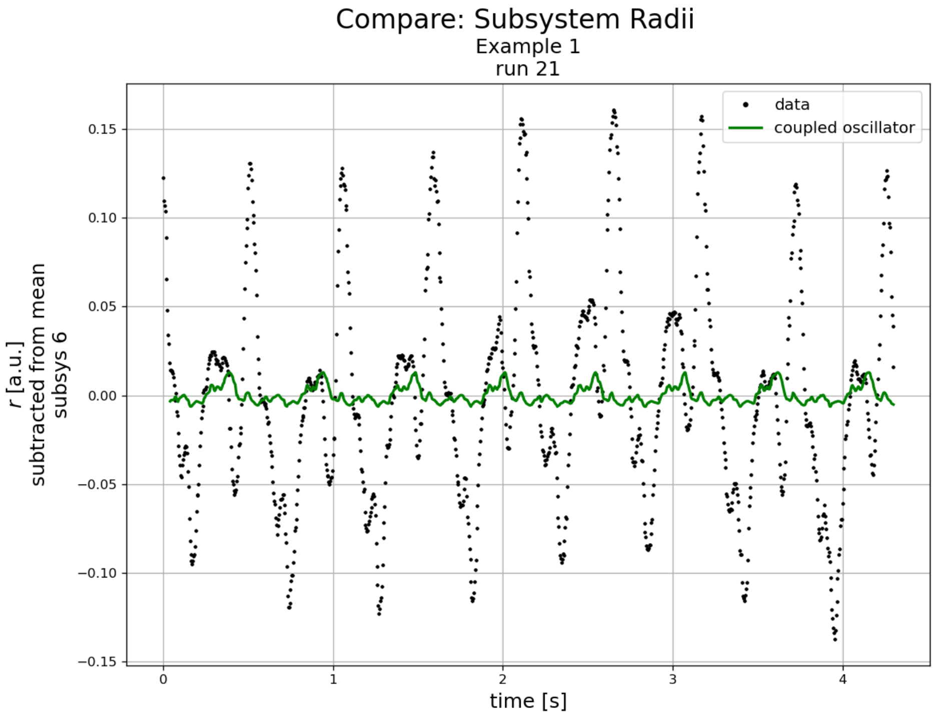
$$\tilde{\boldsymbol{\varphi}}^k[n+s] = \left(\omega s \Delta t + \angle \hat{\varphi}^k[n]\right) + \hat{\mathbf{F}}[\theta, s]\hat{\boldsymbol{\beta}}^k[n]$$

- Where $\tilde{\boldsymbol{\varphi}}^k = [\tilde{\phi}_1^k, \ldots, \tilde{\phi}_{N_s}^k]$

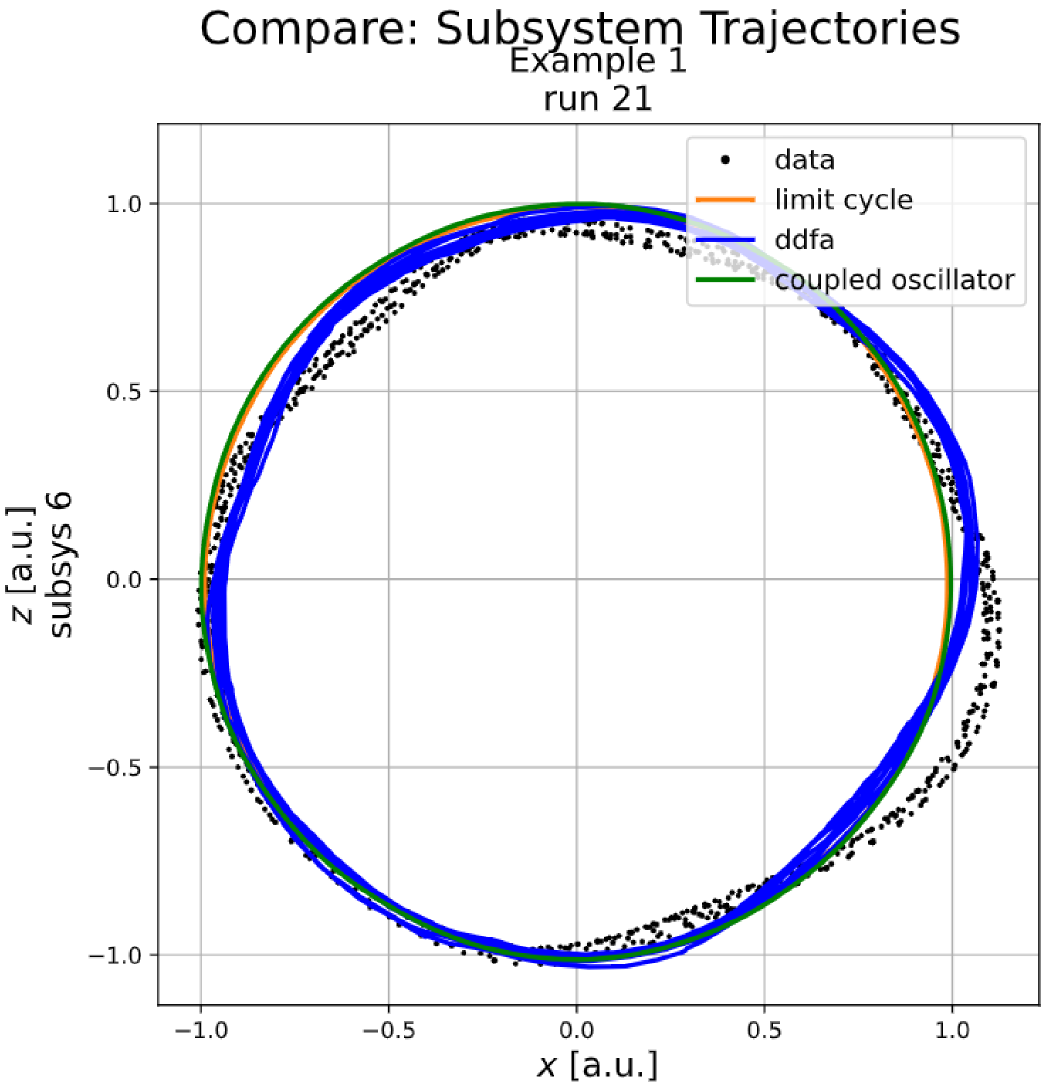
- We then similarly use N_s Fourier series $\hat{\psi}_i$ as limit cycle estimations of each leg where $\tilde{\mathbf{x}}_{2,i}^k[n+s] = \hat{\psi}_i\left(\tilde{\phi}_i^k[n+s]\right)$:

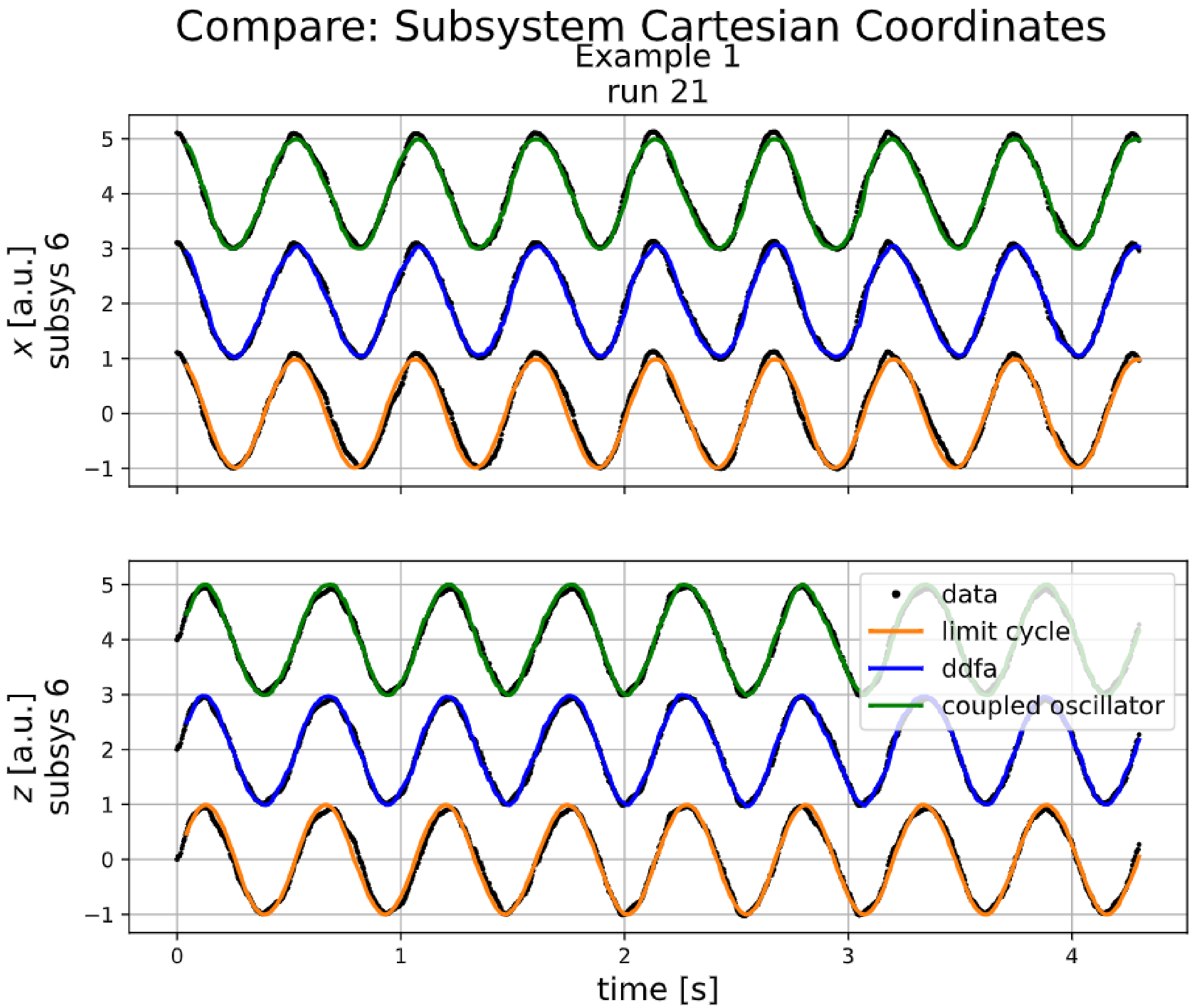
$$\tilde{\mathbf{x}}_2^k[n+s] = \hat{\boldsymbol{\psi}}\left(\tilde{\boldsymbol{\varphi}}^k[n+s]\right)$$





Model Comparison





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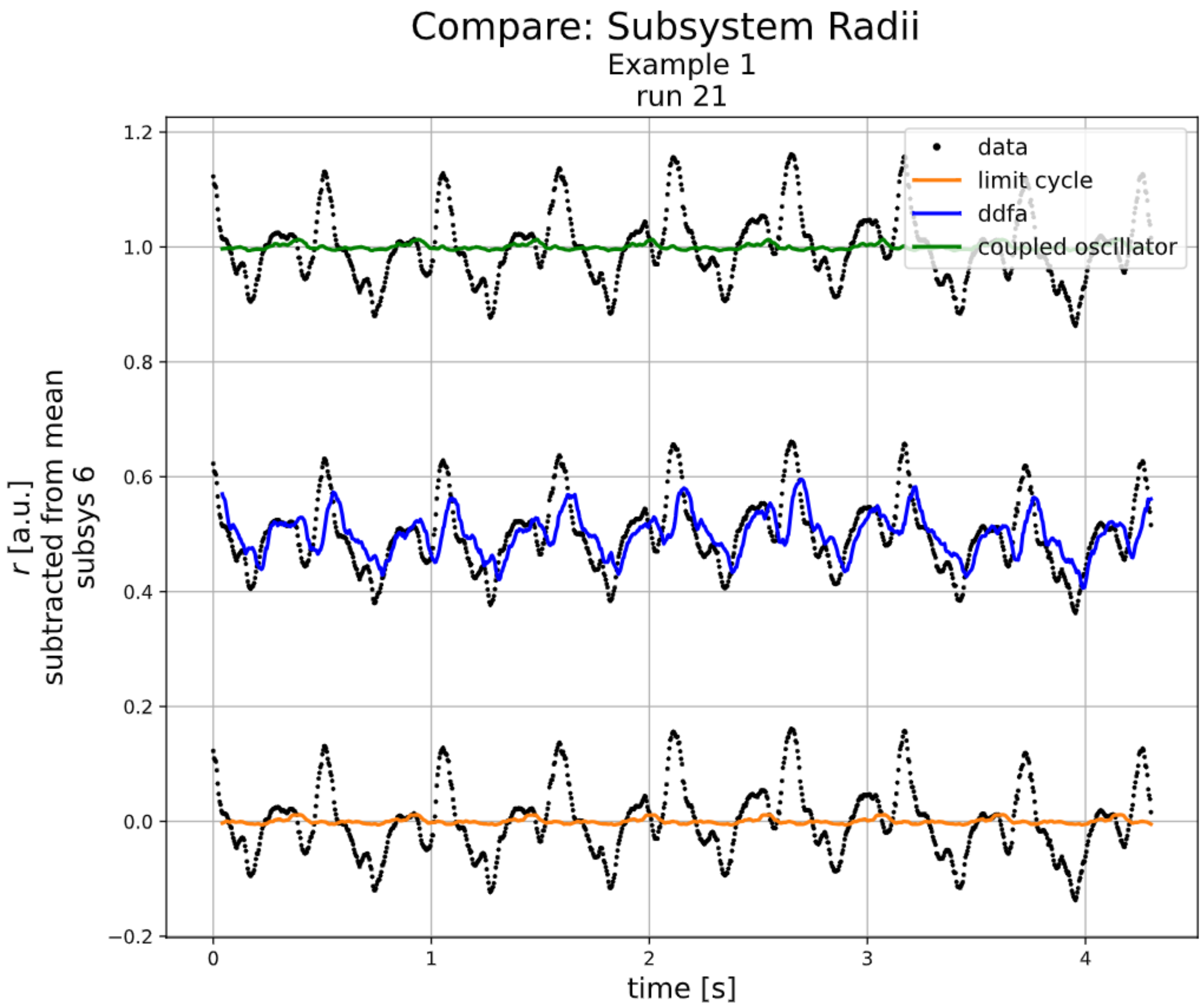
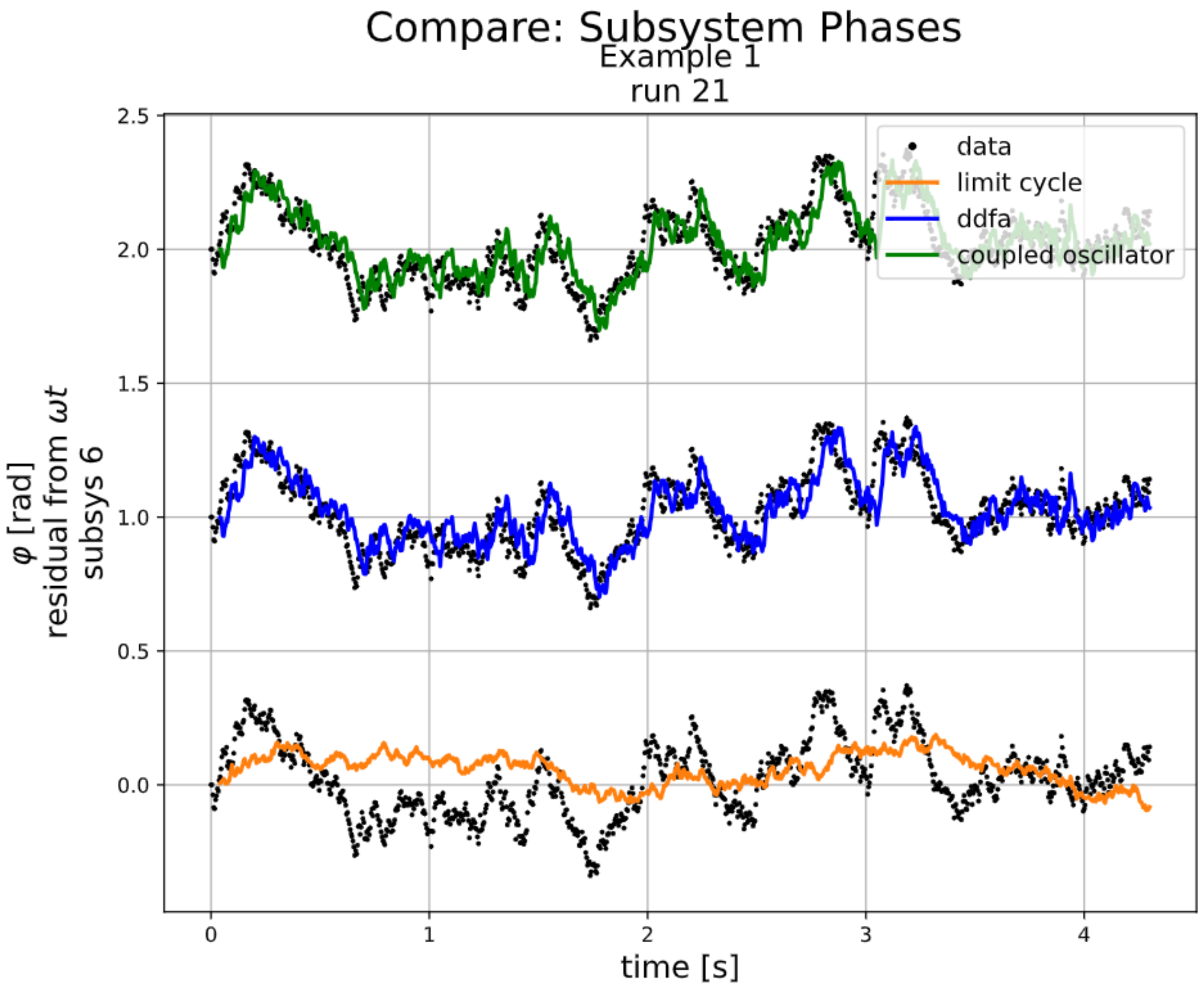
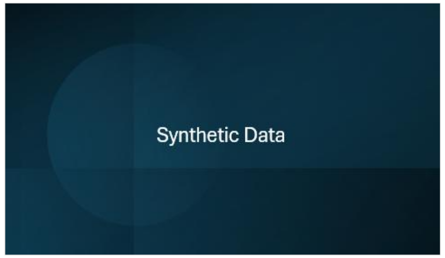


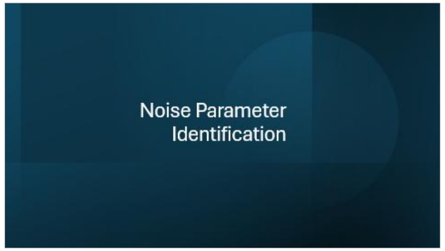


Main Research Contributions

Produced a synthetic dataset of motions based on coupled Hopf oscillators.



Chose the random perturbation parameters to mimic statistics from real robot’s noisy data.



Compared our coupled-oscillators model to a phase oscillator driving a limit cycle; and a Data-Driven Floquet Analysis model.

Synthetic Data

Dataset

- Research is based on data collected from Octobot – an 8-legged modular robot developed by our collaborators at the BIRDS lab at the University of Michigan.
- This was used to create a synthetic dataset as a tool for comparing models.

Why Synthetic Data?

- Parameters are not known for real data (such as coupling).
- Gives us a **ground truth** to compare to.
- Allows for easy configuration of working environment.

Main Research Contributions

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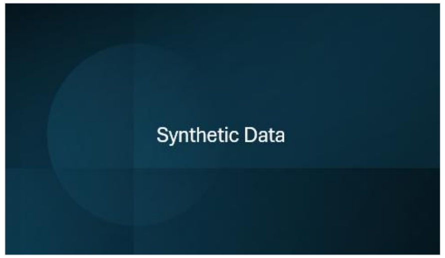

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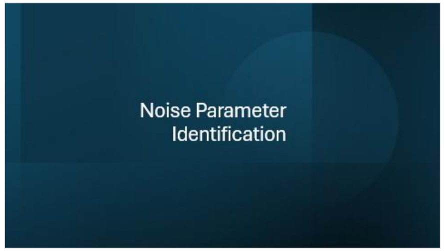

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Noise Parameter Identification

Why Extract Noise Parameters?

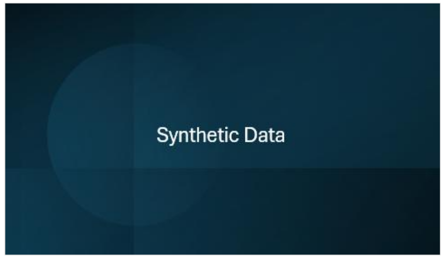
- Statistical power depends on noise.
- Too little noise – not enough perturbations.
- Too much noise – not enough correlation.
- Parameters such as phase standard deviation should track real-world data.

Noise Parameter Identification

- Phase is a circular variable: usual mean and variance cannot be used.
- We estimated noise parameters using **directional statistics**.
    - Directional statistics – statistics of directions and rotations on a unit circle.

Main Research Contributions

Produced a synthetic dataset of motions based on coupled Hopf oscillators.



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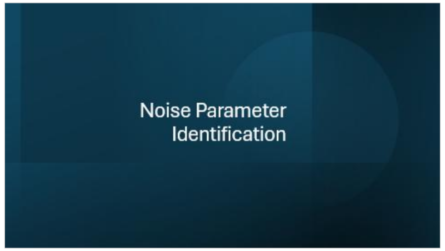

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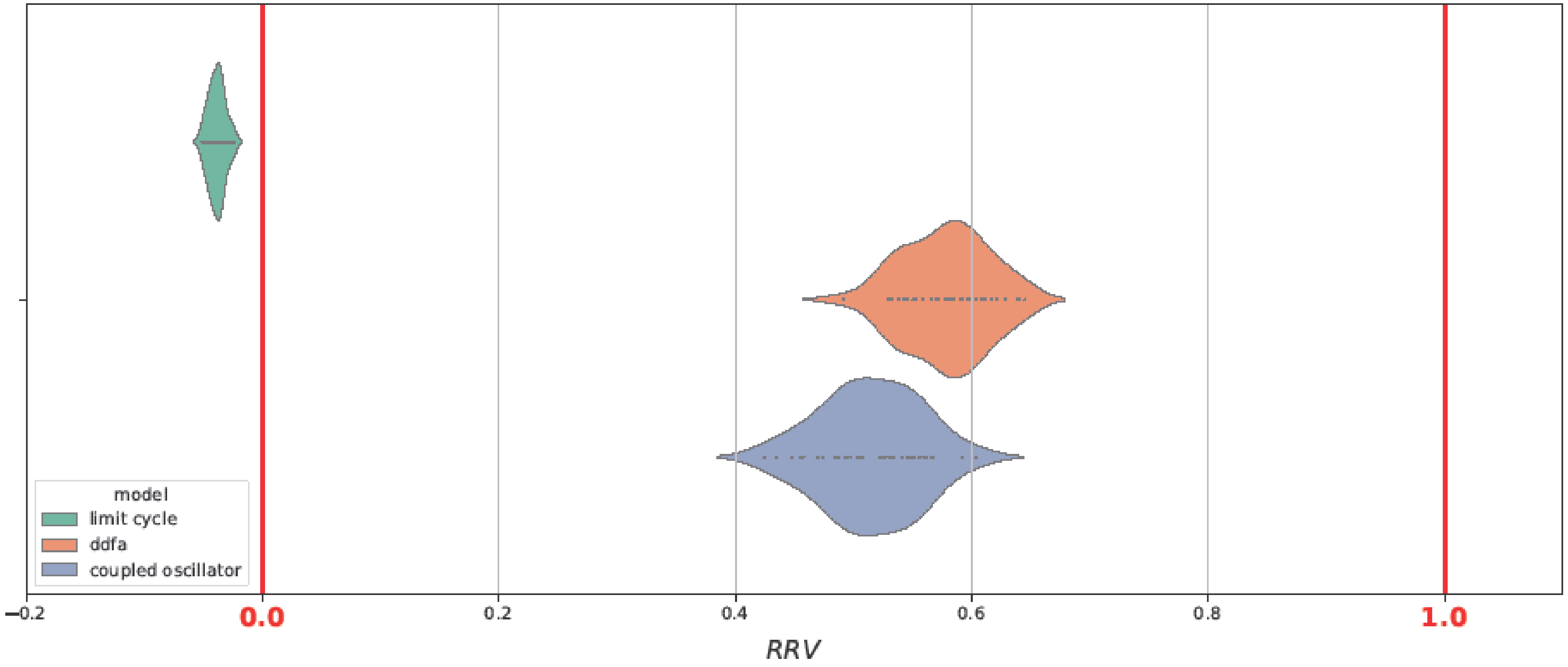
Results

Model Comparison

Relative Remaining Variance – variance of prediction residuals relative to variance of data (from limit cycle) [4].

[4] H.M. Maus et. al., 2015, *Constructing predictive models of human running*

Cartesian *RRV*



Model Comparison Average RRV

Model	Phase RRV	Radius RRV	Cartesian RRV
Limit Cycle	0.273	-0.013	-0.038
DDFA	0.764	0.334	0.578
Coupled Oscillator	0.766	0.008	0.515

Relative Remaining Variance – variance of prediction residuals relative to variance of data (from limit cycle)[4].

[4] H.M. Maus et. al., 2015, *Constructing predictive models of human running*

Summary

- Multi-legged robots can be modeled as both a phase oscillator or as subsystems of coupled phase oscillators.
- In theory model of coupled phase-oscillators can be as good as full Floquet analysis.
- There is a range of parameters that the robot works with for our model to be accurate.

Thank You

Questions

Mathematical Background

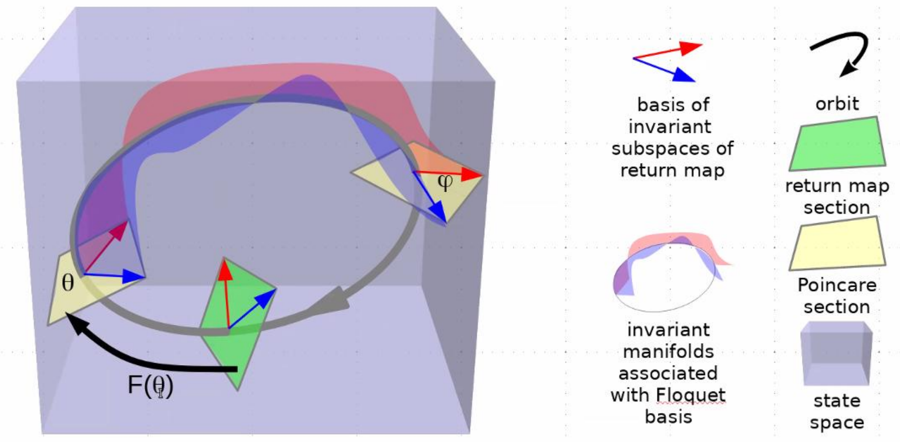
Floquet Theory

- Given a LTP system with period T: $\dot{\mathbf{x}}(t) = \mathbf{A}(t)\mathbf{x}(t)$, $\mathbf{A}(t+T) = \mathbf{A}(t)$

- For a fundamental solution matrix $\mathbf{X}(t)$ of the system:

$$\mathbf{X}(t+T) = \mathbf{X}(t)\mathbf{X}(0)^{-1}\mathbf{X}(T)$$

- In the theory of oscillators $\mathbf{X}(T)$ is called the **Monodromy** matrix with its eigenvalues the **characteristic multipliers** determining stability of the system.



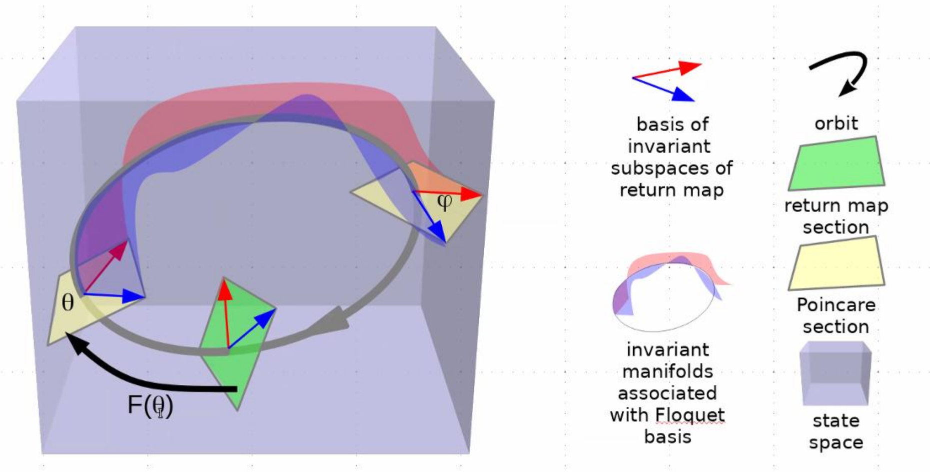
Floquet Theory

- There exists a periodic matrix $\mathbf{P}(t)$ with period T, and constant matrix $\mathbf{R}$ both non-singular such that:

$$\mathbf{X}(t) = \mathbf{P}(t)e^{\mathbf{R}t}$$

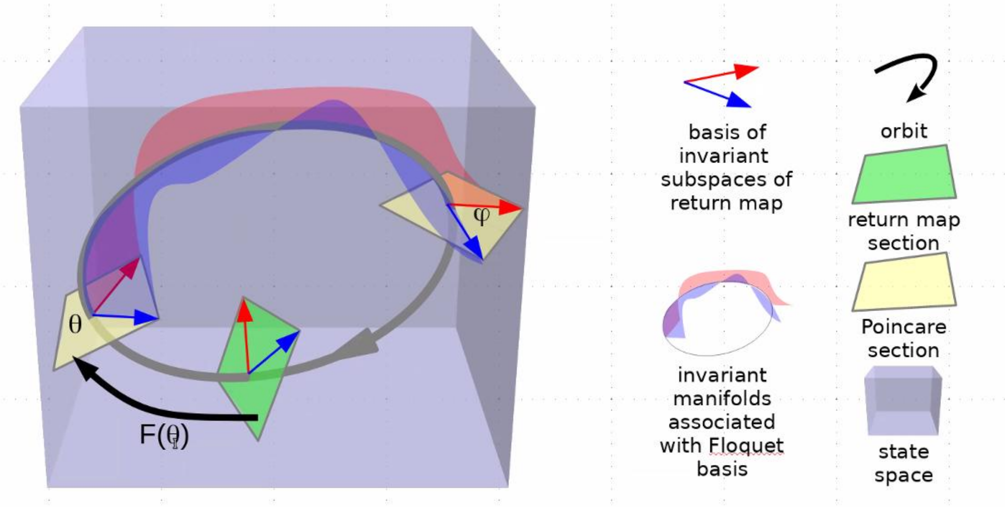
- There also exists a coordinate transformation with periodic matrix $\mathbf{Z}(t)$:

$$\mathbf{y}(t) = \mathbf{Z}(t)\mathbf{x}(t)$$



Floquet Theory

- This allows for the creation of new coordinates based on the Floquet multipliers that is periodic and coincides with the system limit cycle.
- Each Floquet multiplier affects the magnitude of perturbation in the direction of the matching eigenvector. Thus, operating as modes.



Floquet Theory

- Our system has a **global phase** $\varphi(t)$ such that $\varphi(t) = e^{i\omega t}\varphi(0)$, defining the limit cycle $\boldsymbol{\gamma}\big(\varphi(t)\big)$.
- In our case the system $\mathbf{x}(t)$ isn't always at the limit cycle so we assume some perturbations $\boldsymbol{\delta}(t)$ from the limit cycle $\boldsymbol{\gamma}\big(\varphi(t)\big)$:

$$\mathbf{x}(t) = \boldsymbol{\gamma}\big(\varphi(t)\big) + \boldsymbol{\delta}(t)$$

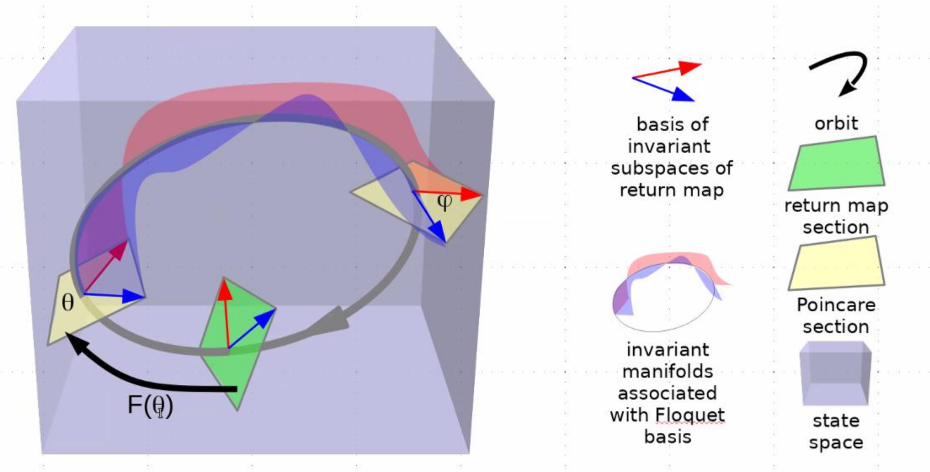
- With the condition that $\varphi(0)$ is the phase of $\mathbf{x}(0)$.

Floquet Theory

- Thus, our dynamical system can be defined as:

$$d\varphi(t) = i\omega\varphi(t)dt$$
$$d\boldsymbol{\delta}(t) = \mathbf{H}(\varphi)\boldsymbol{\delta}(t)dt$$

- For some non-singular matrix $\mathbf{H}(\varphi)$.



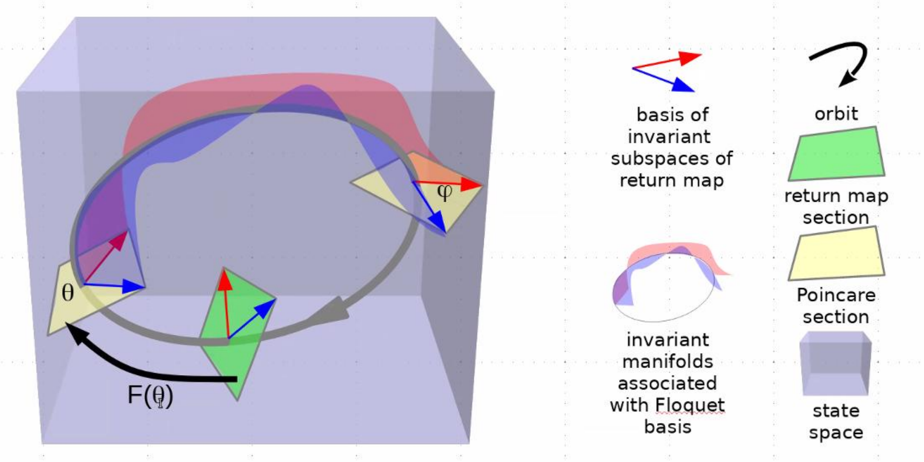
Floquet Theory

- Let $\mathbf{F}$ be the fundamental solution matrix to the LTP $\dot{\mathbf{F}} = \mathbf{H}\left(e^{i\omega t}\right)\mathbf{F}, \mathbf{F}(0) = \mathbf{I}$ then:

$$\varphi(t) = e^{i\omega t}\varphi(0)$$
$$\boldsymbol{\delta}(t) = \mathbf{F}_{\varphi(0)}(t)\boldsymbol{\delta}(0)$$

- Where we define: $\mathbf{F}_\theta(t) \coloneqq \mathbf{F}\left(t + \frac{\arg\theta}{\omega}\right)\mathbf{F}^{-1}\left(\frac{\arg\theta}{\omega}\right)$

- In practice we need to estimate these equations using a DDFA.



General Theory

- We identify the 1-dimensional torus $\mathbb{T}^1$ with the circle $\mathcal{S}^1$, and consider those to be the complex unit circle $\{z \in \mathbb{C}\ s.t. |z| = 1\}$.

Modeling

- A sufficiently smooth dynamical system $\Phi: \mathbb{R} \times \mathbf{X} \to \mathbf{X}$.
- For $t, s \in \mathbb{R}, \mathbf{x} \in \mathbf{X}$: $\Phi^0\mathbf{x} = \mathbf{x}, (\Phi^t \circ \Phi^s)\mathbf{x} = \Phi^{\mathrm{t+s}}\mathbf{x}$
- Assume it is an oscillator therefore de define 4 different parameters with the system.

Modeling

1. A "global phase" $\varphi: \mathbf{X} \to \mathbb{T}^1$ and $\omega > 0$ such that for all $t, \mathbf{x}$: $\varphi(\Phi^t \mathbf{x}) = e^{j\omega t}\varphi(\mathbf{x})$

This phase is not unique.

2. A "limit cycle" $\boldsymbol{\gamma}: \mathbb{T}^1 \to \boldsymbol{\Gamma} \subseteq \mathbf{X}$ such that: $\boldsymbol{\gamma}\left(e^{j\omega t}\right) = \Phi^t\left(\boldsymbol{\gamma}(1)\right)$ and $\boldsymbol{\gamma}$ is onto.

3. A "phase projection" $\mathbf{P}: \mathbf{X} \to \boldsymbol{\Gamma}$ such that $\mathbf{P}(\mathbf{x}) \coloneqq \boldsymbol{\gamma}\big(\varphi(\mathbf{x})\big)$.

It follows that for all t: $\mathbf{P} \circ \Phi^t = \Phi^t \circ \mathbf{P}$.

4. The limit cycle is exponentially stable. There exits $\alpha > 0$ such that for all $t, \mathbf{x}$:

$$\|(\Phi^t \mathbf{x}) - \mathbf{P}(\Phi^t \mathbf{x})\| < e^{-\alpha t}\|\mathbf{x} - \mathbf{P}(\mathbf{x})\|$$

Data Driven Framework

- To obtain a model of $\Phi^{\Delta t}$ we use pairs of sampled states, where Δt is the sampling interval.
- We denote at time $n\Delta t$ the state on trajectory k as $\hat{\mathbf{x}}^k[n]$.
- An ideal (noise-free) dataset is assumed to be pairs $\left(\hat{\mathbf{x}}^k[n], \hat{\mathbf{x}}^k[n+1]\right)$.
- We assume the actual data has a memoryless Gaussian noise process ν that is added to the time evolution: $\Phi^* \coloneqq \Phi^{\Delta t} + \nu$.

Phase Estimation

- We rely on a phase estimation method, which given the dataset $\{\hat{\mathbf{x}}^k[n]\}$ provides an estimated phase $\{\varphi^k[n]\}$ for each data point.
- The phase estimation must be consistent across multiple trajectories in the dataset.
- We used the **Phaser** algorithm[1] which estimates the instantaneous phase for each sample point in a given noisy system dataset.

[1] Revzen S, Guckenheimer JM. Estimating the phase of synchronized oscillators. Physical Review E—Statistical, Nonlinear, and Soft Matter Physics. 2008 Nov;78(5):051907.

Assumptions

The Data-Driven Models we construct rely on some assumptions about the noise and the limit cycle. Let T be the oscillation period of Γ, L be the Euclidean length of Γ, and $\sigma^2(\mathbf{X})$ be the variance of the noise process $\boldsymbol{\nu}$ at point $\mathbf{x} \in \mathbf{X}$. We assume

1. σ^2 does not change much, i.e. for any $\mathbf{x}, \mathbf{y} \in \mathbf{X}$ $\sigma^2(\mathbf{x})/(\sigma^2(\mathbf{x}) + \sigma^2(\mathbf{y}))$ is sufficiently close to $1/2$.

2. $\sqrt{\sigma^2} \ll L$.

3. $1000 > T/\Delta t > 10$.

4. $T > 3/\alpha$ for α the exponential convergence rate bound.

5. Each experiment should be at least $2T$ or more.

DDFA

Because $\mathbf{\Phi}$ is smooth, we can develop $\mathbf{\Phi}^t(\mathbf{x})$ into a first order expansion:

$$\mathbf{\Phi}^t(\mathbf{x}) := \mathbf{\Phi}^t(\mathbf{P}(\mathbf{x}) + \boldsymbol{\delta}) = \mathbf{\Phi}^t(\mathbf{P}(\mathbf{x})) + D\mathbf{\Phi}^t(\mathbf{P}(\mathbf{x}))\boldsymbol{\delta} + r(\mathbf{x}, \boldsymbol{\delta}), \tag{2.6}$$

where the residual r satisfies $\lim_{\boldsymbol{\delta}\to 0} r(\mathbf{x}, \boldsymbol{\delta})/\|\boldsymbol{\delta}\| = 0$. We can now define $\mathbf{M} : \mathbb{T}^1 \times \mathbb{R} \times \mathsf{T}\mathbf{X} \to \mathsf{T}\mathbf{X}$ as

$$\mathbf{M}[\theta, t] := D\mathbf{\Phi}^t(\gamma(\theta)), \tag{2.7}$$

DDFA

The DDFA model consists of estimating $\mathbf{M}$ and using this estimate $\hat{\mathbf{M}}$ to predict $\hat{\boldsymbol{\delta}}^k[n+s]$.

We selected N_ϕ evenly spaced phases $\phi_m := \mathbf{e}^{j2\pi\frac{m}{N_\phi}}$ and define for any $\phi \in \mathbb{T}^1$ that $\lfloor\phi\rceil$ is the closest ϕ_m to ϕ. Using this we computed $\hat{\mathbf{M}}[\lfloor\theta\rceil, s]$ as the least squares solution of:

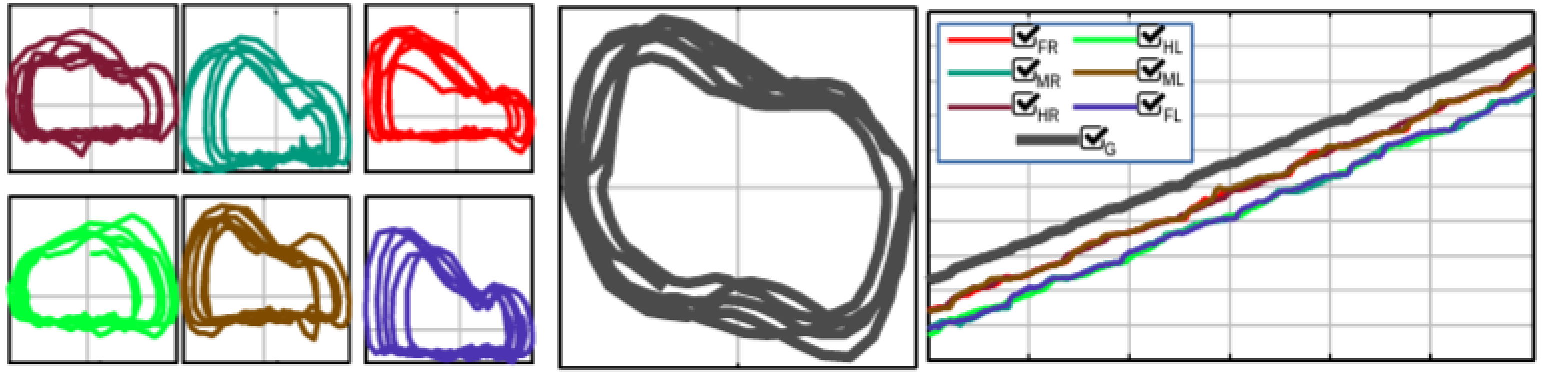
$$\hat{\mathbf{M}}[\lfloor\theta\rceil, s]\hat{\boldsymbol{\delta}}^k[n] = \hat{\boldsymbol{\delta}}^k[n+s] \text{ s.t. } \left\lfloor\hat{\varphi}^k[n]\right\rceil = \lfloor\theta\rceil, \tag{2.8}$$

Coupled Phase Oscillator Model

- Instead of modeling the entire path we only model the phase of each leg using a coupling term and stochastic noise:

$$\dot{\phi}_i(t) = \omega + \sum_{j=1}^{N_s} c_{ji}(\varphi)\left(\phi_j(t) - \phi_i(t)\right) + d\nu$$

- For simplicity we use phases as real numbers from now on.
- N_s is the number of legs.



Revzen S. and Kvalheim M. Locomotion as an Oscillator. Bioinspired Legged Locomotion. 2017:97-110

Coupled Phase Oscillator Model

- We define the residual phases $\beta_i(t) \coloneqq \phi_i(t) - \angle\varphi(t)$ where the **global phase** is $\varphi(t) = e^{j\omega t}\varphi(0)$ such that:

$$\dot{\beta}_i(t) = \sum_{j=1}^{N_s} c_{ji}(\varphi)\left(\beta_j(t) - \beta_i(t)\right) + d\nu$$

- Rewriting:

$$\dot{\beta}_i(t) = \sum_{j=1}^{N_s} c_{ji}(\varphi)\beta_j(t) - \beta_i(t)\sum_{j=1}^{N_s} c_{ji}(\varphi) + d\nu$$

Coupled Phase Oscillator Model

- Let us define the two coupling matrices $\mathbf{C}(\varphi), \mathbf{D}(\varphi), \mathbf{A}(\varphi)$:

$$\mathbf{C}(\varphi) = \begin{bmatrix} c_{11}(\varphi) & \cdots & c_{1N_s}(\varphi) \\ \vdots & \ddots & \vdots \\ c_{N_s1}(\varphi) & \cdots & c_{N_sN_s}(\varphi) \end{bmatrix} \qquad \mathbf{D} = \begin{bmatrix} \sum_j^{N_s} c_{1j}(\varphi) & 0 & 0 \\ 0 & \ddots & 0 \\ 0 & 0 & \sum_j^{N_s} c_{N_sj}(\varphi) \end{bmatrix}$$

$$\mathbf{A}(\varphi) = \mathbf{C}^T(\varphi) - \mathbf{D}(\varphi)$$

- Which gives the equation: $\dot{\boldsymbol{\beta}}(t) = \mathbf{A}(\varphi)\boldsymbol{\beta}(t)$

Coupled Phase Oscillator Model

- We now have an LTP system: $\dot{\boldsymbol{\beta}}(t) \approx \mathbf{A}(\varphi)\boldsymbol{\beta}(t)$
- Solving using Floquet theory: $\boldsymbol{\beta}(t) = \mathbf{F}[\theta, t]\boldsymbol{\beta}(0)$
- Where we have a matrix $\mathbf{F}[\theta, t]$ which is the flow matrix that is solved according to Floquet theory.

Extra Slides

1. The Limit Cycle

For sample paths of a stochastic differential equation:

- $\{\hat{\mathbf{x}}[n]\}^k$ - the $k = 1, \ldots, N$ sampled paths, with $n = 1, \ldots, T_k$ sample points for each path, a sampling interval Δt, and a sufficiently tame noise process.
- $\{\hat{\varphi}[n]\}^k$ - corresponding instantaneous phases.
- $\hat{\boldsymbol{\gamma}}(\hat{\varphi})$ - estimated limit cycle as a fitted Fourier series of order N_{ord}.

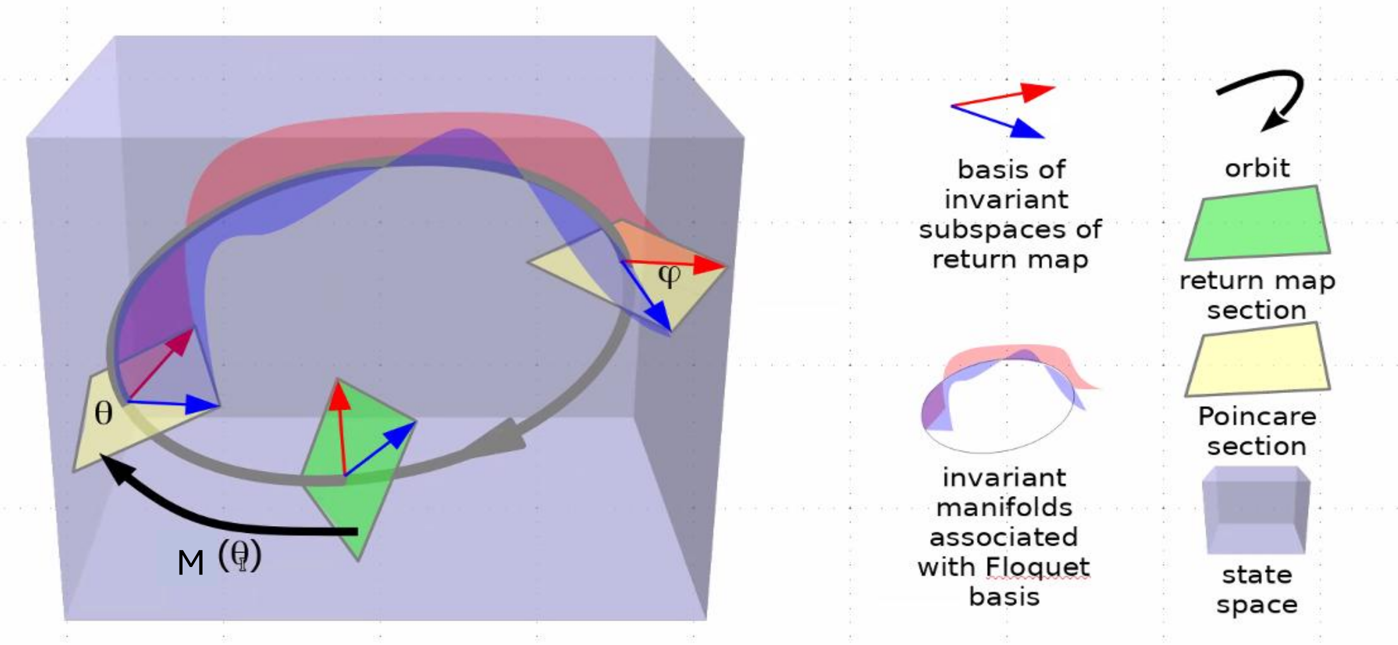
$$\hat{\boldsymbol{\gamma}}(\theta) := \sum_{m=-N_{ord}}^{N_{ord}} c_m e^{jm\theta}$$

2. Data-Driven Floquet Analysis

- Our dynamical system can be defined using $\varphi(t)$ and $\boldsymbol{\delta}(t)$ with $\mathbf{M}_\theta(t)$ the matrix representing the flow of perturbations from some initial phase θ.

$$\begin{cases} \boldsymbol{\delta}(t) = \mathbf{M}_\theta(t)\boldsymbol{\delta}(0) \\ \varphi(t) = e^{j\omega t}\underbrace{\theta}_{\varphi(0)} \end{cases}$$

$$\mathbf{M}_\theta(t) \coloneqq \mathbf{M}\left(t + \frac{\arg\theta}{\omega}\right)\mathbf{M}^{-1}\left(\frac{\arg\theta}{\omega}\right)$$



S. Revzen

Synthetic Data

- We use a “Hopf” oscillator to simulate the motion of each robot leg.

- Dynamics are governed by the following equations in polar coordinates:

$$\dot{r}_i(t) = \alpha\big(1 - r_i(t)\big) + \dot{\eta}_i\big(\theta_i(t)\big) + \delta\nu_r$$

$$\dot{\theta}_i(t) = \omega + \sum_{j=1}^{N_s} C_{ij}\left(\theta_j(t) - \theta_i(t)\right) + \delta\nu_\theta$$

- Here the phase of each subsystem is coupled with all other subsystems.

Synthetic Data