A SIMPLE STABILIZATION METHOD FOR AN ACCELERATING ROCKET

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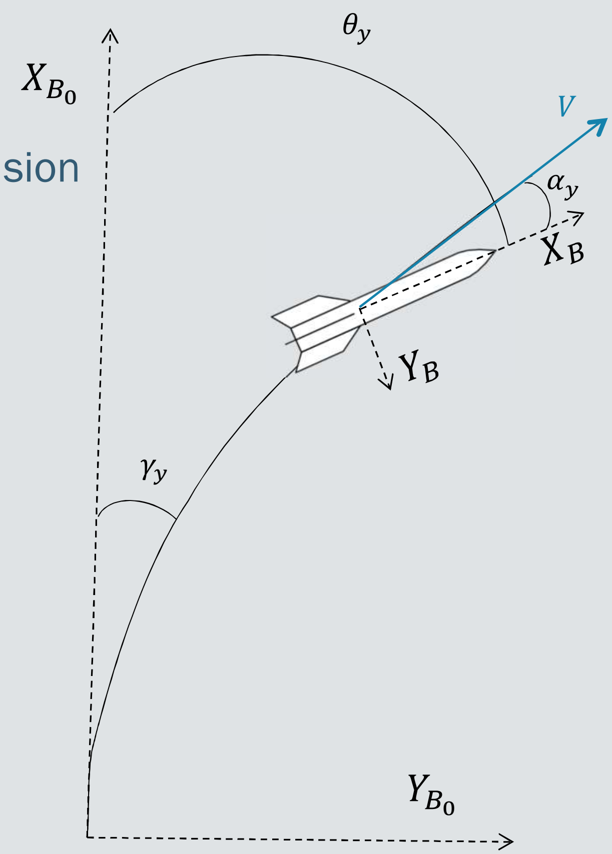
Full Dynamic Model

- This work is focused on the *horizontal* plane but the extension to the longitudinal plane is quite straightforward.
- System of equations:
  - *Forces perpendicular to velocity vector, lateral plane*
  - *Moments around imaginary axis, Y, lateral plane*
  - *Forces parallel the total velocity vector*



Full Dynamic Model

$$\begin{cases} \dot{V} = \frac{T}{m} - g\sin(\theta_{z_0}) - \frac{\rho S C_D}{2m} V^2 \\ \dot{\gamma}_y = \frac{T}{mV}\sin\left(\alpha_y + \beta_{m_y}\right) + \frac{\rho S C_{L\alpha}}{2m}\left(\alpha_y + \frac{w}{V}\right) V \\ \ddot{\theta}_y = C^*_{M\alpha}\frac{1}{d^2}\left(\alpha_y + \frac{w}{V}\right) V^2 + C^*_{M_0}\frac{1}{d^2}V^2 + C^*_{m_q}\frac{1}{2d}\dot{\theta}_y V - \frac{TR_{m_y}}{I_Z} \\ \theta_y = \gamma_y + \alpha_y \end{cases}$$

- Total velocity

- Forces perpendicular to velocity, on lateral plane

- Moments around Y axis, on lateral plane

$$C^*_{M_\alpha} = \frac{\rho S d^3}{2I_Z} C_{M_\alpha}$$

$$C^*_{M_0} = \frac{\rho S d^3}{2I_Z} C_{M_0}$$

$$C^*_{m_q} = \frac{\rho S d^3}{2I_Z} C_{M_q}$$

- From this point on - all angles and misalignment elements in discussion are in Y axis
  - *Except for θ_{Z_0}*

Assumptions

- Small angles
- Acceleration is constant: $V = G_{eff}t$
  - $\dot{V} = G_{eff}$ - *average acceleration*
  - *Attribute all the acceleration to the thrust*
- Lift and dynamic restoring moments are negligible
- No thrust or structure misalignments

Davis L. Jr., Follin J. W., Blitzer L., “EXTERIOR BALLISTICS OF ROCKETS”, 1958

New Independent Variable [1 6]

- Wavelength and time period: $\sigma \triangleq \frac{2\pi d}{\sqrt{-C^*_{M_\alpha}}}$; $t_\sigma \triangleq \sqrt{\frac{2\sigma}{G_{eff}}}$
- Velocity approximation: $V_\sigma = G_{eff} t_\sigma$

↓

- New unit-free time variable: $\boldsymbol{\xi} = \frac{\boldsymbol{t}}{\boldsymbol{t_\sigma}}$
- Downrange distance variable: $\boldsymbol{s} = \boldsymbol{\sigma\xi^2}$

1 McCoy R. L., "Modern Exterior Ballistics“, 1998
6 Davis L. Jr., Follin J. W., Blitzer L., “EXTERIOR BALLISTICS OF ROCKETS”, 1958

Control Free Model

Let

$$\xi = \frac{t}{t_\sigma} \quad \downarrow$$

$$\begin{cases} \xi\gamma' - \alpha = 0 \\ \theta'' + 16\pi^2\xi^2\left(\alpha + \frac{w}{V}\right) = 0 \\ \theta = \gamma + \alpha \end{cases}$$

$$\omega = \alpha\xi \quad \downarrow$$
$$s = \sigma\xi^2$$

$$\boxed{\frac{d^2\omega}{ds^2} + \left(\frac{2\pi}{\sigma}\right)^2 \omega + \left(\frac{2\pi}{\sigma}\right)^2 \frac{w}{V_\sigma} = 0}$$

Controlled Model

- Homogenous model with controller

$$\begin{cases} \xi\gamma' - \alpha = 0 \\ \theta'' + 16\pi^2\xi^2\left(\alpha + \dfrac{w}{V}\right) = \underbrace{\boldsymbol{16\pi^2\xi^2\eta\delta}}_{\substack{aerodynamic \\ controller}} \\ \theta = \gamma + \alpha \end{cases}$$

- Where $\eta \triangleq \dfrac{c_{M\delta}}{c_{M\alpha}}$
- Finally

$$\boxed{\frac{d^2\omega}{ds^2} + \left(\frac{2\pi}{\sigma}\right)^2 \omega + \left(\frac{2\pi}{\sigma}\right)^2 \frac{w}{V_\sigma} = \left(\frac{2\pi}{\sigma}\right)^2 \boldsymbol{\eta\omega_\delta}}$$

- With $\boldsymbol{\omega_{\delta}} = \xi\,\boldsymbol{\delta}$

Controller Design

$$\frac{d^2\omega}{ds^2} + \left(\frac{2\pi}{\sigma}\right)^2 \omega + \left(\frac{2\pi}{\sigma}\right)^2 \frac{w}{V_\sigma} = \left(\frac{2\pi}{\sigma}\right)^2 \boldsymbol{\eta\omega_\delta}$$

- In state space

$$\frac{d}{ds}x = \underbrace{\begin{bmatrix} 0 & 1 \\ -\left(\frac{2\pi}{\sigma}\right)^2 & 0 \end{bmatrix}}_{A} x + \underbrace{\begin{bmatrix} 0 \\ -\left(\frac{2\pi}{\sigma}\right)^2 \end{bmatrix}}_{b} \underbrace{\frac{w}{V_\sigma}}_{d} + \underbrace{\begin{bmatrix} 0 \\ -\left(\frac{2\pi}{\sigma}\right)^2 \end{bmatrix}}_{b} \underbrace{\eta\omega_\delta}_{u}$$

$$x = \begin{bmatrix} \omega \\ \frac{d}{ds}\omega \end{bmatrix}$$

Controller Design

- PD controller

$$\omega_\delta = k_1\omega + k_2\frac{d\omega}{ds}$$

$$\omega = \alpha\xi$$

$$\frac{d\omega}{ds} = \frac{d\alpha}{ds}\xi + \alpha\frac{d\xi}{ds}$$

$$\omega_\delta = k_1\alpha\xi(s) + k_2\frac{\alpha}{2\sigma\xi(s)} + k_2\frac{d\alpha}{ds}\xi(s)$$

$$\delta(s) = \frac{\omega_\delta}{\xi(s)} = k_1\alpha + k_2\frac{\alpha}{2s} + k_2\frac{d\alpha}{ds}$$

$$\delta(t) = (k_1 + k_2\frac{t_\sigma^2}{2\sigma t^2})\alpha + k_2\frac{t_\sigma^2}{2\sigma t}\frac{d\alpha}{dt}$$

Controller Design

- PID controller

$$\omega_\delta = k_1\omega + k_2\frac{d\omega}{ds} + k_3\int \omega ds$$

$$\omega = \alpha\xi$$

$$\delta(s) = \frac{\omega_\delta}{\xi(s)} = k_1\alpha + k_2\frac{\alpha}{2s} + k_2\frac{d\alpha}{ds} + k_3\frac{1}{\xi(s)}\int \alpha\xi(s)ds$$

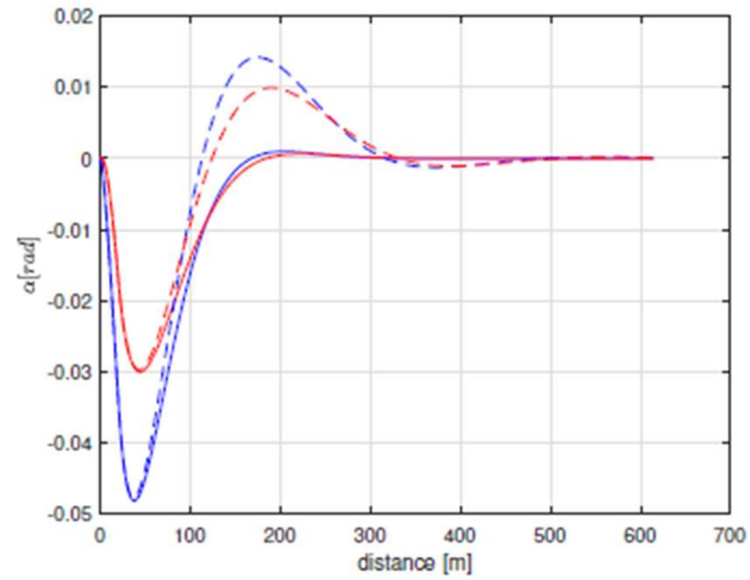
$$\delta(t) = (k_1 + k_2\frac{t_\sigma^2}{2\sigma t^2})\alpha + k_2\frac{t_\sigma^2}{2\sigma t}\frac{d\alpha}{dt} + k_3\frac{2\sigma}{tt_\sigma^2}\int \alpha t^2 dt$$

Results

	Value	Units		Value	Units
l_p	4	$[m]$	t_b	1.26	$[s]$
m_0	97.4	$[kg]$	S_b	614.6	$[m]$
m_p	33.9	$[kg]$	γ_0, θ_0	0	$[^o]$
$\bar{I}_{sp}$	250	$[s]$	$\dot{\theta}_0$	0	$[\frac{rad}{s}]$
d	0.16	$[m]$	θ_{z0}	53	$[^o]$
I_Z	68.2	$[kg \cdot m^2]$	C_D	0.4	
β_m	0	$[rad]$	C_{M_α}	-27.5	
R_m	0	$[m]$	$C^*_{M_\alpha}$	$-2 \cdot 10^-5$	
G_{eff}	778.3	$[\frac{m}{s^2}]$	$C^*_{M_\delta}$	$2 \cdot 10^-5$	
$\bar{\rho}$	1.225	$[\frac{kg}{m^3}]$	C_{L_α}	7	
S	0.02	$[m^2]$	C_{L_δ}	-1.5	
σ	223.5	$[m]$	C_{M_q}	-800	

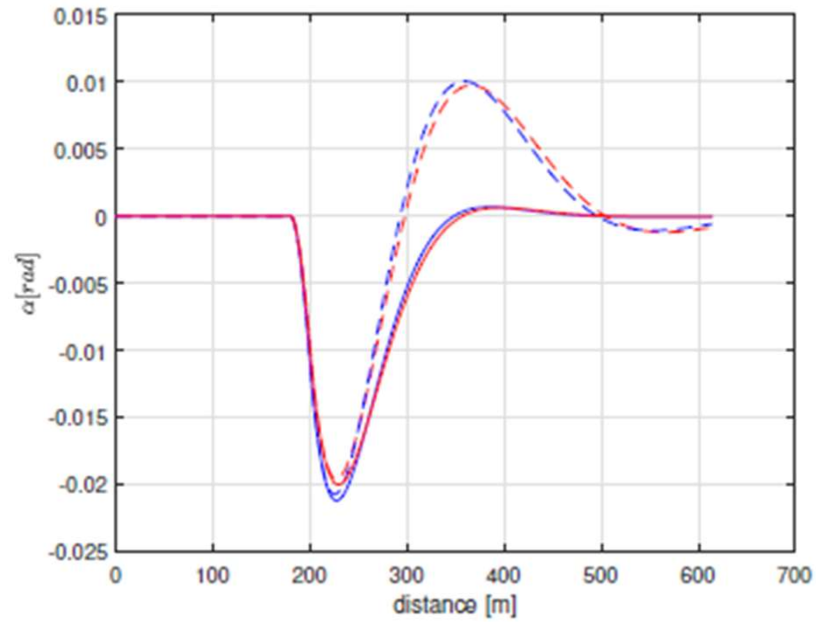
Table 1 Parameters of simulated rocket

Results



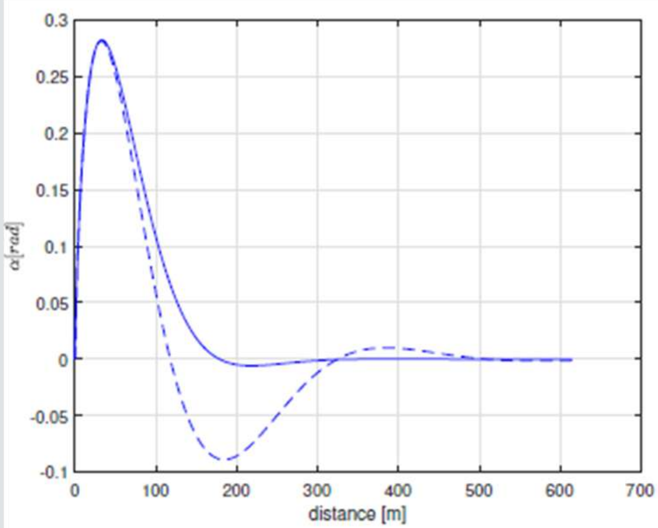
1^{st} scenario, gust response, blue- approximate; red -full; PD (dashed) ; PID (solid) ;

Results



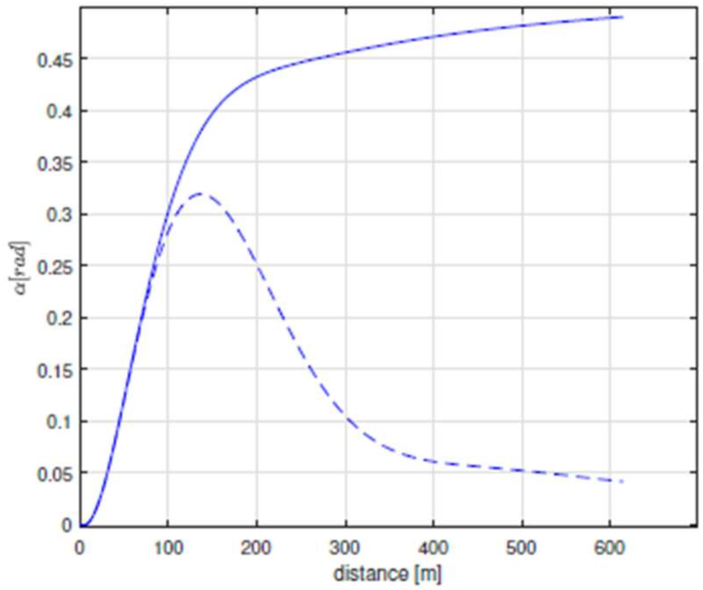
2^{nd} **scenario, gust response, blue- approximate; red -full; PD (solid) ; PID (dashed) ;**

Results



4^{th} **scenario, initial tip-off PD (solid) ; PID (dashed)**

Results



5^{th} **scenario, thrust misalignment PD (solid) ; PID (dashed)**

Conclusion

- A straightforward method for designing the control of accelerating rockets has been suggested, relying on an approximate time-invariant linear system with distance taken as the independent variable
- Simple PD and PID controls have been used for demonstration
- Transforming the control laws to time domain results in linear time-varying feedback control laws
- The control laws were verified against the complete nonlinear dynamics of the rocket